MATICAL SERIES

COMPLETE ARITHMETIC

BY
FRANK·H·HALL

WERNER SCHOOL BOOK COMPANY
EDUCATIONAL PUBLISHERS
NEW YORK · CHICAGO · BOSTON

HALL'S MATHEMATICAL SERIES

A

COMPLETE ARITHMETIC

ORAL AND WRITTEN

BY

FRANK H. HALL

AUTHOR OF "THE WERNER ARITHMETICS," "THE ARITHMETIC READERS," ETC.

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PREFATORY NOTE.

In Part I. of this book (pp. 11–149), classification is made subordinate to gradation. Every problem is selected, not with reference to the place it occupies in a scientific classification of mathematical topics, but rather with reference to the supposed thought-power of the pupil. But systematic arrangement of the matter presented is not ignored. Seven topics are treated. These appear on the first ten pages of the book, and each topic is re-presented in each ten-page group. Compare pages 15, 25, 35, 45, etc.; pages 17, 27, 37, 47, etc.

In Part II. (pp. 151–369), as in Part I., each page is a unit of the greater ten-page unit. The first *six pages* of every ten-page group are devoted to some general topic. Upon the *seventh and eighth* pages the algebraic phase of this topic appears; upon the *ninth*, elementary work in geometry, and upon the *tenth*, miscellaneous problems. This arrangement makes the book convenient for reference and review, and, it is believed, will greatly aid the pupil in properly correlating his own mathematical knowledge.

In Part III. (pp. 371–442), the matter is arranged under four general heads, viz.: Denominate Numbers, Short Methods in Multiplication and Division, Practical Approximations, and Miscellaneous Problems. With the denominate number tables, many practical problems in measurements are presented. The "short methods" are, for pedagogical reasons, placed near the close of the book. The miscellaneous problems include many typical sets of "examination questions" supplied to the author for this use. Altogether, Part III. provides for a complete and thorough application of the principles presented in Parts I. and II. F. H. H.

JACKSONVILLE, ILLINOIS, March, 1899.

CONTENTS — PART I.

THE FOUNDATION.

To THE PUPIL.—Read each problem and (a) tell its meaning, (b) solve it, and (c) tell the suggested number story. Do this until you can easily give the meaning of all problems similar to these, solve them, and tell the suggested number stories without reference to the notes that follow.

$6\text{¢} + 2\text{¢} = *$

$\frac{2}{3} + \frac{1}{4} = (1)$

$6\frac{3}{4} + 5\frac{1}{8} = (6)$

$$\begin{array}{r} 2 \text{ bu. } 3 \text{ pk.} \\ \underline{2 \text{ bu. } 2 \text{ pk.}} \\ (11) \end{array}$$

$6\text{¢} - 2\text{¢} = *$

$\frac{3}{4} - \frac{1}{3} = (2)$

$6\frac{3}{4} - 5\frac{1}{8} = (7)$

$$\begin{array}{r} 5 \text{ bu. } 1 \text{ pk.} \\ \underline{1 \text{ bu. } 3 \text{ pk.}} \\ (12) \end{array}$$

$6\text{¢} \times 2 = *$

$\frac{3}{4}$ bu. $\times 7 = (3)$

$12\text{¢} \times \frac{3}{4} = (8)$

$\$\frac{3}{5} \times \frac{1}{2} = (13)$

$12\text{¢} \times 2\frac{3}{4} = (16)$

$12\frac{1}{2} \times 3 = (19)$

$12\frac{1}{2} \times 2\frac{1}{2} = (22)$

$6\text{¢} \div 2\text{¢} = *$

$4 \div \frac{1}{3} = (4)$

$\frac{2}{3} \div \frac{1}{6} = (9)$

$7\frac{1}{2} \div 2\frac{1}{2} = (14)$

2 ft.)$\underline{46 \text{ ft.}}$ (17)

$2\frac{1}{2}$ ft.)$\underline{15 \text{ ft.}}$ (20)

$2\frac{1}{4}$ bu.)$\underline{13\frac{1}{2} \text{ bu.}}$
(23)

$6\text{¢} \div 2 = *$

$\frac{1}{3}$ bu. $\div 4 = (5)$

$6\frac{3}{4}$ yd. $\div 2 = (10)$

$5\frac{3}{4}$ tons $\div 2 = (15)$

2)$\underline{46 \text{ ft.}}$
(18)

3)$\underline{27\frac{1}{2} \text{ ft.}}$
(21)

1. $\frac{2}{3}$ plus $\frac{1}{4}$, means, $\frac{2}{3}$ *and* $\frac{1}{4}$. I can change thirds and fourths to twelfths; $\frac{2}{3}$ = —— twelfths; $\frac{1}{4}$ = —— twelfths; —— twelfths and —— twelfths = —— twelfths. *Story—Harry played ball $\frac{2}{3}$ of an hour, and "hide and seek" $\frac{1}{4}$ of an hour; in all he played —— twelfths of an hour.* (See First Book, pages 125, 135, 145, 147, 149.)

2. $\frac{3}{4}$ minus $\frac{1}{3}$, means, $\frac{3}{4}$ *less* $\frac{1}{3}$. I can change fourths and thirds to twelfths. $\frac{3}{4}$ = —— twelfths; $\frac{1}{3}$ = —— twelfths; —— twelfths less

* See Elementary Arithmetic, pp. 89 and 153.

—— twelfths = —— twelfths. *Story—William had a piece of wire $\frac{3}{4}$ of a foot long; he cut from it a piece $\frac{1}{6}$ of a foot long; what remained was —— twelfths of a foot long.*

3. $\frac{3}{4}$ of a bushel multiplied by 7, means, take 7 times three fourth-bushels; 7 times 3 fourth-bushels = —— fourth-bushels, or —— and —— —— bushels. *Story—A farmer fed to his horses $\frac{3}{4}$ bu. of oats each day for a week; in all he fed —— and —— —— bushels.*

4. 4 divided by $\frac{1}{3}$, means, find how many times 1 third is contained in 4. I can change 4 to thirds. 4 is 12 thirds. 1 third is contained in 12 thirds —— times. *Story—The teacher divided 4 oranges among some boys, giving to each $\frac{1}{3}$ of an orange; there were —— boys.*

5. $\frac{1}{3}$ of a bushel divided by 4, means, find 1 fourth of $\frac{1}{3}$ of a bushel. 1 fourth of $\frac{1}{3}$ of a bushel = —— —— of a bushel. *Story—A boy divided $\frac{1}{3}$ of a bushel of oats equally among 4 ponies; each pony received —— —— of a bushel.*

6. $6\frac{3}{4}$ plus $5\frac{1}{8}$, means, $6\frac{3}{4}$ *and* $5\frac{1}{8}$. $6\frac{3}{4}$ and $5\frac{1}{8}$ = —— and —— ——. *Story—In one jar there were $6\frac{3}{4}$ lb. of butter; in another there were $5\frac{1}{8}$ lb.; in both there were —— and —— —— pounds.*

7. $6\frac{3}{4}$ minus $5\frac{1}{8}$, means, $6\frac{3}{4}$ *less* $5\frac{1}{8}$. $6\frac{3}{4}$ less $5\frac{1}{8}$ = —— and —— ——. *Story—A grocer had $6\frac{3}{4}$ lb. cheese, from which he sold $5\frac{1}{8}$ lb.; there remained —— and —— —— pounds.*

8. 12¢ multiplied by $\frac{3}{4}$, means, take $\frac{3}{4}$ of 12¢. 3 fourths of 12¢ = —— cents. *Story—At 12¢ a pound, $\frac{3}{4}$ of a pound of meat costs —— cents.*

9. $\frac{2}{3}$ divided by $\frac{1}{6}$, means, find how many times 1 sixth is contained in 2 thirds. I can change thirds to sixths; 2 thirds = —— sixths; 1 sixth is contained in 4 sixths —— times. *Story—Mrs. Smith divided $\frac{2}{3}$ of a pie among some boys, giving to each $\frac{1}{6}$ of a pie; there were —— boys.*

10. $6\frac{3}{4}$ yd. divided by 2, means, find 1 half of $6\frac{3}{4}$ yards. 1 half of $6\frac{3}{4}$ yards = —— and —— —— yards. *Story—A salesman cut $6\frac{3}{4}$ yd. of ribbon into 2 equal pieces; there were —— and —— —— yards in each piece.*

11. 2 bu, 3 pk. plus 2 bu. 2 pk., means, 2 bu. 3 pk. *and* 2 bu. 2 pk. 2 bu. 3 pk. and 2 bu. 2 pk. = —— bu. —— pk. *Story—A dealer put 2 bu. 3 pk. of oats into one bag and 2 bu. 2 pk. into another bag; in both there were —— bushels —— peck.*

12. 5 bu. 1 pk. minus 1 bu. 3 pk., means, 5 bu. 1 pk. *less* 1 bu. 3 pk. 5 bu. 1 pk. less 1 bu. 3 pk. = —— bu. —— pk. *Story—Mr. Bean had 5 bu. 1 pk. of oats, from which he fed 1 bu. 3 pk.; there remained —— bushels —— pecks.*

13. $\$\frac{3}{5}$ multiplied by $\frac{1}{2}$, means, take 1 half of 3 fifths of a dollar. 1 half of 3 fifths of a dollar = —— —— of a dollar. *Story—At $\frac{3}{5}$ of a dollar a yard, 1 half of a yard of ribbon is worth —— —— of a dollar.*

14. 7½ divided by 2½, means, find how many times 2½ is contained in 7½. I can change 2½ and 7½ to halves; 2½ = —— halves; 7½ = —— halves. —— halves are contained in —— halves —— times. *Story—A farmer put 7½ bushels of oats into bags, putting 2½ bushels in each bag; there were —— bags.*

15. 5¾ tons divided by 2, means, find 1 half of 5¾ tons. 1 half of 5¾ tons = —— and —— —— tons. *Story—5¾ tons of coal were divided equally between two families; each family received —— and —— —— tons.*

16. 12¢ multiplied by 2¾, means, take 2 times 12¢ plus 3 fourths of 12¢ (2¾ times 12¢). 2 times 12¢ plus 3 fourths of 12¢ = —— cents. *Story—At 12¢ a pound 2¾ lb. of cheese cost —— cents.*

17. 46 ft. divided by 2 ft., means, find how many times 2 ft. are contained in 46 ft. 2 ft. are contained in 46 ft. —— times. *Story—A mechanic cut 46 ft. of moulding into pieces, each piece being 2 ft. long; there were —— pieces.*

18. 46 ft. divided by 2, means, find 1 half of 46 ft. 1 half of 46 ft. = —— ft. *Story—Henry divided 46 ft. of wire into 2 equal parts; each part was —— feet long.*

19. 12½ multiplied by 3, means, take 3 times 12½ (3 times ½, plus 3 times 12). 3 times 12½ = ——. *Story—One side of a triangle having equal sides is 12½ ft.; the perimeter of the triangle is —— and —— —— feet.*

20. 15 feet divided by 2½ feet, means, find how many times 2½ feet are contained in 15 feet. I can change 15 ft. and 2½ ft. to half-feet; 15 ft. = —— half-feet; 2½ feet = —— half-feet; —— half-feet are contained in —— half-feet —— times. *Story—A mechanic had 15 feet of moulding which he cut into pieces, each piece being 2½ ft. long; there were —— pieces.*

21. 27½ feet divided by 3, means, find 1 third of 27½ feet. 1 third of 27½ feet = —— and —— —— feet. *Story—The perimeter of a triangle having equal sides is 27½ feet; each side is —— and —— —— feet.*

22. 12½ multiplied by 2½, means, take 2 times 12½, plus ½ of 12½. 2 times 12½, plus ½ of 12½ = —— and —— ——. *Story—At $12½ per ton, 2½ tons of hay cost —— and —— —— dollars.*

23. 13½ bu. divided by 2¼ bu., means, find how many times 2¼ bu. are contained in 13½ bu. I can change 2¼ and 13½ to fourths. 2¼ = —— fourths. 13½ = —— fourths. —— fourths are contained in —— fourths —— times. *Story—A farmer had 13½ bu. of oats which he put into bags, putting 2¼ bu. in each bag. There were —— bags.*

$.6 + .2 = $ *	$.6 - .2 = $ *	$.6 \times 2 = $ *
$.24 + .05 = $ (1)	$.64 - .05 = $ (2)	$.12 \times 9 = $ (3)
$.24 + .5 = $ (6)	$.64 - .5 = $ (7)	$20 \times .2 = $ (8)
$\begin{array}{r} 176.4 \\ 148.75 \\ \hline \end{array}$ (11)	$\begin{array}{r} 276.4 \\ 148.23 \\ \hline \end{array}$ (12)	$20 \times 3.2 = $ (13)
		$.1 \times .1 = $ (16)
		$.3 \times .2 = $ (19)

$.6 \div .2 = $ *	$.6 \div 2 = $ *
$.08 \div .02 = $ (4)	$\$.08 \div 2 = $ (5)
$8 \div .4 = $ (9)	$\$6.36 \div 3 = $ (10)
$4 \div .05 = $ (14)	5.6 tons $\div 4 = $ (15)
$2.5 \div .05 = $ (17)	$5\overline{)\$24.5}$ (18)
$\$.5\overline{)\$24.5}$ (20)	$5\overline{)\$8.2}$ (21)

1. .24 plus .05, means, 24 hundredths *and* 5 hundredths. .24 and .05 = —— hundredths. *Story—William had $.24; he earned $.05; he then had* ———.

2. .64 minus .05, means, 64 hundredths *less* 5 hundredths. .64 less .05 = —— hundredths. *Story—Martha had $.64; she spent $.05; she then had* ———.

3. .12 multiplied by 9, means, take 9 times 12 hundredths. 9 times .12 = —— hundredths, or —— and —— hundredths.

4. .08 divided by .02, means, find how many times 2 hundredths are contained in 8 hundredths. .02 are contained in .08 —— times. *Story—I paid $.08 for oranges at $.02 each; I bought —— oranges.*

5. $.08 divided by 2, means, find 1 half of 8 hundredths of a dollar. One half of $.08 = ——. *Story—I paid $.08 for 2 lemons; one lemon cost* ——.

6. .24 plus .5, means, 24 hundredths *and* 5 tenths. 5 tenths = 50 hundredths. .24 and .50 = —— hundredths. *Story—John had $.24; Alfred had $.5; together they had* ———.

7. .64 minus .5, means, 64 hundredths *less* 5 tenths. 5 tenths = 50 hundredths. .64 less .50 = —— hundredths. *Story—Sarah had $.64; Mary had $.5; Sarah had $—— more than Mary.*

* See Elementary Arithmetic, pp. 153 and 155.

8. 20 multiplied by .2, means, find 2 tenths of 20. One tenth of 20 = ——; 2 tenths of 20 = ——. *Story—At $20 an acre, .2 of an acre of land would cost —— dollars.*

9. 8 divided by .4, means, find how many times 4 tenths are contained in 8. 8 = 80 tenths. 4 tenths are contained in 80 tenths —— times. *Story—I paid $8 for potatoes at $.4 (4 dimes) a bushel; I bought —— bushels.*

10. $6.36 divided by 3, means, find 1 third of $6.36. One third of $6.36 = ——. *Story—I paid $6.36 for 3 barrels of apples; 1 barrel cost ——.*

11. The sum of 176.4 and 148.75 is ——.

12. The difference of 276.4 and 148.23 is ——.

13. 20 multiplied 3.2, means, take 3 times 20, plus 2 tenths of 20. 3 times 20 = ——. 2 tenths of 20 = ——. 3 times 20, plus 2 tenths of 20 = ——. *Story—At $20 an acre, 3.2 acres of land are worth —— dollars.*

14. 4 divided by .05, means, find how many times 5 hundredths are contained in 4. 4 = 400 hundredths. 5 hundredths are contained in 400 hundredths —— times. *Story—I paid $4 for tablets at $.05 (5¢) each; I bought —— tablets.*

15. 5.6 tons divided by 4, means, find 1 fourth of 5.6 tons. One fourth of 5.6 = —— and —— —— tons. *Story—A farmer sold 4 loads of hay, the entire weight of the hay was 5.6 tons; the loads averaged —— and —— —— tons.*

16. .1 multiplied by .1, means, find 1 tenth of 1 tenth. One tenth of 1 tenth = —— ——.

17. 2.5 divided by .05, means, find how many times 5 hundredths are contained in 2.5. 2.5 = 250 hundredths. 5 hundredths are contained in 2.5 (250 hundredths) —— times. *Story—I paid $2.5 for pencils at $.05 (5¢) each; I bought —— pencils.*

18. $24.5 divided by 5, means, find 1 fifth of $24.5. One fifth of $24.5 = ——. *Story—I paid $24.5 for 5 tons of coal; 1 ton cost ——.*

19. .3 multiplied by .2, means, find 2 tenths of 3 tenths. One tenth of 1 tenth = ——. One tenth of 3 tenths = ——. Two tenths of 3 tenths = ——.

20. $24.5 divided by $.5, means, find how many times 5 tenths of a dollar are contained in $24.5 (245 tenths dollars). 5 tenths are contained in 24.5 (245 tenths) —— times. *Story—I paid $24.5 for apples at $.5 (5 dimes) a bushel; I bought —— bushels.*

21. $8.2 divided by 5, means, find 1 fifth of $8.2. One fifth of $8.2 ($8.20) = ——. *Story—I paid $8.2 for 5 yards of cloth; 1 yard cost ——.*

SUGGESTIONS TO TEACHERS.

1.

ORDER OF PROCEDURE IN PART I.

STEP 1.—Prepare the pupil by means of oral instruction for the work of a given page. This preparation may be, in part, the slow reading to the pupil of the *figure problems** upon the page, the teacher hesitating at each blank for the pupil to supply the word. In this preparatory work, no book should be in the hands of the pupil. New words should be first presented through the voice of the teacher in the expression of thought. They may then be written upon the blackboard by the teacher, erased, and the pupil exercised in both oral and written reproduction.

STEP 2.—The book should be put into the hands of the pupil, and he should read, first silently, then orally, the *figure problems* upon any page for which proper preparation has been made.

STEP 3.—The pupil may attempt the *letter problems** at his desk without further assistance. If he is unable to solve these, review the *figure problems* and give others similar to them. In some instances it may be well to have the *figure problems* solved upon the blackboard as preparation for the *letter problems.*

2.

Wherever the word "*story*" follows a problem, as on pp. 12 and 13, require the pupil to make a statement showing how the problem might originate in business or other experience; thus, the story for problem (f), page 12, might be, *A man divided* $4\frac{3}{4}$ *acres of land into lots each containing* $\frac{1}{20}$ *of an acre. There were 95 lots.*

3.

The separatrix, as an aid in "pointing off" in multiplication and division of decimals, is mentioned in a note on page 133. Its use is illustrated on page 143. Its occasional use in blackboard work earlier in the course may be helpful; but a too early reliance upon formal rules is to be avoided if the purpose of the work is the development of the thought-power of the pupil.

* The *figure problems* are those designated by figures; the *letter problems*, those designated by letters.

ARITHMETIC.

PART I.

SIMPLE NUMBERS.

1. Three is an **exact divisor** of 6, of 9, of ——, of ——, of ——, and of ——. Four is an exact divisor of ——.

2. Eighteen is **exactly divisible** by 2, by ——, by ——, and by ——. 28 is exactly divisible by ——.

3. Two is an exact divisor of 4, 6, ——, ——, ——, etc.

4. A number that is exactly divisible by 2 is called an **even number**; 8, 10, ——, ——, ——, are even numbers.

5. Two is not an exact divisor of 1, 3, 5, ——, ——, etc.

6. A number that is not exactly divisible by 2 is called an **odd number**; 9, 11, ——, ——, ——, ——, are odd numbers.

7. 37 is an —— number. 24 is an —— number.

8. Tell which of the following are odd numbers and which are even numbers: 375, 256, 320, 197, 281, 378, 584, 252, 323, 569.

(a) Copy the numbers given in problem 8 and find their sum.*

(b) Find the sum of all the odd numbers from 1 to 15 inclusive.

(c) Find the sum of all the even numbers from 2 to 16 inclusive.

* Problems *designated by letters* are for the slate.

FRACTIONS.

1. When two or more fractions have denominators that are alike, they are said to have a **common denominator.** $\frac{1}{12}$, $\frac{7}{12}$, $\frac{5}{12}$, and $\frac{9}{12}$ have a —— ——. $\frac{2}{3}$ and $\frac{5}{7}$ do not have a —— ——.

Reduce to equivalent fractions having a common denominator.

2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.	(Change to 12ths.)	$\frac{1}{2} =$	$\frac{1}{3} =$	$\frac{1}{4} =$
3. $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{2}$.	(Change to ——.)	$\frac{1}{5} =$	$\frac{1}{4} =$	$\frac{1}{2} =$
4. $\frac{1}{5}$, $\frac{1}{7}$.	(Change to ——.)	$\frac{1}{5} =$	$\frac{1}{7} =$	
5. $\frac{1}{4}$, $\frac{1}{7}$.	(Change to ——.)	$\frac{1}{4} =$	$\frac{1}{7} =$	

(a) $\frac{3}{4}$ and $\frac{2}{7}$. (b) $\frac{3}{5}$ and $\frac{5}{7}$. (c) $\frac{3}{8}$ and $\frac{2}{5}$.

6. Add $\frac{5}{8}$ and $\frac{1}{3}$. I can change eighths and thirds to ——ths. $\frac{5}{8} = \frac{}{24}$. $\frac{1}{3} = \frac{}{24}$. —— twenty-fourths and —— twenty-fourths = —— twenty-fourths.

(d) Find the sum of $576\frac{5}{8}$ and $239\frac{1}{3}$.

7. From $\frac{5}{7}$ subtract $\frac{1}{3}$. I can change sevenths and thirds to ——sts. $\frac{5}{7} = \frac{}{21}$. $\frac{1}{3} = \frac{}{21}$. —— twenty-firsts less —— twenty-firsts = —— twenty-firsts.

(e) Find the difference of $572\frac{5}{7}$ and $368\frac{1}{3}$.

8. Divide $\frac{3}{4}$ by $\frac{1}{20}$. This means, **find how many times $\frac{1}{20}$ is contained in $\frac{3}{4}$.** I can change $\frac{3}{4}$ to twentieths. $\frac{3}{4} = \frac{}{20}$. 1 twentieth is contained in —— twentieths —— times. *Story.**

(f) Find the quotient of $4\frac{3}{4}$ divided by $\frac{1}{20}$. (Change $4\frac{3}{4}$ to twentieths.) *Story.*†

(g) $147\frac{2}{7} + 38\frac{2}{3}$. (h) $374\frac{5}{8} - 181\frac{1}{3}$. (i) $27\frac{1}{2}$ bu. ÷ $2\frac{1}{2}$ bu.‡

* See Note (9) page 6. † See paragraph 2, page 10.

‡ Tell the meaning. Change $27\frac{1}{2}$ and $2\frac{1}{2}$ to halves. *Story.* See note (14) page 7.

DECIMALS—THOUSANDTHS.

1. .125 and .006 are —— thousandths.
2. .125 and .06 are —— thousandths.
3. .125 and .6 are —— thousandths.
4. .275 less .006 are —— thousandths.
5. .275 less .06 are —— thousandths.

6. 2.4 and 3.104 are —— and —— thousandths.
(a) Add 27.006, 14.8, and 25.06.

7. 36.025 less 1.008 equals —— and —— thousandths.
(b) From 8.175 subtract 3.023.

8. 3 times 2.005 is —— and —— thousandths.
(c) Multiply 26.008 by 4.

9. Divide .035 by .007. This means —— 4* ——.
(d) Find the quotient of .375 divided by .005. *Story.*†

10. Divide .035 by 7. This means —— 5* ——.
(e) Find the quotient of 35.049 acres divided by 7. *Story.*

11. At $.05 each, for $1.25 I can buy —— pencils. 1.25 ÷ .05 =

12. At $.05 each, for $2 I can buy —— pencils. 2 ÷ .05 =

(f) Add.	(g) Subtract.	(h) Multiply.	(i) Divide.	(j) Divide.
46.27	54.75	7.054‡	$.005)$.385§	5)$.385§
25.308	8.326	5		

* These figures refer to notes on page 8.

† See page 10, paragraph 2.

‡ Write the decimal point in the product *immediately after writing the tenths' figure of the product.*

§ "Point off" in division of decimals by thinking what the problems mean. Problem (i) means, *find how many times 5 thousandths are contained in 385 thousandths.* 5 thousandths are contained in 385 thousandths 77 times. Problem (j) means, *find 1 fifth of 385 thousandths.* ⅕ of 385 thousandths is 77 thousandths, or .077.

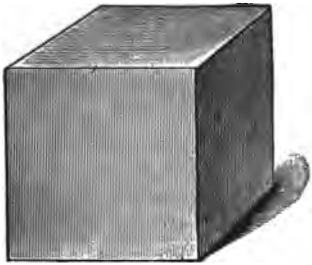
DENOMINATE NUMBERS.

2000 lb. = 1 ton.

1. 500 lb. are —— —— of a ton.
2. 1500 lb. are —— —— of a ton.
3. 1000 lb. are —— —— of a ton.
4. 200 lb. are —— —— of a ton.
5. I weigh —— pounds.
6. My father's horse weighs —— pounds.
7. A keg of nails weighs 100 pounds.
8. One ton of nails is —— kegs.
9. A load of hay weighs from —— to —— pounds.*
10. A load of coal weighs from —— to —— pounds.*
11. When hay is $12 a ton, 1000 lb. cost ————.
12. When coal is $8 a ton, 1500 lb. cost ————.
13. When meal is $16 a ton, 500 lb. cost ————.
14. When hay is $6 a ton, 200 lb. cost ————.
15. When hay is $7 a ton, 200 lb. cost ————.
16. When hay is $5 a ton, 200 lb. cost ————.
17. When hay is $8 a ton, 200 lb. cost ————.
18. When hay is $6 a ton, 2200 lb. cost ————.
19. When hay is $8 a ton, 2200 lb. cost ————.
20. When hay is $5 a ton, 2200 lb. cost ————.
21. 2500 lb. are —— and —— —— tons.
22. 3000 lb. are —— and —— —— tons.
23. 4500 lb. are —— and —— —— tons.
24. 5000 lb. are —— and —— —— tons.
25. When hay is $12 a ton, 2500 lb. cost ————.
26. When hay is $10 a ton, 3000 lb. cost ————.
27. When hay is $8 a ton, 4500 lb. cost ————.

* When you see a load of hay or a load of coal, courteously ask the man in charge of it to tell you how much it weighs.

MEASUREMENTS.



1-inch cube.

1. A 2-inch square contains —— square inches.
2. A 2-inch cube contains —— cubic inches.
3. A 3-inch square contains —— square inches.
4. A 3-inch cube contains —— cubic inches.

5. A 2-foot square contains —— square feet.
6. A 2-foot cube contains —— cubic feet.
7. A 3-foot square contains —— square feet.
8. A 3-foot cube contains —— cubic feet.

9. The area of a 2-inch square is —— —— inches.
10. The solid content of a 2-inch cube is ———.
11. The area of a 3-inch square is —— —— inches.
12. The solid content of a 3-inch cube is ———.

13. The area of a 2-foot square is —— —— feet.
14. The solid content of a 2-foot cube is ———.
15. Find the area of— Find the solid content of—*

Find the area of—	Find the solid content of—*
(1) A 4-inch square.	(a) A 4-inch cube.
(2) A 5-inch square.	(b) A 5-inch cube.
(3) A 6-inch square.	(c) A 6-inch cube.
(4) A 12-inch square.	(d) A 12-inch cube.

*Problems designated by letters are for the slate.

RATIO AND PROPORTION.

1. One fourth of 20 is ——. 20 is $\frac{1}{4}$ of ——.
2. One half of 11 is ——. 11 is $\frac{1}{2}$ of ——.
3. One third of 10 is ——. 10 is $\frac{1}{3}$ of ——.
4. One fifth of 45 is ——. 45 is $\frac{1}{5}$ of ——.

5. Two thirds of 60 are ——. 60 is $\frac{2}{3}$ of ——.*
6. Two thirds of 36 are ——. 36 is $\frac{2}{3}$ of ——.*
7. Two thirds of 15 are ——. 15 is $\frac{2}{3}$ of ——.
8. Two thirds of 21 are ——. 21 is $\frac{2}{3}$ of ——.

9. 10 is —— —— of 15. 15 is —— ——† of 10, or —— times 10.
10. 15 is —— —— of 20. 20 is —— —— of 15, or —— times 15.
11. 20 is —— —— of 25. 25 is —— —— of 20, or —— times 20.
12. 25 is —— —— of 30. 30 is —— —— of 25, or —— times 25.

13. Three fourths of 16 are 2 thirds of ——.
14. Two thirds of 15 are 1 half of ——.
15. Three fourths of 24 are 2 thirds of ——.
16. Two thirds of 12 are 1 third of ——.

17. Eight is —— —— of 12. A man can earn —— —— as much in 8 days as he can earn in 12 days. If he can earn $21 in 12 days, in 8 days he can earn —— dollars.

18. Fifteen is —— and —— —— times 6. A man can earn —— times as much in 15 days as he can earn in 6 days. If he can earn $10 in 6 days, in 15 days he can earn —— dollars.

(a) If a man can earn $104 in 6 weeks, how much can he earn in 15 weeks.

* It may be well, after the pupil has solved "mentally" such problems as Nos. 5 and 6, to *require him* to solve them with a pencil—this, as a preparation for similar problems in which larger numbers are used.

† Allow the pupil to say, "15 is 3 halves of 10," but remind him that the expression "3 halves of 10," means, *3 times 1 half of 10.*

PERCENTAGE.*

$$50 \text{ per cent} = .50 = \tfrac{1}{2}.$$
$$25 \text{ per cent} = .25 = \tfrac{1}{4}.$$
$$33\tfrac{1}{3} \text{ per cent} = .33\tfrac{1}{3} = \tfrac{1}{3}.$$
$$20 \text{ per cent} = .20 = \tfrac{1}{5}.$$

(*1*)†	(*2*)†
1. 50 per cent of 10 =	10 is 50% of ——.
2. 25 per cent of 12 =	12 is 25% of ——.
3. $33\frac{1}{3}$ per cent of 12 =	12 is $33\frac{1}{3}$% of ——.
4. 20 per cent of 10 =	10 is 20% of ——.
5. 50 per cent of 8 =	8 is 50% of ——.
6. 25 per cent of 8 =	8 is 25% of ——.
7. 20 per cent of 15 =	15 is 20% of ——.
8. $33\frac{1}{3}$ per cent of 15 =	15 is $33\frac{1}{3}$% of ——.

(*3*)†	(*3*)†
9. 5 is —— per cent of 10.	5 is ——% of 20.
10. 5 is —— per cent of 15.	5 is ——% of 25.
11. 4 is —— per cent of 8.	4 is ——% of 12.
12. 4 is —— per cent of 16.	4 is ——% of 20.
13. 3 is —— per cent of 6.	3 is ——% of 15.

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14. Henry had 25 cents; he spent 20 per cent of his money; he spent —— cents.

15. Peter spent 10 cents; this was 25 per cent of all he earned; he earned —— cents.

16. Roscoe earned 60 cents and spent 30 cents; he spent —— per cent of what he earned.

*To THE TEACHER.—Frequently review this page, and *give many similar oral problems* as a preparation for page 27.

†These figures designate the *percentage cases* to which the problems below them belong.

PERCENTAGE.

(1)

1. 50% of 18 is ——. 50% of 19 is ——.
(a) Find 50 per cent of 724; (b) of 725.

2. 25% of 16 is ——. 25% of 17 is ——.
(c) Find 25 per cent of 896; (d) of 897.

3. $33\frac{1}{3}$% of 18 is ——. $33\frac{1}{3}$% of 19 is ——.
(e) Find $33\frac{1}{3}$ per cent of 726; (f) of 727.

4. 20% of 25 is ——. 20% of 26 is ——.
(g) Find 20 per cent of 875; (h) of 877.

(2)

5. 7 is 50% of ——. $7\frac{1}{2}$ is 50% of ——.
(i) 376 is 50% of what? (j) $376\frac{1}{2}$ is 50% of what?

6. 9 is 25% of ——. $9\frac{1}{2}$ is 25% of ——.
(k) 524 is 25% of what? (l) $524\frac{1}{2}$ is 25% of what?

7. 8 is $33\frac{1}{3}$% of ——. $8\frac{1}{2}$ is $33\frac{1}{3}$% of ——.
(m) 652 is $33\frac{1}{3}$% of what? (n) $652\frac{1}{2}$ is $33\frac{1}{3}$% of what?

(3)

8. 11 is ——% of 22. 11 is ——% of 33.
9. 11 is ——% of 44. 11 is ——% of 55.
10. 15 is ——% of 30. 30 is ——% of 60.

(1)

11. One per cent (1%) of a number is 1 hundredth of the number; 2% is 2 hundredths; 3% is 3 hundredths.

12. One per cent of 500 is ——. 2% of 500 is ——.
13. One per cent of 600 is ——. 3% of 600 is ——.

REVIEW.

1. Tell which of the following are odd numbers and which are even numbers: 14, 17, 19, 20, 24, 27.

2. The fractions $\frac{1}{15}$, $\frac{7}{15}$, and $\frac{11}{15}$ have a —— denominator.

3. Change $\frac{2}{3}$ and $\frac{4}{5}$ to equivalent fractions having a common denominator. I can change thirds and fifths to ——.
$\frac{2}{3} =$ $\frac{4}{5} =$

4. Add $\frac{2}{7}$ and $\frac{1}{3}$. I can change 7ths and 3ds to ——.
$\frac{2}{7} = \frac{}{21}$. $\frac{1}{3} = \frac{}{21}$. —— twenty-firsts and —— twenty-firsts = —— twenty-firsts.

5. Divide $\frac{3}{5}$ by $\frac{1}{20}$. This means, find how many times $\frac{1}{20}$ is contained in $\frac{3}{5}$. I can change 5ths to ——. $\frac{3}{5} = \frac{}{20}$. One twentieth is contained in —— twentieths —— times.

6. Multiply 6.05 by 3. Three times ————.

(a) Find the product of 34.02 multiplied by 4.

7. Divide $6.28 by 2. This means, —— 10* ——.

(b) Find the quotient of $475.65 divided by 5.

8. When hay is $12 a ton, 1500 lb. cost —— dollars.

9. When coal is $7 a ton, 3000 lb. cost —— dollars.

10. A 2-inch square is how many times as large as a 1-inch square?

11. A 2 inch cube is how many times as large as a 1-inch cube?

(c) How many square inches in an oblong 9 inches by 23 inches?

12. Three fourths of 20 are 1 half of ——.

(d) Three fourths of 84 are 1 half of what number?

* See note 10, page 9.

MISCELLANEOUS.

1. Mr. A owned $15\frac{1}{2}$ acres of land; he sold $6\frac{1}{4}$ acres; he then had —— acres.

(a) Mr. B owned $546\frac{1}{2}$ acres of land; he sold $228\frac{1}{4}$ acres How many acres had he remaining?

2. A bushel of oats weighs 32 lb.; 2 bushels of oats weigh —— pounds; 3 bushels weigh —— pounds.

(b) Sixty bushels of oats weigh how many pounds less than one ton?

3. Two pounds of coffee @ \$.23 a pound cost ——.

(c) Find the cost of 84 pounds of coffee @ \$.23 a pound.*

4. From a piece of cloth containing 12 yards there were sold $5\frac{1}{2}$ yards and $2\frac{1}{2}$ yards; there were left —— yards.

(d) From a piece of cloth containing 55 yards, there were sold $24\frac{5}{8}$ yd. and $17\frac{3}{4}$ yd. How many yards were left?

5. A boarding-school uses one gallon of milk each day; at 5¢ a quart the milk for one week costs ——.

(e) A boarding-school uses 4 gal. of milk each day; at 5¢ a quart how much will the milk for the month of October cost?

6. John bought 20 oranges at 3¢ each and sold them at 5¢ each; he gained —— cents.

(f) John's father bought 25 bushels of potatoes at 32¢ a bushel and sold them at 45¢ a bushel. How much did he gain?

* In multiplication of decimal fractions, require the pupil to locate the decimal point in the product *immediately after* he has written the figure to the right of the decimal point in any whole or partial product. Thus, in the multiplication of \$.23 by 84 he writes (and thus locates) the decimal point of the product immediately after writing the figure 9 in the first partial product. This will be quite clear to the pupil if he understands that he first finds 4 times .23; then 80 times .23; then the sum of these two partial products.

$$\begin{array}{r} \$.23 \\ 84 \\ \hline .92 \\ 18.4 \\ \hline \$19.32 \end{array}$$

SIMPLE NUMBERS.

1. Seventeen is an **integral number.**

2. Five eighths is a **fractional number.**

3. Seventeen and five eighths is a **mixed number.**

4. 25 is an —— number. $5\frac{1}{2}$ is a —— number

5. $\frac{21}{24}$ is a —— number. 3.5 is a —— number.

6. .8 is a —— number. 18 is an —— number.

7. *A number is exactly divisible by* 2 *if the right-hand figure is* 0, 2, 4, 6, *or* 8. Tell which of the following are exactly divisible by 2: 241, 136, 274, 393, 247, 826.

8. *A number is exactly divisible by* 5 *if its right-hand figure is* 0 *or* 5. Tell which of the following are exactly divisible by 5: 184, 275, 320, 145.

9. The following are exactly divisible by 2: 16, 38, 54, 68, ——, ——, ——, ——.

10. The following are exactly divisible by 5: 85, 140, 175, 180, ——, ——, ——, ——.

11. The following are exactly divisible by 10: 30, 40, 80, 120, ——, ——, ——, ——.

12. There are twice as many 5's as there are 10's in a number. 90 is —— tens. 90 is —— fives.

(a) Divide $2150 by 50.* (b) Divide $215.00 by 50.
(c) Divide $2295 by 51. (d) Divide $290.70 by 51.†
(e) Divide $2058 by 49. (f) Divide $156.80 by 49.
(g) Divide $3276 by 52. (h) Divide $322.40 by 52.
(i) Divide $2496 by 48. (j) Divide $254.40 by 48.

* This means, find 1 fiftieth of $2150. 1 fiftieth of $2150 is —— dollars. *Story*—If 50 acres of land cost $2150, 1 acre cost ——.

† In solving such problems as (b), (d), etc., require the pupil to write the decimal point in the quotient, *immediately after writing the units' figure* of the quotient. If the pupil thinks what the problem means, he will easily "point off" correctly.

COMMON FRACTIONS.

Reduce to equivalent fractions having a common denominator:

1. $\frac{1}{4}$ and $\frac{1}{9}$. (Change to ——.) $\frac{1}{4} =$ $\frac{1}{9} =$

(a) $\frac{2}{5}$ and $\frac{2}{9}$. (b) $\frac{3}{4}$ and $\frac{5}{9}$. (c) $\frac{3}{5}$ and $\frac{7}{9}$.

2. Add $\frac{8}{9}$ and $\frac{1}{5}$. (Change to ——ths.)*

(d) Find the sum of $248\frac{8}{9}$ and $467\frac{1}{5}$.

3. From $\frac{3}{4}$ subtract $\frac{2}{9}$. (Change to ——ths.)†

(e) Find the difference of $837\frac{3}{4}$ and $284\frac{2}{9}$.

4. Divide 6 by $\frac{2}{3}$. This means, *find how many times* $\frac{2}{3}$ *are contained in* 6. I can change 6 to thirds. $6 = \frac{\quad}{3}$. 2 thirds are contained in —— thirds —— times. *Story.*‡

(f) Find the quotient of 48 divided by $\frac{2}{3}$. *Story.*§

5. Divide $8\frac{1}{2}$ feet by 2. This means, *find* 1 *half of* $8\frac{1}{2}$ *feet.* 1 half of $8\frac{1}{2}$ feet = *Story.*

(g) Find the quotient of $86\frac{1}{2}$ miles divided by 2. *Story.*‖

(h) $3\frac{1}{2}$ miles)154 miles. This means, ***find how many times $3\frac{1}{2}$ miles are contained in 154 miles.*** (Change $3\frac{1}{2}$ and 154 to halves.) *Story*—A canal-boat moved at the rate of $3\frac{1}{2}$ miles an hour; to move 154 miles would require —— hours.

(i) 3)$165\frac{1}{2}$ miles. This means, ***find 1 third of $165\frac{1}{2}$ miles.*** *Story*—A train moved $165\frac{1}{2}$ miles in 3 hours; it moved at the rate of —— miles an hour.

* See page 12, problem 6.

† See page 12, problem 7.

‡ See note (4), page 6.

§ 48 feet of ribbon was cut into pieces $\frac{2}{3}$ of a foot long; there were —— pieces.

‖ A train moved $86\frac{1}{2}$ miles in 2 hours; this was at the rate of —— miles an hour.

DECIMAL FRACTIONS.

1. One thousandth of a dollar is —— mill.
2. Four thousandths of a dollar are 4 ——.
3. Four hundredths of a dollar are 4 ——.
4. Four tenths of a dollar are 4 ——.

5.	.1 of \$60 =	.1 of \$4 =	.1 of \$64 =*
6.	.1 of \$.1 =	.1 of \$.5 =	.1 of \$.7 =
7.	.1 of \$6 =	.1 of \$8.5 =	.1 of \$5.3 =
8.	.1 of \$.01 =	.1 of \$.03 =	.1 of \$.07 =
9.	.1 of \$6.42 =	.1 of \$7.56 =	.1 of \$8.42 =
10.	.1 of \$875 =	.1 of \$74.2 =	.1 of \$5.35 =*

(a) Multiply \$374 by .3. This means, *find* 3 *tenths of* 374.

Operation.

\$374†
.3
\$112.2

Explanation.

One tenth of \$374 =
Three tenths of \$374 =

NUMBER STORY.

If one acre of land is worth \$374,
1 tenth of an acre of land is worth ——.
3 tenths of an acre of land are worth ——.

(b)	(c)	(d)	(e)	(f)
Multiply.	Multiply.	Multiply.	Divide.	Divide
346	23.5	2.73	\$.05)\$.85‡	5)\$.85§
.3	.3	3		

* By means of problems 5 to 10 and other similar exercises, lead the pupil to see that he can obtain 1 tenth of an integral number by "pointing off" one figure on the right, and of a mixed decimal by removing the point one place to the left.

† The pupil should understand that he multiplies \$37.4 (not \$374) by 3, and should be taught to write the decimal point in the product *immediately after writing the tenths' figure of the product.* See page 10, paragraph 3.

‡ This means, *find how many times 5 hundredths are contained in 85 hundredths.*

§ This means, *find 1 fifth of 85 hundredths.*

DENOMINATE NUMBERS.

A bushel of oats weighs 32 lb.
A bushel of wheat weighs 60 lb.
A bu. of corn (shelled) weighs 56 lb.
A bu. of corn (not shelled) weighs 70 lb.*

1. Two bushels of oats weigh —— lb.; 5 bu. weigh ——.

(a) How much more than 1 ton do 75 bu. of oats weigh?

2. Two bu. of wheat weigh ——; 3 bu. weigh ——.

(b) Seventy-five bushels of wheat weigh how much more than two tons? (c) How much less than three tons?

3. Two bushels of shelled corn weigh ——; 3 bu. weigh ——.

(d) Forty-five bushels of shelled corn weigh how much more than $1\frac{1}{4}$ tons? (e) How much less than $1\frac{1}{2}$ tons?

4. Two bushels of corn (not shelled) weigh ——; 3 bushels weigh ——; 4 bushels weigh ——.

(f) How much more than three fourths of a ton do 25 bushels of corn (unshelled) weigh? (g) How much less than one ton?

5. One hundred pounds of oats are —— bushels and —— pounds, or —— and —— 32nds bushels.

(h) Five hundred pounds of oats are how many bushels?

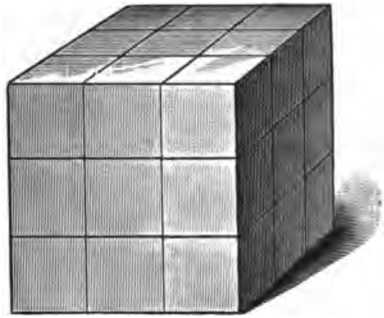
(i) How many bushels in one and one half tons of wheat?

(j) How many bushels in two tons of unshelled corn?

(k)	(l)	(m)	(n)
32 lb.)768 lb.	32 lb.)769 lb.	32 lb.)770 lb.	32 lb.)772 lb.

* Explain to the pupil that the expression "a bushel of corn (not shelled)," means, *the amount of unshelled corn required to make 1 bushel of shelled corn.*

MEASUREMENTS.



1. A 3-foot square (or its equivalent) is called a square yard. It contains —— square feet.

2. A 3-foot cube (or its equivalent) is called a cubic yard. It contains —— cubic feet.

3. The area of a 4-foot square is —— square feet, or —— and —— —— square yards.

4. The solid content of a 4-foot cube is —— cubic feet,

(a) How many cubic yards in a 4-foot cube?

5. The area of a 5-foot square is —— square feet. or —— and —— —— square yards.

6. The solid content of a 5-foot cube is —— cubic feet.

(b) How many cubic yards in a 5-foot cube?

7. The area of a 6-foot square is —— square feet, or —— square yards.

8. The solid content of a 6-foot cube is —— cubic feet.

(c) How many cubic yards in a 6-foot cube?

9. Find the area of—

(1) A 7-foot square.

(2) An 8-foot square.

(3) A 9-foot square.

Find the solid content of—

(d) A 7-foot cube.

(e) An 8-foot cube.

(f) A 9-foot cube.

RATIO AND PROPORTION.

1. One fifth of 25 is ——. 25 is $\frac{1}{5}$ of ——.
2. One fourth of 21 is ——. 21 is $\frac{1}{4}$ of ——.
3. One third of 22 is ——. 22 is $\frac{1}{3}$ of ——.
4. One half of 45 is ——. 45 is $\frac{1}{2}$ of ——.

5. Three fourths of 24 are ——. 24 is $\frac{3}{4}$ of ——.
6. Three fourths of 36 are ——. 36 is $\frac{3}{4}$ of ——.

(a) Find $\frac{3}{4}$ of 96. (b) 96 is $\frac{3}{4}$ of what number?
(c) Find $\frac{2}{3}$ of 96. (d) 96 is $\frac{2}{3}$ of what number?
(e) Find $\frac{1}{2}$ of 96. (f) 96 is $\frac{1}{2}$ of what number?

7. 12 is —— —— of 18. 18 is —— —— of 12.
8. 18 is —— —— of 24. 24 is —— —— of 18.
9. 24 is —— —— of 30. 30 is —— —— of 24.

10. Two thirds of 18 are 1 half of ——.
11. Three fourths of 24 are 2 thirds of ——.
12. Two thirds of 30 are 1 half of ——.

(g) One third of 132 is 1 half of what number?
(h) Two thirds of 252 are 1 half of what number?
(i) Three fourths of 96 are 2 thirds of what number?
(j) Two thirds of 96 are 3 fourths of what number?

13. Nine is —— —— of 12. A man can earn —— —— as much in 9 days as he can earn in 12 days. If he can earn $40 in 12 days, in 9 days he can earn —— dollars.

(k) If a man can earn $896 in 12 months, how many dollars can he earn in 9 months?

(l) If Mr. Conrad's horses consume 726 bushels of oats in a year, how many bushels will be required to feed them 8 months?

PERCENTAGE.

$16\frac{2}{3}$ per cent $= .16\frac{2}{3} = \frac{1}{6}$.
$14\frac{2}{7}$ per cent $= .14\frac{2}{7} = \frac{1}{7}$.
$12\frac{1}{2}$ per cent $= .12\frac{1}{2} = \frac{1}{8}$.

	(1)	(2)
1.	$16\frac{2}{3}$ per cent of 12 =	12 is $16\frac{2}{3}\%$ of ——.
2.	$14\frac{2}{7}$ per cent of 21 =	21 is $14\frac{2}{7}\%$ of ——.
3.	$12\frac{1}{2}$ per cent of 16 =	16 is $12\frac{1}{2}\%$ of ——.
4.	$16\frac{2}{3}$ per cent of 18 =	18 is $16\frac{2}{3}\%$ of ——.
5.	$14\frac{2}{7}$ per cent of 28 =	28 is $14\frac{2}{7}\%$ of ——.
6.	$12\frac{1}{2}$ per cent of 24 =	24 is $12\frac{1}{2}\%$ of ——.
7.	50 per cent of 12 =	12 is 50% of ——.
8.	25 per cent of 16 =	16 is 25% of ——.
9.	20 per cent of 25 =	25 is 20% of ——.
10.	$33\frac{1}{3}$ per cent of 18 =	18 is $33\frac{1}{3}\%$ of ——.

	(3)	(3)
11.	2 is —— per cent of 6.	2 is ——% of 4.
12.	2 is —— per cent of 8.	2 is ——% of 14.
13.	2 is —— per cent of 12.	2 is ——% of 10.
14.	2 is —— per cent of 16.	1 is ——% of 2.

15. There were 40 pears on a tree; 25% of them fell off; —— pears remained on the tree.

16. William sold 7 melons; these were $12\frac{1}{2}\%$ of all the melons he raised; he raised —— melons.

17. Mary had 30 little chickens; a hawk killed five of them; the hawk killed —— per cent of her chickens.

18. Twelve and one half per cent of \$48 is —— dollars

(a) Find $12\frac{1}{2}$ per cent of \$992.

(b) Find $16\frac{2}{3}$ per cent of \$852.

PERCENTAGE.

(1)

1. $16\frac{2}{3}\%$ of 24 = $16\frac{2}{3}\%$ of 25 =

(a) Find $16\frac{2}{3}$ per cent of 342; (b) of 343.

2. $12\frac{1}{2}\%$ of 24 = $12\frac{1}{2}\%$ of 25 =

(c) Find $12\frac{1}{2}$ per cent of 976; (d) of 978.

3. $14\frac{2}{7}\%$ of 28 = $14\frac{2}{7}\%$ of 30 =

(e) Find $14\frac{2}{7}$ per cent of 994; (f) of 996.

(2)

4. 5 is $12\frac{1}{2}\%$ of ——. $5\frac{1}{2}$ is $12\frac{1}{2}\%$ of ——.

(g) 246 is $12\frac{1}{2}\%$ of what? (h) $246\frac{1}{2}$ is $12\frac{1}{2}\%$ of what?

5. 5 is $14\frac{2}{7}\%$ of ——. $5\frac{1}{2}$ is $14\frac{2}{7}\%$ of ——.

(i) 351 is $14\frac{2}{7}\%$ of what? (j) $356\frac{1}{4}$ is $14\frac{2}{7}\%$ of what?

6. 5 is $16\frac{2}{3}\%$ of ——. $5\frac{1}{2}$ is $16\frac{2}{3}\%$ of ——.

(k) 239 is $16\frac{2}{3}\%$ of what? (l) $241\frac{1}{3}$ is $16\frac{2}{3}\%$ of what?

(3)

7. 12 is ——% of 24. 12 is ——% of 36.
8. 12 is ——% of 48. 12 is ——% of 60.
9. 12 is ——% of 72. 12 is ——% of 84.
10. 12 is ——% of 96. 15 is ——% of 45.

(1)

11. One per cent of $300 is ——. 2% of $300 =
12. One per cent of $320 is ——. 2% of $320 =
13. One per cent of $325 is ——. 2% of $325 =
14. One per cent of $342 is ——. 2% of $342 =

(m) Find 3% of $342. (n) Find 4% of $342.
(o) Find 5% of $342. (p) Find 6% of $342.
(q) Find 3% of $536. (r) Find 4% of $536.

REVIEW.

1. Two is an exact divisor of 28, 274, ——, ——.

2. Five is an exact divisor of 75, 230, ——, ——.

3. Ten is an exact divisor of 80, ——, ——, ——.

4. Twenty is an integral number. $\frac{3}{4}$ and .5 are —— numbers. $5\frac{1}{4}$ and 3.2 are —— numbers.

5. The fractions $\frac{3}{20}$, $\frac{9}{20}$, ——, ——, have a common denominator. $\frac{2}{7}$ and —— do not have a common denominator.

6. The sum of .135 and .6 is ——. .135 − .06 =

7. Divide .045 by .009. (This means —— 4* ——.)

(a) Find the quotient of .875 divided by .005.

8. Divide .045 by 9. (This means —— 5* ——.)

(b) Find the quotient of 54.063 divided by 9.

9. When hay is $8 a ton, 2500 lb. cost ——.

10. Five bushels of wheat weigh —— pounds.

(c) Sixty-five bushels of wheat weigh how much more than 65 bushels of oats?

11. The area of a 10-inch square is —— square inches.

(d) How many square feet in a 15-foot square?

(e) How many square yards in a 15-foot square?

12. The solid content of a 10-inch cube is —— cu. in.

(f) How many cubic feet in a 15-foot cube?

(g) How many cubic yards in a 15-foot cube?

13. Three fourths of 60 are ——. 60 is $\frac{3}{4}$ of ——.

(h) Find $\frac{3}{4}$ of 132. (i) 132 is $\frac{3}{4}$ of what number?

14. Two thirds of 27 are 1 half of ——.

(j) Two thirds of 132 are 1 half of what number?

* These figures refer to notes on page 8.

MISCELLANEOUS PROBLEMS.

1. Byron bought 6 melons for 54¢; he sold them at 15¢ each; on each melon he gained —— cents.

(a) A merchant bought 6 barrels of apples for $13.50, and sold them at $2.75 a barrel. How much did he gain on each barrel?

2. James carried 4 dozen eggs to market; he sold them for 12¢ a dozen; the dealer paid James for the eggs with sugar at 5¢ a lb.; James should receive —— and —— —— pounds of sugar.

(b) A farmer took four cords of wood to market; he sold it for $6.75 a cord and took its value in coal at $6 a ton. How many tons of coal should he receive?

3. At 5¢ a quart, 1 gallon of milk is worth —— cents. $2\frac{1}{2}$ gallons are worth —— cents.

(c) At 5¢ a quart, how much are $45\frac{1}{2}$ gallons of milk worth?

4. Two and 1 half feet are —— inches. 4 weeks are —— days.

(d) Forty-six and 1 half feet are how many inches?

(e) Fifty-two weeks are how many days?*

(f) Seventy-five gallons are how many quarts?

5. William had 30¢; he spent $\frac{3}{5}$ of his money for ink and pencils and the remainder for paper; the paper cost —— cents.

(g) William's father had $585; he spent $\frac{3}{5}$ of his money for a carriage and harness and the remainder for horses. How much did the horses cost?

* The pupil should see that in the solution of this problem he may take fifty-two 7's or seven 52's. If he adopts the latter method his thought may be—*One* day in each week would make 52 days, and *seven* days in each week would make 7 times 52 days; or he may think that fifty-two 7's are equal to seven 52's, and that, as a matter of convenience, he finds seven 52's.

SIMPLE NUMBERS.

1. A number that has no exact integral divisors except itself and 1, is called a **prime number.** 1, 2, 3, 5, 7, 11, ——, ——, ——, ——, ——, are prime numbers.

2. Integral numbers that are not prime are said to be **composite.** 4, 6, 8, 9. ——, ——, ——, ——, are composite numbers.

3. 24 is a —— number. 23 is a —— number.
29 is a —— number. 26 is a —— number.
28 is a —— number. 27 is a —— number.
31 is a —— number. 37 is a —— number.

4. If an integral number is expressed by two or more figures, and the right-hand figure is 5, the number is ——.

5. Every even number, except 2, is ——.

6. Some odd numbers are ——, and some are ——. 9 is ——. 11 is ——. 13 is ——. 21 is ——.

(a) Write all the prime numbers from 1 to 53 inclusive, and find their sum.

(b)	(c)	(d)	(e)	(f)
Add.	Subtract.	Multiply.	Divide.	Divide.
346	5423	$754\frac{1}{2}$	\$52)\$7436*	52)\$7436†
275	$1896\frac{5}{9}$	26		
142				
879	(g)	(h)	(i)	(j)
27	$7462\frac{4}{5}$	846	53 bu.)2438 bu.	53)2438 bu.
624	1827	$35\frac{1}{2}$		

* This means, find how many times \$52 are contained in \$7436. \$52 are contained in \$7436, 143 times. *Story—A man paid \$7436 for land at \$52 an acre; there were 143 acres.*

† This means, find 1 fifty-second of \$7436. 1 fifty-second of \$7436 is \$143. *Story—A man paid \$7436 for 52 acres of land; one acre cost \$143.*

COMMON FRACTIONS.

Reduce to their **lowest terms**:

1. $\frac{12}{40} =$ $\frac{18}{36} =$ $\frac{5}{35} =$ $\frac{8}{30} =$ $\frac{8}{36} =$ $\frac{10}{42} =$

(a) $\frac{75}{120} =$ (b) $\frac{24}{120} =$ (c) $\frac{36}{120} =$

Reduce to whole or mixed numbers:

2. $\frac{15}{4} =$ $\frac{40}{7} =$ $\frac{32}{8} =$ $\frac{63}{9} =$ $\frac{46}{9} =$ $\frac{46}{5} =$ $\frac{37}{6} =$

(d) $\frac{175}{5} =$ (e) $\frac{346}{4} =$ (f) $\frac{534}{6} =$

Reduce to equivalent fractions having a common denominator:

3. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$. (Change to ——.)* $\frac{1}{2} =$ $\frac{1}{3} =$ $\frac{1}{8} =$

(g) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$. (h) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$. (i) $\frac{7}{8}$, $\frac{1}{3}$, $\frac{3}{4}$.

4. Add $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{4}$. (Change to ——ths.)

(j) Find the sum of $75\frac{3}{8}$, $86\frac{2}{3}$, and $47\frac{1}{4}$.

5. From $\frac{7}{8}$ subtract $\frac{1}{3}$. (Change to ——ths.)

(k) Find the difference of $946\frac{5}{8}$ and $388\frac{1}{3}$.

6. Divide 7 by $\frac{3}{4}$. This means ———. *Story.*†

(l) Find the quotient of 36 divided by $\frac{3}{4}$. *Story.*‡

(m) $2\frac{1}{4}$ gallons)126 gallons. This means, ***find how many times 2¼ gallons are contained in 126 gal.*** (Change $2\frac{1}{4}$ and 126 to fourths.) *Story.*§

(n) 4)$276\frac{1}{2}$ gallons. This means, ***find 1 fourth of 276½ gallons.*** *Story*—In 4 days Mr. Smith sold $276\frac{1}{2}$ gallons of milk; this was at the rate of —— gallons per day.

* The pupil is expected to find by trial that he can change halves, thirds, and eighths, to twenty-fourths.

† See note (4) page 6, and problem 4, page 22.

‡ Mr. Brown put 36 bushels of peaches into $\frac{3}{4}$-bushel baskets; there were —— baskets.

§ Put 126 gal. of milk into $2\frac{1}{4}$-gal. cans.

DECIMAL FRACTIONS.

1. One tenth of \$2.45 is ——. .2 of \$2.45 =

(a) Multiply \$2.45 by 2.3. This means, *find* 2 *times* \$2.45 *plus* 3 *tenths of* \$2.45.

Operation.	*Explanation.*
\$2.45	One tenth of \$2.45 is ——.
2.3	Three tenths of \$2.45 are ——.
\$.735*	Two times \$2.45 are ——.
\$4.90	\$.735 + \$4.90 = \$5.635.
\$5.635	

NUMBER STORY.

If 1 ton of coal is worth \$2.45,
1 tenth of a ton of coal is worth ——.
3 tenths of a ton of coal are worth ——.
2 tons of coal are worth ——.
2.3 tons of coal are worth ——.

(b) Multiply \$3.65 by 2.4.* (.1 of \$3.65 is \$.365.)
(c) Multiply \$52.8 by 3.2. (.1 of \$52.8 is \$5.28.)

2. I bought $7\frac{2}{3}$ yards of print at 6¢ a yd. and gave the salesman half a dollar; I should receive in change ——.

(d) I bought 5.3 tons of coal at \$4.20 a ton and gave the salesman 3 10-dollar bills. How much change should I receive?

(e)	(f)	(g)	(h)	(i)
Add.	Subtract.	Multiply.	Divide.	Divide.
$64\frac{2}{5}$	78	3.75	\$.5)\$47.5†	5)\$47.5‡
28.37	24.42	2.6		

* Write the decimal point in each partial product *immediately after making the figure that represents the tenths in that product.*

† This means, *find how many times 5 tenths are contained in 475 tenths.*

‡ This means, *find 1 fifth of \$47.5.*

DENOMINATE NUMBERS.

1. One ton is —— pounds.
2. One tenth of a ton is —— pounds.
3. Three tenths of a ton are —— pounds.
4. Nine tenths of a ton are —— pounds.

5. One hundredth of a ton is —— pounds.
6. Three hundredths of a ton are —— pounds.
7. Seven hundredths of a ton are —— pounds.

8. One thousandth of a ton is —— pounds.
9. Two thousandths of a ton are —— pounds.
10. Six thousandths of a ton are —— pounds.

11. 2400 lb. = 1 and —— tenths (1.2) tons.
12. 2800 lb. = 1 and —— tenths (   ) tons.
13. 3200 lb. = 1 and —— tenths (   ) tons.
14. 2460 lb. = 1 and —————— tons.
15. 2468 lb. = 1 and —————— tons.

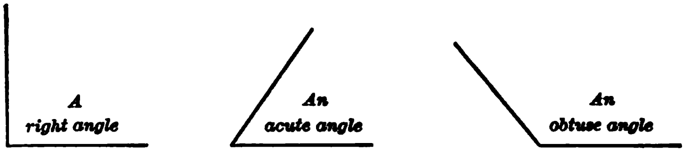
(a) Change 4870 lb. to tons. (b) 4980 lb.
(c) Change 5260 lb. to tons. (d) 5750 lb.
(e) Change 6480 lb. to tons. (f) 7260 lb.

16. 2 tons are —— lb. .3 of a ton are —— lb.
17. 2.3 tons are —— lb. 2.4 tons are —— lb.
18. .03 of a ton are —— lb. .04 of a ton are —— lb.
19. 2.04 tons are —— lb. 2.03 tons are —— lb.
20. 2.34 tons are —— lb. 3.23 tons are —— lb.

(g) Change 3.52 tons to lb. (h) 4.37 tons.
(i) Change 5.18 tons to lb. (j) 3.72 tons.
(k) Change 1.48 tons to lb. (l) 2.324 tons.

(m) At $\frac{1}{2}$ a cent a pound, find the cost of 3.45 tons of scrap iron.

MEASUREMENTS.



1. When two lines meet at a point, they are said to form **an angle.**

2. When a jackknife is half way open, the handle and blade form a **right angle.** If it is less than half way open, they form an —— angle. If it is more than half way open, but not fully open, they form an —— angle.

3. A square □ has 4 right angles.

4. An oblong ▭ has —— right angles.

5. **Rectangular** means right-angled.

6. A rectangular figure, or **rectangle,** may be a *square,* or it may be an *oblong.*

7. A rectangle 6 inches by 6 inches is a ——.

8. A rectangle 4 in. by 6 in. is an ——.

9. A rectangular surface 4 feet by 6 feet is an ——; its area is —— square feet and its perimeter is —— feet.

10. A rectangular surface 8 feet by 8 feet is a ——; its area is —— square feet; its perimeter is —— ft.

11. Which has the greater area, a 5-in. square, or an oblong 4 in. by 6 in.? Compare their perimeters.

Find the area and the perimeter of each of the following rectangular surfaces:

(a) 23 feet by 6 feet.
(b) 25 inches by 7 inches.
(c) 15 yards by 8 yards.
(d) 32 feet by 23 feet.
(e) 30 inches by 20 inches.
(f) 21 yards by 24 yards.

RATIO AND PROPORTION.

1. One sixth of 12 is ——.	12 is $\frac{1}{6}$ of ——.
2. One fifth of 12 is ——.	12 is $\frac{1}{5}$ of ——.
3. One fourth of 11 is ——.	11 is $\frac{1}{4}$ of ——.
(a) Find 1 third of 275.	(b) 275 is 1 third of what?
(c) Find 1 half of 377.	(d) 377 is 1 half of what?
4. Two fifths of 30 are ——.	30 is $\frac{2}{5}$ of ——.
5. Three fifths of 30 are ——.	30 is $\frac{3}{5}$ of ——.
(e) Find $\frac{2}{5}$ of 90.	(f) 90 is $\frac{2}{5}$ of what number?
(g) Find $\frac{3}{5}$ of 90.	(h) 90 is $\frac{3}{5}$ of what number?
6. 14 is —— —— of 21.	21 is —— —— of 14.
7. 21 is —— —— of 28.	28 is —— —— of 21.
8. 28 is —— —— of 35.	35 is —— —— of 28.

9. Two fifths of 30 are 1 half of ——.

10. Three fifths of 30 are 2 thirds of ——.

(i) Two fifths of 90 are 1 half of what number?

(j) Three fifths of 90 are 2 thirds of what number?

11. Twelve is —— —— of 8, or —— and —— —— times 8. A man can earn —— and —— —— times as much in 12 days as he can earn in 8 days. If he can earn \$20 in 8 days, in 12 days he can earn ——.

(k) If a man can earn \$73 in 8 months, how many dollars can he earn in 12 months?

12. If 8 lb. of sugar are worth 50¢, 12 lb. are worth —— cents.

(l) If 8 tons of coal are worth \$34.20, how much are 12 tons worth?

(m) If 12 barrels of apples are worth \$27.60, how much are 8 barrels worth?

PERCENTAGE.

$11\frac{1}{9}$ per cent $= .11\frac{1}{9} = \frac{1}{9}$.
10 per cent $= .10 = \frac{1}{10}$.

	(1)	(2)
1.	$11\frac{1}{9}$ per cent of 27 =	27 is $11\frac{1}{9}$% of ——.
2.	10 per cent of 20 =	20 is 10% of ——.
3.	$11\frac{1}{9}$ per cent of 18 =	18 is $11\frac{1}{9}$% of ——.
4.	10 per cent of 30 =	30 is 10% of ——.
5.	50 per cent of 3 =	3 is 50% of ——.
6.	25 per cent of 9 =	9 is 25% of ——.
7.	$33\frac{1}{3}$ per cent of 7 =	7 is $33\frac{1}{3}$% of ——.
8.	20 per cent of 11 =	11 is 20% of ——.
9.	$16\frac{2}{3}$ per cent of 19 =	19 is $16\frac{2}{3}$% of ——.
10.	$14\frac{2}{7}$ per cent of 15 =	15 is $14\frac{2}{7}$% of ——.

	(3)	(3)
11.	6 is —— per cent of 24.	6 is ——% of 18.
12.	6 is —— per cent of 12.	6 is ——% of 30.
13.	3 is —— per cent of 24.	3 is ——% of 21.
14.	3 is —— per cent of 18.	3 is ——% of 27.

15. Helen bought a piece of flannel that was 40 inches long; by washing it shrank 10% in length; after washing it was —— inches long.

16. Before washing, a piece of flannel was 40 inches in length; after washing it was 35 inches long; it shrank by washing —— per cent.*

17. A dealer had 25 bu. of apples; he lost 20% of them by decay; there remained —— bushels.

(a) A dealer had 2375 bu. of apples; he lost 20% of them by decay. How many bushels remained?

* It shrank *what part* of its original length?

PERCENTAGE.

(1)

1. $11\frac{1}{9}\%$ of 36 = $11\frac{1}{9}\%$ of 38 =

(a) Find $11\frac{1}{9}$ per cent of 1044; (b) of 1047.

2. 10% of 50 = 10% of 51 = 10% of 52 =

(c) Find 10% of 870; (d) of 874.

(2)

3. 7 is $11\frac{1}{9}\%$ of ——. $7\frac{1}{2}$ is $11\frac{1}{9}\%$ of ——.

(e) 371 is $11\frac{1}{9}\%$ of what? (f) $371\frac{1}{3}$ is $11\frac{1}{9}\%$ of what?

4. 8 is 10% of ——. $8\frac{1}{3}$ is 10% of ——.

(g) 89 is 10% of what? (h) 89.2 is 10% of what?

(3)

5. 12 is ——% of 24. $12\frac{1}{2}$ is ——% of 50.

6. 6 is ——% of 60. 6 is ——% of 54.

7. $3\frac{1}{2}$ is ——% of 7. $3\frac{1}{3}$ is ——% of 10.

8. 36 is ——% of 72. 72 is ——% of 144.

(1)

9. One per cent of \$700 is ——. 2% of \$700 =

10. One per cent of \$730 is ——. 2% of \$730 =

11. One per cent of \$732 is ——. 2% of \$732 =

(i) Find 3% of \$732. (j) Find 4% of \$732

(k) Find 5% of \$732. (l) Find 6% of \$732.

(m) Find 7% of \$320. (n) Find 7% of \$326.

(2)

12. Two dollars are 1 hundredth of \$200.

13. Two dollars are 1% of ——. \$4 are 2% of ——.

14. Six dollars are 3% of ——. \$8 are 4% of ——.

15. Five dollars are 1% of ——. \$10 are 2% of ——.

REVIEW.

1. All integral numbers are either prime or composite. 17 is ——. 31 is ——. 95 is ——. 242 is ——. 370 is ——.

2. Reduce each of the following improper fractions to a whole number or to a mixed number: $\frac{17}{4}$, $\frac{35}{6}$, $\frac{37}{9}$, $\frac{18}{7}$, $\frac{25}{6}$.

(a) $\frac{324}{5}$. (b) $\frac{185}{5}$. (c) $\frac{256}{4}$. (d) $\frac{391}{7}$.

3. Reduce each of the following to its lowest terms:

$\frac{8}{20}$ $\frac{9}{27}$ $\frac{12}{24}$ $\frac{15}{25}$ $\frac{11}{33}$ $\frac{12}{48}$ $\frac{10}{100}$

(e) $\frac{65}{125}$. (f) $\frac{39}{120}$. (g) $\frac{130}{170}$. (h) $\frac{72}{120}$.

4. Reduce the following to equivalent fractions having a common denominator:

$\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{2}$. (Change to ——.) $\frac{1}{5} =$ $\frac{1}{6} =$ $\frac{1}{2} =$

(i) $\frac{3}{5}$, $\frac{5}{6}$, $\frac{1}{2}$. (j) $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{2}$. (k) $\frac{3}{8}$, $\frac{2}{5}$, $\frac{1}{2}$.

5. If $\frac{1}{2}$ of a bushel of potatoes costs 20¢, $2\frac{1}{2}$ bushels will cost ——.

(l) If $\frac{1}{2}$ of a ton of straw costs $2.25, how much will 3.2 tons cost?

6. Change the following to tons: 4200 lb.

(m) 5750 lb. (n) 7320 lb. (o) 3150 lb.

7. Find the area and the perimeter of each of the following rectangular surfaces: 7 feet by 5 feet.

(p) 27 feet by 12 feet. (q) 15 inches by 15 inches.

8. If 2 gallons of molasses are worth 60¢, 3 gallons are worth —— cents.

(r) If 2 loads of brick are worth $12.60, how much are 3 loads worth?

MISCELLANEOUS PROBLEMS.

1. In a school there are 35 pupils; $\frac{3}{7}$ of the pupils are boys; —— —— of the pupils are girls; there are —— boys and —— girls.

(a) In a school there are 392 pupils; $\frac{3}{7}$ of the pupils are boys. How many girls are in the school?

2. If 3 melons cost 36¢, at the same rate 5 melons will cost —— cents.

(b) If 3 acres of land cost $525, how much will 5 acres cost at the same rate?

3. Harry exchanged 5 lb. of butter at 20¢ a pound for coffee at 25¢ a pound; he should receive —— pounds of coffee.

(c) Harry's father exchanged 6 cords of wood at $5.20 a cord for cedar posts at 20¢ each. How many posts should he receive?

4. $3+2+6+4+5+8+1+7+9+4+7+3+2=$

(d) $275+361+554+732+598+236+347+256=$

5. If Mark saves $4 a month, in 1 year he will save —— dollars.

(e) If Mark's father saves $21.50 a month, how much will he save in 1 year?

6. A common brick is 8 inches long, 4 in. wide, and 2 in. thick; it has two faces each of which is 4 in. by 8 in., two faces each of which is —— in. by —— in., and two faces each of which is —— in. by —— in.

(f) Find the sum of the areas of all the faces of a common brick.

(g) Change 674 inches to feet and inches.

SIMPLE NUMBERS.

1. Any exact integral divisor of a number (except the number itself and 1) is called a **factor** of the number. The factors of 6 are —— and ——. The factors of 10 are —— and ——.

2. A factor that is itself a prime number is called a **prime factor.** A factor that is itself a composite number is called a **composite factor.**

2 and 3 are —— factors of 24.
4 and 6 are —— factors of 24.

3. Every composite number may be resolved into prime factors.

The prime factors of 12 are 2, 2, and 3.
The prime factors of 18 are 2, 3, and 3.
The prime factors of 15 are —— and ——.
The prime factors of 14 are —— and ——.
The prime factors of 30 are ——, ——, and ——.

4. From the above it will be seen that a number is equal to the product of its prime factors. 3 and 7 are the prime factors of ——; 2, 2, and 7, of ——.

2)30	The prime factors of	2)50	The prime factors of
3)15	30 are 2, 3, and 5.	5)25	50 are 2, 5, and 5.
5	$2 \times 3 \times 5 = 30$.	5	$2 \times 5 \times 5 = 50$.

(a) What are the prime factors of 40? (b) Of 60? (c) Of 65? (d) Of 72? (e) Of 86? (f) Of 85?

Multiply.	Divide.	Divide.
(g) 724 by 28.	(h) 748 lb. by 32 lb.	(i) 1254 lb. by 9.
(j) 846 by 23.	(k) 834 lb. by 32 lb.	(l) 1046 lb. by 9.
(m) 637 by 25.	(n) 928 lb. by 32 lb.	(o) 1134 lb. by 9.
(p) 926 by 27.	(q) 796 lb. by 32 lb.	(r) 1341 lb. by 9.

COMMON FRACTIONS.

Reduce to improper fractions:

1. $7\frac{1}{2} = \frac{}{2}$. $8\frac{3}{4} = \frac{}{4}$. $9\frac{3}{5} = \frac{}{5}$. $7\frac{5}{6} = \frac{}{6}$.

(a) $27\frac{1}{2} =$ (b) $48\frac{3}{4} =$ (c) $75\frac{3}{5} =$

Reduce to whole or mixed numbers:

2. $\frac{17}{6} =$ $\frac{35}{8} =$ $\frac{39}{4} =$ $\frac{38}{3} =$ $\frac{45}{9} =$ $\frac{39}{5} =$

(d) $\frac{285}{5} =$ (e) $\frac{524}{7} =$ (f) $\frac{387}{6} =$

Reduce to equivalent fractions having a common denominator:

3. $\frac{1}{2}, \frac{1}{4}, \frac{1}{7}$. (Change to ——.) $\frac{1}{2} =$ $\frac{1}{4} =$ $\frac{1}{7} =$

(g) $\frac{1}{2}, \frac{3}{4}, \frac{5}{7}$. (h) $\frac{5}{6}, \frac{2}{3}, \frac{3}{8}$. (i) $\frac{3}{5}, \frac{3}{4}, \frac{1}{2}$.

4. Add $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{2}$. (Change to ——.)
(j) Find the sum of $86\frac{2}{5}$, $95\frac{3}{4}$, and $87\frac{1}{2}$.

5. From $\frac{5}{7}$ subtract $\frac{1}{4}$. (Change to ——.)
(k) Find the difference of $873\frac{6}{7}$ and $249\frac{3}{4}$.

6. Divide $\frac{3}{4}$ by $\frac{1}{3}$. This means, *find how many times $\frac{1}{3}$ is contained in $\frac{3}{4}$*. I can change fourths and thirds to ——ths. $\frac{3}{4} = \frac{}{12}$. $\frac{1}{3} = \frac{}{12}$. —— twelfths are contained in —— twelfths —— times. *Story*—James can husk a row of corn in $\frac{1}{3}$ of an hour; in $\frac{3}{4}$ of an hour he can husk —— and —— —— rows.

(l) $\frac{4}{5} \div \frac{1}{4}$. (m) $\frac{2}{3} \div \frac{1}{4}$. (n) $\frac{4}{5} \div \frac{1}{3}$.

(o) $5\frac{1}{3}$ acres)288 acres. This means, *find how many times $5\frac{1}{3}$ acres are contained in 288 acres.*

(p) 5)$625\frac{1}{2}$ acres. This means, *find 1 fifth of $625\frac{1}{2}$ acres. Story.*

DECIMAL FRACTIONS.

1. One hundredth of \$500 is ——.		.02 of \$500 =
2. One hundredth of \$540 is ——.		.02 of \$540 =
3. One hundredth of \$542 is ——.		.02 of \$542 =
4. .01 of \$600 =	.01 of \$60 =	.01 of \$660 =
5. .01 of \$.1 =	.01 of \$.5 =	.01 of \$.7 =
6. .01 of \$6.4 =	.01 of \$7.5 =	.01 of \$3.2 =
7. .01 of \$24.2 =	.01 of \$37.1 =	.01 of \$53.1 =

(a) Multiply \$374 by .03. This means, *find* 3 *hundredths of* \$374.

Operation.	*Explanation.*
\$374*	One hundredth of \$374 is ——.
.03	Three hundredths of \$374 are ——.
\$11.22	

NUMBER STORY.

If one acre of land is worth \$374,
1 hundredth of an acre of land is worth ——.
3 hundredths of an acre of land are worth ——.

(b) Multiply \$347 by .03. (c) \$537 × .04.

(d) Multiply \$24.6 by .03. (e) \$39.4 × .04.

(f) At \$875 an acre, how much will .04 of an acre of land cost?

(g) Add.	(h) Subtract.	(i) Multiply.	(j) Divide.	(k) Divide.
286.3	$146\frac{2}{5}$	356	\$.04)\$5.76†	4)\$5.76‡
$184\frac{4}{5}$	78.2	.05		

* The pupil should understand that he multiplies \$3.74 (not \$374) by 3, and should be taught to write the decimal point in the product *immediately after* writing the tenths' figure of the product.

† This means, *find how many times 4 hundredths are contained in 576 hundredths.*

‡ This means, *find 1 fourth of \$5.76.*

DENOMINATE NUMBERS.

1. 4000 lb. are —— tons.
2. 4200 lb. are —— tons.
3. 4400 lb. are —— tons.
4. 4600 lb. are —— tons.

(a) At $5.20 a ton, how much will 4600 lb. of coal cost?

Operation.	*Explanation.*
4600 lb. = 2.3 tons.	**One ton costs $5.20.**
$5.20	**1 tenth of a ton costs $.52.**
2.3	**3 tenths of a ton cost ——.**
$1.560	**2 tons cost ——.**
$10.40	**2.3 tons cost ——.**
$11.960	

Find the cost:

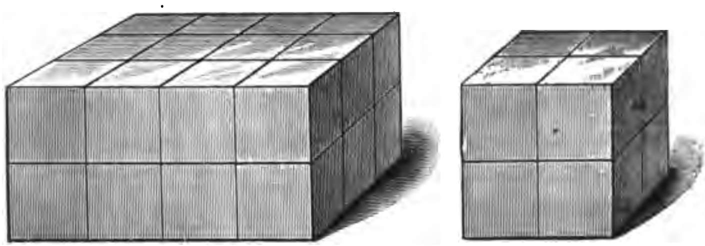
(b) 4800 lb. @ $5.40 per ton.
(c) 4200 lb. @ $5.60.
(d) 6200 lb. @ $5.35 per ton.
(e) 6400 lb. @ $5.25.
(f) 5600 lb. @ $5.70 per ton.
(g) 5400 lb. @ $5.80.

5. Three yd. are —— feet. 3 yd. are —— inches.
6. Three hr. are —— minutes. 3 yr. are —— months
7. Three min. are —— seconds. 3 wk. are —— days.
8. Three bu. are —— quarts. 3 gal. are —— quarts.
9. Three lb. are —— ounces. 3 pk. are —— pints.

(h) Change 148 yd. to feet.
(i) Change 15 yd. to inches.
(j) Change 24 hr. to min.
(k) Change 24 hr. to seconds.
(l) Change 35 yr. to mo.
(m) Change 52 wk. to days.
(n) Change 37 lb. to oz.
(o) Change 49 gal. to quarts.
(p) Change 43 bu. to qts.
(q) Change 56 pk. to pints.

(r) How many square inches in a 5-foot square?

MEASUREMENTS—RECTANGULAR SOLID



1. A rectangular solid has —— faces. Each face is a rectangle.

2. Some or all of the faces of a rectangular solid may be squares.

3. If each face of a rectangular solid is a square, the solid is called a ——.

4. If some of the faces of a rectangular solid are oblongs, the solid is not a cube.

5. The area of a rectangular surface 3 inches by 4 inches is —— square inches.

6. The solid content of a rectangular solid 3 in. by 4 in. by 2 in. is —— cubic inches.

7. The area of a rectangular surface 3 inches by 5 inches is —— square inches.

8. The solid content of a rectangular solid 3 in. by 5 in. by 2 in. is —— cubic inches.

9. The area of a rectangular surface 2 inches by 5 inches is —— square inches.

10. The solid content of a rectangular solid 2 in. by 5 in. by 3 in. is —— cubic inches.

RATIO AND PROPORTION.

1. One seventh of 28 is ——. 28 is $\frac{1}{7}$ of ——.
2. One fifth of 28 is ——. 28 is $\frac{1}{5}$ of ——.
3. One third of 28 is ——. 28 is $\frac{1}{3}$ of ——.
(a) Find 1 fourth of 387. (b) 387 is 1 fourth of what?
(c) Find 1 fifth of 724. (d) 724 is 1 fifth of what?

4. Five sixths of 60 are ——. 60 is $\frac{5}{6}$ of ——.
5. Four fifths of 40 are ——. 40 is $\frac{4}{5}$ of ——.
(e) Find $\frac{5}{6}$ of 420. (f) 420 is $\frac{5}{6}$ of what number?
(g) Find $\frac{4}{5}$ of 920. (h) 920 is $\frac{4}{5}$ of what number?

6. 16 is —— —— of 24. 24 is —— —— of 16.
7. 24 is —— —— of 32. 32 is —— —— of 24.
8. 32 is —— —— of 40. 40 is —— —— of 32.
9. 40 is —— —— of 48. 48 is —— —— of 40.

10. Five sixths of 30 are 1 third of ——.
11. One sixth of 48 is 2 thirds of ——.
(i) Five sixths of 366 are 1 half of what number?
(j) One sixth of 852 is 2 thirds of what number?

12. Twelve is —— —— of 9, or —— and —— —— times 9. A man can earn —— and —— —— times as much in 12 days as he can earn in 9 days. If he can earn $24 in 9 days, in 12 days he can earn —— dollars.

(k) If a man can earn $840 in 9 months, how many dollars can he earn in 12 months.

13. If 9 lb. of nails are worth 33 cents, 12 lb. are worth —— cents.

(l) If 9 cords of wood are worth $42.75, how much are 12 cords worth?

PERCENTAGE.

$66\frac{2}{3}$ per cent = $.66\frac{2}{3} = \frac{2}{3}$.
75 per cent = $.75 = \frac{3}{4}$.

	(1)	*(2)*
1.	$66\frac{2}{3}$ per cent of 12 =	12 is $66\frac{2}{3}\%$ of ——.
2.	75 per cent of 12 =	12 is 75% of ——.
3.	$66\frac{2}{3}$ per cent of 24 =	24 is $66\frac{2}{3}\%$ of ——.
4.	75 per cent of 24 =	24 is 75% of ——.
5.	50 per cent of 5 =	5 is 50% of ——.
6.	25 per cent of 13 =	13 is 25% of ——.
7.	$33\frac{1}{3}$ per cent of 10 =	10 is $33\frac{1}{3}\%$ of ——.
8.	20 per cent of 16 =	16 is 20% of ——.
9.	$16\frac{2}{3}$ per cent of 25 =	25 is $16\frac{2}{3}\%$ of ——.
10.	$14\frac{2}{7}$ per cent of 22 =	22 is $14\frac{2}{7}\%$ of ——.
11.	$12\frac{1}{2}$ per cent of 33 =	33 is $12\frac{1}{2}\%$ of ——.
12.	$11\frac{1}{9}$ per cent of 19 =	19 is $11\frac{1}{9}\%$ of ——.
13.	10 per cent of 21 =	21 is 10% of ——.

	(3)	*(3)*
14.	4 is —— per cent of 24.	7 is ——% of 35.
15.	4 is —— per cent of 28.	7 is ——% of 28.
16.	4 is —— per cent of 32.	7 is ——% of 21.
17.	4 is —— per cent of 36.	7 is ——% of 14.
18.	4 is —— per cent of 40.	8 is ——% of 12.
19.	9 is —— per cent of 12.	12 is ——% of 18.

20. Twenty-five per cent of 80 sheep are —— sheep.
21. Twenty-one sheep are 25% of —— sheep.
22. Twenty-five sheep are —— per cent of 75 sheep.
23. Forty sheep are —— per cent of 60 sheep.

PERCENTAGE.

1. 75 per cent of 36 = 66$\frac{2}{3}$ per cent of 36 =

(a) Find 75% of 796. (b) Find 66$\frac{2}{3}$% of 822.

(c) Find 3% of \$375. This means, *find* 3 *hundredths of* 375.

Operation.	*Explanation.*
\$375	One per cent of \$375 = \$3.75.
.03	Three per cent of \$375 = \$11.25.
\$11.25	

NUMBER STORY.

Mr. A collected money for Mr. B. It was agreed that Mr. A should keep 3% of all he might collect to pay him for his trouble. He collected \$375; he should keep \$11.25 and "pay over" the remainder to Mr. B.

(d) How much should Mr. A "pay over" to Mr. B?

(e) Find 7% of \$465. (f) Find 9% of \$324.

(g) Find 3% of \$422. (h) Find 7% of \$538.

2. 36 is 75% of ——. 36 is 66$\frac{2}{3}$% of ——.

(i) 453 is 75% of what? (j) 562 is 66$\frac{2}{3}$% of what?

(k) Forty-eight dollars are 3% of what? This means, \$48 *are* 3 *hundredths of how many dollars?*

Operation.	*Explanation.*
$\$48 \div 3 = \$16.$	One hundredth of the unknown number is
$\$16 \times 100 = \$1600.$	\$16. 100 hundredths (the whole) are ——.

NUMBER STORY.

A lawyer collected some money for 3% of the amount collected; his share (commission) was \$48; the amount collected was \$1600.

(l) How much should the lawyer pay over to the man for whom he collected the money?

REVIEW.

1. The prime factors of 45 are ——, ——, and ——.

(a) What are the prime factors of 100? (b) Of 125?

2. Two, 3, and 5 are the prime factors of ——.

(c) Of what number are 3, 3, 2, and 7 the prime factors?

3. Reduce $\frac{12}{18}$ to its lowest terms. $\frac{12}{18} =$

(d) Reduce $\frac{125}{240}$ and $\frac{48}{144}$ to their lowest terms.

4. Reduce $8\frac{2}{3}$ to an improper fraction. $8\frac{2}{3} =$

(e) Reduce $57\frac{2}{3}$ and $72\frac{3}{4}$ to improper fractions.

5. Reduce $\frac{38}{7}$ to a mixed number. $\frac{38}{7} =$

(f) Reduce $\frac{324}{7}$ and $\frac{543}{7}$ to mixed numbers.

6. Reduce $\frac{2}{9}$ and $\frac{3}{4}$ to equivalent fractions having a common denominator. $\frac{2}{9} =$ $\frac{3}{4} =$

(g) Reduce $\frac{7}{12}$ and $\frac{5}{9}$ to equivalent fractions having a common denominator.

7. Multiply 60 by .7. This means, *find* 7 *tenths of* 60. One tenth of 60 = ——; 7 tenths of 60 = ——.

(h) Multiply $537 by .07. This means, *find* 7 *hundredths of* $537. (See page 43.)

8. At $1 per ton, 4240 lb. of coal cost ——.

(i) Find the cost of 4240 lb. of coal at $7 per ton.

9. The volume of a rectangular solid 5 inches by 4 inches by 2 inches is —— cubic inches.

(j) Find the volume of a rectangular solid 9 inches by 7 inches by 7 inches.

10. If 9 lb. of tea are worth $6, 12 lb. are worth ——.

(k) If 9 acres of land are worth $346.50, how much are 12 acres worth at the same rate? (12 is $1\frac{1}{3}$ times 9.)

MISCELLANEOUS PROBLEMS.

1. From June, 1881, to June, 1899, it is —— years.

(a) How many years from Aug., 1492, to Aug., 1897?

2. A man sold a horse for one hundred twenty dollars; this was seventeen dollars and twenty cents more than the horse cost him; the horse cost him ——.

(b) A man sold a farm for fourteen thousand seven hundred fifty dollars; this was eight hundred seventy-five dollars more than he paid for the farm. How much did the farm cost him?

3. From 5460 lb. of coal there were sold 2 tons. —— pounds remained.

(c) From 18940 lb. of coal there were sold $8\frac{3}{4}$ tons. How many pounds were left?

4. From Mendota to Galesburg it is 80 miles; a train going 30 miles an hour, that leaves Mendota at 8:30, should arrive at Galesburg at ——.

(d) From Chicago to Denver it is about 1000 miles. If a train leaves Chicago for Denver at 9 o'clock Monday morning and goes at the rate of 30 miles an hour, when will it arrive at Denver?

5. If butter is 25¢ a pound and coffee is 30¢ a pound, 3 lb. butter will pay for —— lb. coffee.

(e) Fifteen and one half pounds of butter at 24¢ a pound, will pay for how many pounds of coffee at 32¢ a pound?

6. If $\frac{3}{4}$ of a yard of lace is worth 15¢, $2\frac{1}{4}$ yards are worth —— cents.

(f) If $\frac{3}{4}$ of a yard of cloth costs $1.05, how much will $27\frac{1}{2}$ yards cost?

SIMPLE NUMBERS.*

1. Four, 6, 8, 10, 12, etc., are **multiples** of 2.

2. Six, 9, 12, 15, 18, etc., are multiples of 3.

3. Twelve is a multiple of 2. 12 is also a multiple of 3, and of 4, and of 6.

4. Twelve is a **common multiple** of 2, 3, ——, and ——.

5. Fifteen is a common multiple of —— and ——.

6. Common multiples of 4 and 6 are, 12, 24, ——, etc. The **least common multiple** of 4 and 6 is 12.

7. Common multiples of 6 and 8 are, 24, 48, ——, etc. The least common multiple of 6 and 8 is ——.

8. Common multiples of 8 and 12 are, ——, ——, etc. The least common multiple of 8 and 12 is ——.

9. Common multiples of 6 and 9 are, ——, ——, etc. The least common multiple of 6 and 9 is ——.

10. The prime factors of 18 are ——, ——, and ——.

11. The prime factors of 70 are ——, ——, and ——.

(a) What are the prime factors of 140. (b) Of 160? (c) Of 135? (d) Of 175? (e) Of 250? (f) Of 225?

Multiply.	Divide.	Divide.
(g) 635 by 53.	(h) 944 lb. by 56 lb.†	(i) 1536 lb. by 12.†
(j) 728 by 54.	(k) 846 lb. by 56 lb.	(l) 1445 lb. by 12.
(m) 834 by 52.	(n) 739 lb. by 56 lb.	(o) 1374 lb. by 12.
(p) 947 by 51.	(q) 873 lb. by 56 lb.	(r) 1653 lb. by 12.
(s) 836 by 55.	(t) 965 lb. by 56 lb.	(u) 1738 lb. by 12.

* Do much oral work in preparation for this page. *By using these terms*, make the pupil as familiar with *multiple*, *common multiple*, and *least common multiple*, as he is with *house*, *schoolhouse*, and *stone schoolhouse*.

† Tell the meaning. Tell a number story.

COMMON FRACTIONS.

L. c. m. is the abbreviation for *least common multiple.*

1. Add $\frac{5}{8}$ and $\frac{11}{12}$.

L. c. m. of 8 and 12 is ——.

$\frac{5}{8} = \frac{\quad}{24}$. $\frac{11}{12} = \frac{\quad}{24}$.

$\frac{15}{24} + \frac{22}{24} = \frac{\quad}{24} = 1\frac{\quad}{24}$.

2. From $\frac{8}{9}$ subtract $\frac{1}{6}$.

L. c. m. of 9 and 6 is ——.

$\frac{8}{9} = \frac{\quad}{18}$. $\frac{1}{6} = \frac{\quad}{18}$.

$\frac{16}{18} - \frac{3}{18} = \frac{\quad}{18}$.

3. Divide $\frac{11}{12}$ by $\frac{1}{8}$.

L. c. m. of 12 and 8 is ——.

$\frac{11}{12} = \frac{\quad}{24}$. $\frac{1}{8} = \frac{\quad}{24}$.

$\frac{22}{24} \div \frac{3}{24} =$

4. Multiply 12 by $2\frac{3}{4}$.
This means ————————.
$2\frac{3}{4}$ times 12 = ——. *Story.*

5. Multiply 12 by $\frac{3}{4}$.
This means ————————.
$\frac{3}{4}$ of 12 = ——. *Story.*

(a) $\frac{5}{6} + \frac{8}{9}$.
(b) $\frac{3}{4} + \frac{5}{6}$.
(c) $\frac{5}{6} + \frac{3}{8}$.
(d) $\frac{3}{5} + \frac{2}{7}$.

(e) $\frac{5}{6} - \frac{1}{4}$.
(f) $\frac{7}{8} - \frac{1}{6}$.
(g) $\frac{7}{12} - \frac{3}{8}$.
(h) $\frac{5}{7} - \frac{2}{5}$.

(i) $\frac{7}{8} \div \frac{1}{8}$.
(j) $\frac{8}{9} \div \frac{1}{6}$.
(k) $\frac{5}{8} \div \frac{1}{6}$.
(l) $\frac{5}{7} \div \frac{1}{5}$.

(m) $48 \times 2\frac{3}{4}$.
(n) $252 \times 3\frac{3}{4}$.
(o) $175 \times 2\frac{1}{5}$.

(p) $96 \times \frac{3}{4}$.
(q) $84 \times \frac{2}{3}$.
(r) $95 \times \frac{3}{5}$.

(s) $6\frac{1}{4}$ dollars)375 dollars. This means, *find how many times \$$6\frac{1}{4}$ are contained in \$375.*
(Change $6\frac{1}{4}$ and 375 to fourths.) *Story.*

(t) 6)756 dollars. This means, *find 1 sixth of \$756.* *Story.*

DECIMAL FRACTIONS.

1. One tenth of $432 is ——. .2 of $432 =
2. One hundredth of $432 is ——. .02 of $432 =

(a) Multiply $432 by .25. This means, *find* 2 *tenths of* $432, *plus* 5 *hundredths of* $432.

Operation.	*Explanation.*
$432	One hundredth of $432 is ——.
.25	5 hundredths of $432 are ——.
$21.60*	One tenth of $432 is ——.
$86.4 †	2 tenths of $432 are ——.
$108.00	$21.60 + $86.4 =

NUMBER STORY.

If 1 acre of land is worth $432,
1 hundredth of an acre is worth ——.
5 hundredths of an acre are worth ——.
1 tenth of an acre is worth ——.
2 tenths of an acre are worth ——.
25 hundredths of an acre are worth ——.

(b) Multiply $325 by .23. (c) $482 × .32.
(d) Multiply $278 by .43. (e) $356 × .36.
(f) Multiply $536 by .07. (g) $351 × .7.
(h) Multiply $249 by 2.6. (i) $426 × .45.

(j)	(k)	(l)	(m)	(n)
Add.	Subtract.	Multiply.	Divide.	Divide.
8.75	$56\frac{1}{8}$	675	$.05)$6.	5)$6.
7.324	12.9	.36		

* The pupil should understand that he multiplies $4.32 (not $432) by 5, and should be taught to write the decimal point in the partial product *immediately after writing the tenths' figure*, 6, of the partial product.

† The pupil should understand that he multiplies $43.2 (not $432) by 2, and should be taught to write the decimal point in the partial product *immediately after writing the tenths' figure*, 4, of the partial product.

DENOMINATE NUMBERS.

1. *Sixteen and one half feet are* 1 *rod.*

2. *Three hundred twenty rods are* 1 *mile.*

3. One rod is —— and —— —— yards.*

4. Two rods are —— feet. 4 rods are —— feet.

5. Six rods are —— feet. 10 rods are —— feet.

6. One mile is —— rods. $\frac{1}{2}$ mile is —— rods.

7. $\frac{1}{4}$ mile is —— rods. $\frac{1}{8}$ mile is —— rods.

8. The telegraph poles along the line of a railroad are usually ten rods apart; they are —— feet apart. From the first telegraph pole to the third it is —— rods; from the first to the fifth it is —— rods.

(a) How far is it from the first telegraph pole to the thirty-third?

9. The roads in the country are usually 4 rods wide. A 4-rod road is —— feet wide.

10. A 100-foot street is —— rods and —— foot wide.

11. From the schoolhouse to ——, it is —— —— of a mile, or —— rods.

(b) Change 5 mi. to rods. (c) Change 40 rd. to feet.

(d) Change 28 rd. to yards. (e) Change 7 mi. to rods.

REVIEW.

12. When hay is $12 a ton, 3000 lb. cost ——.

13. When oats are 30¢ a bushel, 96 lb. cost ——.

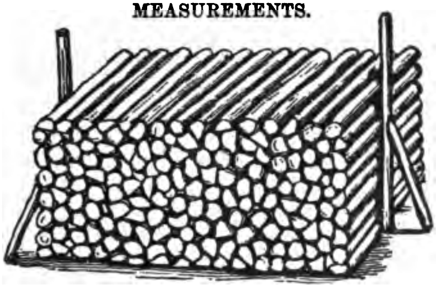
14. When wheat is 80¢ a bushel, 180 lb. cost ——.

(f) When corn (not shelled) is 45¢ a bushel, how much will 3640 lb. cost?

(g) When coal is $5.40 a ton, how much will 2600 lb. cost?

* Discover by actual measurement the number of yards in a rod.

MEASUREMENTS.



1. *A pile of wood* 8 *feet long,* 4 *feet wide, and* 4 *feet high (or its equivalent) is called a cord.*

(a) How many cubic feet in a rectangular solid 8 feet by 4 feet by 4 feet?

2. A pile of wood 16 feet long, 4 feet wide, and 4 feet high contains —— cords.

(b) How many cubic feet in 2 cords?

3. A pile of wood 8 feet long, 4 feet wide, and 6 feet high contains —— cords.

(c) How many cubic feet in $1\frac{1}{2}$ cords? (d) In $\frac{3}{4}$ of a cord?

(e) How many cubic feet in $5\frac{1}{4}$ cords? (f) In $6\frac{3}{4}$ cords?

REVIEW.

4. A 1-inch square = —— —— of a 2-inch square.

5. A 1-inch cube = —— —— of a 2-inch cube.

6. A 2-inch square = —— —— of a 3-inch square.

7. A 2-inch cube = —— —— of a 3-inch cube.

(g) Find the area of a surface 12 ft. by 24 ft.*

(h) Find the volume of a solid 12 ft. by 8 ft. by 4 ft.†

*Take care that the pupil understands that he does not (cannot) multiply 24 ft by 12 ft., but 24 square feet by 12, or 12 square feet by 24.

†In this problem the pupil multiplies 12 cubic feet by 8, and the product thus obtained by 4.

RATIO AND PROPORTION.

1. One eighth of 24 is ——. 24 is $\frac{1}{8}$ of ——.

2. One seventh of 24 is ——. 24 is $\frac{1}{7}$ of ——.

3. One fifth of 24 is ——. 24 is $\frac{1}{5}$ of ——.

(a) Find $\frac{1}{6}$ of 49.26. (b) 49.26 is $\frac{1}{6}$ of what?

(c) Find $\frac{1}{7}$ of $9.31. (d) $9.31 is $\frac{1}{7}$ of what?

4. Four sevenths of 56 are ——. 56 is $\frac{4}{7}$ of ——.

5. Three sevenths of 42 are ——. 42 is $\frac{3}{7}$ of ——.

(e) Find $\frac{4}{7}$ of 875. (f) 876 is $\frac{4}{7}$ of what number?

(g) Find $\frac{3}{7}$ of 924. (h) 927 is $\frac{3}{7}$ of what number?

6. 18 is —— —— of 27. 27 is —— —— of 18.

7. 27 is —— —— of 36. 36 is —— —— of 27.

8. 45 is —— —— of 54. 54 is —— —— of 45.

9. Three sevenths of 28 are 2 thirds of ——.

10. Six sevenths of 28 are 3 fourths of ——.

(i) Three sevenths of 364 are 1 half of what number?

(j) One seventh of 434 is 2 thirds of what number?

11. Twenty is —— —— of 25. If 25 bags of salt are worth $15, 20 bags of salt are worth —— dollars.

(k) If 25 acres of land are worth $640, how much are 20 acres worth at the same rate?

12. Twenty-five is —— —— of 20, or —— and —— times 20. If 20 bushels of apples are worth $12, 25 bushels of apples are worth —— dollars.

(l) If 20 barrels of salt are worth $22.40, how much are 25 barrels of salt worth at the same rate?

13. If 12 qt. of nuts are worth 40¢, 9 qt. of nuts are worth —— cents.

PERCENTAGE.

$$40 \text{ per cent} = .40 = \tfrac{2}{5}.$$
$$60 \text{ per cent} = .60 = \tfrac{3}{5}.$$

	(1)	(2)
1.	40 per cent of 50 =	50 is 40% of ——.
2.	60 per cent of 15 =	15 is 60% of ——.
3.	40 per cent of 45 =	16 is 40% of ——.
4.	60 per cent of 45 =	18 is 60% of ——.
5.	75 per cent of 28 =	18 is 75% of ——.
6.	$66\frac{2}{3}$ per cent of 21 =	18 is $66\frac{2}{3}$% of ——.
7.	40 per cent of 60 =	20 is 40% of ——.
8.	60 per cent of 60 =	24 is 60% of ——.
9.	75 per cent of 32 =	27 is 75% of ——.
10.	$66\frac{2}{3}$ per cent of 36 =	22 is $66\frac{2}{3}$% of ——.

	(3)	(3)
11.	8 is —— per cent of 32.	8 is ——% of 24.
12.	8 is —— per cent of 40.	8 is ——% of 16.
13.	8 is —— per cent of 64.	8 is ——% of 56.
14.	8 is —— per cent of 48.	8 is ——% of 72.
15.	8 is —— per cent of 80.	8 is ——% of 20.
16.	9 is —— per cent of 15.	18 is ——% of 27.
17.	18 is —— per cent of 24.	$4\frac{1}{3}$ is ——% of 13.

18. Russel earned 60¢; he spent 10% of his money for a tablet and 20% of it for a book; the tablet cost —— cents and the book cost —— cents.

19. Ten per cent of the sheep in a certain flock were black; there were 8 black sheep; in all there were —— sheep.

PERCENTAGE.

(1)

1. 40 per cent of 55 = 60 per cent of 55 =

(a) Find 40% of 575. (b) Find 60% of 365.

(c) Find 75% of 676. (d) Find $66\frac{2}{3}$% of 591.

(e) Find 3% of $254. (f) Find 7% of $254.

(g) A lawyer's commission for collecting money was 7%; he collected $635. How much of the money should he keep and how much should he "pay over" to the man for whom he collected the money?

(2)

2. 18 is 40% of ——. 18 is 60% of ——.

(h) 346 is 40% of what? (i) 345 is 60% of what?

(j) 534 is 75% of what? (k) 534 is $66\frac{2}{3}$% of what?

(l) $36 is 3% of what? (m) $84 is 7% of what?

(n) A lawyer's commission for collecting money was 7%; his commission amounted to $63. How much did he collect and how much should he "pay over" to the man for whom he collected the money?

(3)

3. 18 is —— per cent of 45. 33 is ——% of 55.

4. 33 is —— per cent of 44. 18 is ——% of 27.

(o) Eighteen dollars are what per cent of $600? This means, *find how many hundredths of* $600, $18 *are.*

Operation.	*Explanation.*
$600 ÷ 100 = $6.	One per cent of $600 is ——.
$18 ÷ $6 = 3 times.	$18 are as many per cent of $600 as $6 are contained times in $18.
$18 are 3% of $600.	

5. $84 are ——% of $1200. $72 are ——% of $800.

REVIEW.

1. The prime factors of 63 are ——, ——, and ——.

(a) What are the prime factors of 215? (b) Of 470?

2. Three, 3, 3, and 2 are the prime factors of ——.

(c) Of what number are 3, 5, 2, and 23 the prime factors?

3. Add $\frac{1}{8}$ and $\frac{2}{3}$. The l. c. m. of 8 and 3 is ——.

(d) Add $\frac{7}{9}$, $\frac{5}{12}$, and $\frac{2}{3}$. The l. c. m. of 9, 12, and 3 is ——.

(e) Divide $\frac{8}{9}$ by $\frac{1}{12}$. The l. c. m. of 9 and 12 is ——.

4. Multiply $324 by $\frac{1}{4}$. This means ——.

(f) Multiply $324 by .25.

5. One half of a rod is —— and —— —— feet.

(g) Twenty-six and one half rods are how many feet?

6. Three eighths of a mile are —— rods.

(h) How many rods in $7\frac{3}{8}$ miles? (i) In $4\frac{3}{4}$ miles?

7. Two rods are —— yards. 3 rd. are —— yards.

(j) How many yards in 42 rods? (k) In 64 rods?

8. When wheat is 70¢ a bushel, 240 lb. cost ——.

(l) When wheat is 90¢ a bushel, how much will one ton of wheat cost?

9. A pile of wood 32 feet long, 4 feet wide, and 4 feet high contains —— cords.

(m) How many cu. ft. in a pile of wood 32 ft. by 4 ft. by 2 ft.?

10. If 12 lb. of sugar are worth 45¢, at the same rate 8 lb. are worth —— cents.

(n) If 12 boxes of soap are worth $22.50, how much are 8 boxes worth at the same rate?

MISCELLANEOUS PROBLEMS.

1. The perimeter of a rectangular surface 7 feet by 9 feet is —— feet.

(a) How many rods of fence are required to enclose a rectangular field 38 rods by 54 rods?

2. If the sun rises at 7 o'clock and sets at 4:30, from sunrise to sunset it is —— hours and —— minutes.

(b) If the sun rises at 7:14 and sets at 4:28, how long is it from sunrise to sunset?

3. A man bought 16 dozen eggs at 10¢ a dozen; he lost 25 per cent of them by decay; he sold the remainder at 12¢ a dozen; he lost —— cents.

(c) A man bought 360 dozen eggs at 9¢ a dozen; he lost 25% of them by decay; he sold the remainder at 11¢ a dozen. How much money did he lose?

4. From midnight Monday to midnight Wednesday it is —— hours.

(d) How many hours in a week? (e) In the month of January?

5. At 45¢ each, 3 copies of "Robinson Crusoe" will cost —— dollar and —— cents; for $1.80 I can buy —— copies.

(f) How much will 7 copies of "Robinson Crusoe" cost?

(g) How many copies of "Robinson Crusoe" can I buy for $10.35? (h) For $4.95?

(i) Add four thousand three hundred twenty-four and twenty-five thousandths, and seven hundred forty-six and thirty-four hundredths, and seventy-five and eight tenths.

(j) From forty-five hundred subtract forty-five hundredths.

SIMPLE NUMBERS.

1. Ten times 60 are ——. 10 times 40 are ——.
2. Ten times 46 are ——. 10 times 25 are ——.
3. Ten times 300 are ——. 10 times 325 are ——.

4. *Ten times 2 times a number are —— times the number.*

(a) Multiply 347 by 20.

Operation.	*Explanation.*
347	Two times 347 are ——.
20	Ten times 694 are ——.
6940	Ten times 2 times (20 times) 347 are ——.

(b) Multiply 564 by 30. (c) Multiply 468 by 40.

(d) Multiply 735 by 50. (e) Multiply 642 by 60.

5. *One hundred times 2 times a number are —— times the number.*

(f) Multiply 86 by 200. (g) Multiply 94 by 300.

(h) Multiply 75 by 400. (i) Multiply 48 by 700.

REVIEW.

6. The prime factors of 48 are ——, ——, ——, ——, and ——.

(j) What are the prime factors of 290? (k) Of 430?

Multiply.	Divide.	Divide.
(l) 347 by 70.	(m) 8540 lb. by 70 lb.*	(n) 1480 lb. by 20.†
(o) 575 by 70.	(p) 9470 lb. by 70 lb.	(q) 1350 lb. by 20.

*This means, find how many times 70 lb. are contained in 8540 lb. 70 lb. are contained in 8540 lb. —— times. *Story—In 8540 lb. of ear corn there are —— bushels.*

†This means, find 1 twentieth of 1480 lb. 1 twentieth of 1480 lb. is —— lb. *Story—I wish to put 1480 lb. of wheat into 20 sacks; I must put —— lb. in each sack.*

COMMON FRACTIONS.

1. Add $\frac{1}{10}$ and $\frac{1}{12}$.*	(a) $375\frac{3}{10}+256\frac{5}{12}$.	
2. From $7\frac{1}{5}$ subtract $2\frac{4}{5}$.†	(b) $3467\frac{1}{5}-1282\frac{4}{5}$.	
3. Divide $\frac{3}{5}$ by $\frac{1}{10}$.	(c) $3\frac{3}{5}\div\frac{1}{10}$.	| (9)
4. Divide 8 by $\frac{2}{3}$.	(d) $28\div\frac{2}{3}$.	(1)
5. Divide $3\frac{3}{4}$ by $1\frac{1}{4}$.	(e) $47\frac{1}{4}\div1\frac{3}{4}$.	(11)
6. Divide $21\frac{1}{4}$ ft. by 3.	(f) $87\frac{1}{4}$ ft. $\div$ 3.	(21)
7. Divide $1\frac{3}{4}$ ft. by 3.	(g) $5\frac{3}{4}$ ft. $\div$ 3.	(15)
8. Divide $13\frac{3}{4}$ ft by 3.	(h) $745\frac{3}{4}$ ft. $\div$ 3.	(21)
9. Multiply 20¢ by $2\frac{3}{4}$.‡	(i) $96\times2\frac{3}{4}$.	(16)
10. Multiply 20¢ by $\frac{3}{4}$.	(j) $124\times\frac{3}{4}$.	(8)
11. Multiply $\$\frac{3}{5}$ by 6.	(k) $64\frac{3}{5}\times6$.	(19)
12. Multiply $\$\frac{1}{2}$ by $\frac{1}{2}$.	(l) $\$\frac{1}{2}\times26\frac{1}{2}$.	(13)
13. Multiply $\$1\frac{1}{2}$ by $\frac{1}{2}$.	(m) $\$27\frac{1}{2}\times\frac{1}{2}$.	(13)
14. Multiply $\$6\frac{1}{2}$ by $2\frac{1}{2}$.§	(n) $\$86\frac{1}{2}\times2\frac{1}{2}$.	(22)

15. At $\$4\frac{1}{2}$ per ton, 2 tons of coal cost —— dollars; $\frac{1}{2}$ a ton costs ———; $2\frac{1}{2}$ tons cost ———.

16. At $\$8\frac{1}{2}$ per ton, $2\frac{1}{2}$ tons of hay cost ———.

* The pupil is expected to find by trial that the l. c. m. of 10 and 12 is ———.

† Take 1 from the 7, change it to fifths and add it to the $\frac{1}{5}$.

‡ In problems 9 to 13 inclusive, think of the number to be multiplied as the price per yard; this will suggest the following:

Problem 9, $2\frac{3}{4}$ yd. of ribbon at 20¢ a yard.
Problem 10, $\frac{3}{4}$ yd. of ribbon at 20¢ a yard.
Problem 11, 6 yd. of ribbon at $\$\frac{3}{5}$ a yard.
Problem 12, $\frac{1}{2}$ yd. of ribbon at $\$\frac{1}{2}$ a yard.
Problem 13, $\frac{1}{2}$ yd. of ribbon at $\$1\frac{1}{2}$ a yard.

§ This means, 2 times $\$6\frac{1}{2}$ plus $\frac{1}{2}$ of $\$6\frac{1}{2}$. 2 times $\$6\frac{1}{2}$ are ——— dollars. $\frac{1}{2}$ of $\$6\frac{1}{2}$ is ——— dollars. $\$13+\$3\frac{1}{4}=$ ——— dollars. *Story*—At $\$6\frac{1}{2}$ a ton, $2\frac{1}{2}$ tons of coal will cost ———. | See note, page 6.

DECIMAL FRACTIONS.

(a) Multiply $546 by 3.24. This means, *find* 3 *times* $546, *plus* 2 *tenths of* $546, *plus* 4 *hundredths of* $546.

Operation.	*Explanation.*
$546	One hundredth of $546 is ——.
3.24	4 hundredths of $546 are ——.
$21.84	One tenth of $546 is ——.
$109.2	2 tenths of $546 are ——.
$1638	3 times $546 are ——.
$1769.04	$21.84 + $109.2 + $1638 =

NUMBER STORY.

If one acre of land is worth $546,
1 hundredth of an acre is worth ——.
4 hundredths of an acre are worth ——.
1 tenth of an acre is worth ——.
2 tenths of an acre are worth ——.
3 acres are worth ——.
3.24 acres are worth ——.

(b) Multiply $437 by 2.36; (c) $375 by 5.27.

(d) Multiply $352 by 6.21; (e) $284 by 7.32.

(f) Multiply four hundred seventy-three dollars by four and thirty-five hundredths.

(g) When oil meal is $28 a ton, how much will 4680 lb. cost? (4680 lb. = —— tons.)

(h)	(i)	(j)	(k)	(l)
Add.	Subtract.	Multiply.	Divide.	Divide.
.886	8.744	8.2*	$.5)$31	5)$31.
.075	.956	.43		

*One hundredth of 8.2 is .082. The pupil should write the decimal point in the first partial product *immediately after writing the tenths' figure of the partial product.*

DENOMINATE NUMBERS.

No. 186 **CITY SCALE.**

Load of *Hay,* *Dec. 31,* 1896.

From *Clyde H. Hall* *To* *Fay D. Winslow.*

Gross Weight, 4760 *lb.*

Tare, 1960 *lb.*

Net Weight, 2800 *lb.*

Clarence Marshall, *Weigher.*

(a) Find the value of the load of hay, ticket No. 186, at $8.25 a ton.

Find the value of the following:

	Commodity.	Gross Weight.	Tare.	Price per Ton.
(b)	Hay.	5610	1870	$12.50
(c)	Coal.	6380	2140	$6.50
(d)	Bran.	3340	1560	$13.50

---

1. One hundred feet are —— rods and —— foot.
2. Two hundred feet are —— rods and —— feet.
3. Three hundred feet are —— rods and —— feet.
4. Four hundred feet are —— rods and —— feet.
5. Five hundred feet are —— rods and —— feet.

(e) Change 8 miles to rods. (f) Change 30 rods to feet.
(g) Change 36 rods to yards. (h) Change 2160 lb. oats to bu.
(i) Change 5240 lb. to tons. (j) Change 274 ft. to inches.

MEASUREMENTS.

1. *One hundred sixty square rods are one acre.* A piece of land 1 rod wide and 160 rods long is one ——.

2. A piece of land $\frac{1}{2}$ of a mile long and 1 rod wide is —— ——.

3. Land 2 rd. wide and —— rd. long is one acre.

4. Land 4 rd. wide and —— rd. long is one acre.

5. Land 8 rd. wide and —— rd. long is one acre.

6. Land 10 rd. wide and —— rd. long is one acre.

7. Land 5 rd. wide and —— rd. long is one acre.

8. Land 4 rods by 4 rods is —— ——th of an acre.

9. Land 4 rods by 10 rods is —— —— of an acre.

10. Land 8 rods by 10 rods is —— —— of an acre.

11. Land 1 rod wide and 1 mile long is —— acres.

12. Land 2 rods wide and 1 mile long is —— acres.

13. Land 66 ft. wide and 1 mile long is —— acres.

14. Land 5 rods by 16 rods is —— square rods.

(a) How many square rods in a rectangular piece of land 28 rd. by 36 rd.?

REVIEW.

(b) Find the area of a rectangular surface 18 ft. by 26 ft.*

(c) Find the volume of a rectangular solid 15 ft. by 8 ft. by 23 ft.†

15. How many cords in a pile of wood 4 ft. wide, 4 ft. high, and 36 ft. long?

(d) How many cords in a pile of wood 4 ft. high, 4 ft. wide, and 276 ft. long?

(e) How many cords in a pile of wood 4 ft. wide, 4 ft. high, and as long as your schoolroom?

* See foot-note (*), page 55. † See foot-note (†), page 55.

RATIO AND PROPORTION.

1. One ninth of 36 is ——. 36 is $\frac{1}{9}$ of ——.
2. One eighth of 36 is ——. 36 is $\frac{1}{8}$ of ——.
3. One seventh of 36 is ——. 36 is $\frac{1}{7}$ of ——.

(a) Find 1 eighth of 353.6. (b) 353.6 is $\frac{1}{8}$ of what?
(c) Find 1 fifth of 1685. (d) 1685 is $\frac{1}{5}$ of what?

4. Five eighths of 80 are ——. 80 is $\frac{5}{8}$ of ——.
5. Three fifths of 60 are ——. 60 is $\frac{3}{5}$ of ——.

(e) Find $\frac{5}{8}$ of 680. (f) 680 is $\frac{5}{8}$ of what number?
(g) Find $\frac{3}{5}$ of 435. (h) 435 is $\frac{3}{5}$ of what number?

6. 22 is —— —— of 33. 33 is —— —— of 22.
7. 33 is —— —— of 55. 55 is —— —— of 33.
8. 55 is —— —— of 99. 99 is —— —— of 55.
9. 44 is —— —— of 77. 77 is —— —— of 44.

10. Three eighths of 32 are 2 thirds of ——.
11. Five eighths of 32 are 2 thirds of ——.

(i) Three eighths of 528 are 2 thirds of what number?
(j) Five eighths of 528 are 2 thirds of what number?

12. Twenty is —— —— of 16, or —— and —— —— times 16. A man can earn —— and —— —— times as much in 20 days as he can earn in 16 days. If he can earn $44 in 16 days, in 20 days he can earn —— dollars.

(k) If a man can earn $740 in 16 weeks, how much can he earn in 20 weeks?

13. If 20 bushels of apples are worth $12, 15 bushels are worth —— dollars.

(l) If 20 gal. of milk are worth $3.24, how much are 15 gal. worth?

PERCENTAGE.

$$80 \text{ per cent} = .80 = \tfrac{4}{5}.$$
$$83\tfrac{1}{3} \text{ per cent} = .83\tfrac{1}{3} = \tfrac{5}{6}.$$

	(1)	(2)
1.	80 per cent of 60 =	60 is 80% of ——.
2.	$83\frac{1}{3}$ per cent of 60 =	30 is $83\frac{1}{3}$% of ——.
3.	80 per cent of 40 =	40 is 80% of ——.
4.	$83\frac{1}{3}$ per cent of 36 =	35 is $83\frac{1}{3}$% of ——.
5.	75 per cent of 60 =	60 is 75% of ——.
6.	$66\frac{2}{3}$ per cent of 18 =	32 is $66\frac{2}{3}$% of ——.
7.	40 per cent of 35 =	18 is 40% of ——.
8.	60 per cent of 35 =	27 is 60% of ——.

	(3)	(3)
9.	9 is —— per cent of 54.	9 is ——% of 45.
10.	9 is —— per cent of 63.	9 is ——% of 36.
11.	9 is —— per cent of 27.	9 is ——% of 72.
12.	9 is —— per cent of 18.	9 is ——% of 90.
13.	9 is —— per cent of 81.	14 is ——% of 35.
14.	21 is —— per cent of 35.	14 is ——% of 21.
15.	21 is —— per cent of 28.	35 is ——% of 42.
16.	28 is —— per cent of 35.	$12\frac{1}{2}$ is ——% of 25.

17. Mr. Dow had 80 bushels of apples; he lost 25% of them by decay; he lost —— bushels and had —— bushels left.

18. Fourth-of-July night Willie had 10¢; this was $12\frac{1}{2}$ per cent of what his father gave him to spend; his father gave him —— cents.

19. Sarah had 45 chicks; a hawk killed 18 of them; the hawk killed —— per cent of her chickens.

PERCENTAGE.

(1)

1. 80 per cent of 55 = 83⅓ per cent of 48 =

(a) Find 80% of 435. (b) Find 83⅓% of 492.

(c) Find 40% of 435. (d) Find 75% of 492.

(e) Find 7% of \$435. (f) Find 9% of \$492.

(g) A lawyer's commission for collecting money was 9%;* he collected \$834. How much of this money should he keep? and how much should he "pay over" to the man for whom he collected the money?

(2)

2. 20 is 80% of ——. 20 is 83⅓ % of ——.

(h) 280 is 80% of what? (i) 280 is 83⅓% of what?

(j) 276 is 60% of what? (k) 276 is 75% of what?

(l) 224 is 7% of what? (m) 531 is 9% of what?

(n) A lawyer's commission for collecting money was 9%;* his commission amounted to \$54. How much did he collect? and how much should he "pay over" to the man for whom he collected the money?

(3)

3. Fifty-six dollars are what per cent of \$800? This means, \$56 *are how many hundredths of* \$800?

4. \$36 is ——% of \$400. \$55 is ——% of \$500.

5. \$2.40 is ——% of \$80. \$3.50 is ——% of \$50.

6. A lawyer collected \$900; he retained as his commission \$63 of this sum, and paid the remainder, \$837, to the man for whom he collected the money; the lawyer's commission for collecting was —— %.

* 9% of what? 9% of the amount collected.

REVIEW.

1. Ten times 6 times a number are —— times the number. 10 times 6 times 8 are ——. $8 \times 60 =$

2. One hundred times 5 times a number are —— times the number. 9 multiplied by 500 =

(a) Multiply 78 by 80. (b) Multiply 96 by 700.

3. Divide $12\frac{1}{2}$ by $2\frac{1}{2}$. (Change to halves.) *Story.*

(c) Divide $345\frac{3}{5}$ by $2\frac{2}{5}$. (Change to ——.) *Story.*

(d) Divide 345.6 by 2.4. This means, ———. *Story.*

(e) Multiply $338 by 2.4. (f) Multiply $338 by 2.43.

(g) When land is $375 an acre, how much will 3.35 acres cost?

Find the value of the following:

Commodity.	Gross Weight.	Tare.	Price per Ton.
(h) Straw.	4360 lb.	1780 lb.	$4.25
(i) Coal.	6240 lb.	1830 lb.	$6.75

4. One acre is —— square rods. $\frac{1}{2}$ acre =

5. One mile is —— rods. $\frac{1}{2}$ mile =

6. A piece of land $\frac{1}{2}$ of a mile long and 2 rods wide is —— acres.

7. A piece of land 160 rods long and as wide as the schoolroom is about —— acres.

(j) How many square rods in a rectangular piece of land 47 rods by 6 rods? Is this more or less than 1 acre? Is it more or less than 2 acres?

8. One cord is —— cubic feet. $\frac{1}{2}$ cord =

(k) How many cubic feet of wood in a pile 7 feet by 5 feet by 6 feet? Is this more or less than 1 cord? Is it more or less than 2 cords?

MISCELLANEOUS PROBLEMS.

1. If a man can save \$5 a month, in 3 years he can save —— dollars.

(a) If a man can save \$27 in a month, how much can he save in 9 years?

2. Byron bought 2 doz. oranges for 40¢; he sold them at 3¢ each; he gained —— cents.

(b) A merchant bought 35 barrels of apples for \$85; he sold them at \$2.75 a barrel. How much did he gain?

3. Henry had 25 chickens; a hawk caught 20% of them; he sold the remainder at 22¢ each; he received —— dollars and —— cents.

(c) Henry's father had 75 bushels of apples; he lost 20% of them by decay; he sold the remainder at 85¢ a bushel. How much did he receive for them?

4. If coffee costs $\$\frac{2}{5}$ a pound, for 1 dollar I can buy —— pounds. $1 \div \frac{2}{5} =$

(d) If tea costs $\$\frac{3}{5}$ a pound, how many pounds can I buy for \$57? (Change \$57 to fifth-dollars.)

5. The sum of two numbers is 34; one of the numbers is 12; the other number is ——.

(e) The sum of two numbers is 346.2; one of the numbers is 75.36. What is the other number?

6. In a piece of slate 1 foot square and 1 inch thick there are —— cubic inches.

(f) How many cubic inches in a piece of slate 2 feet square and 2 inches thick?

(g) If $\frac{3}{5}$ of a ton of coal is worth \$4.20, how much is 1 ton worth?

SIMPLE NUMBERS.

1. One tenth of 6 is ——. 1 tenth of 40 is ——.

2. One tenth of 46 is ——. 1 tenth of 25 is ——.

3. One tenth of 300 is ——. 1 tenth of 325 is ——.

4. *One half of* 1 *tenth of a number is one* ——*th of the number.* One third of 1 tenth of a number =

(a) Divide 472 by 20.

Operation.	*Explanation.*
$2\not{0})\underline{47^{\vee}2}$ 23.6	One tenth of 472 is ——. One half of 1 tenth ($\frac{1}{10}$) of 472 is ——.

(b) Divide 741 by 30. (c) Divide 548 by 40.

(d) Divide 735 by 50. (e) Divide 960 by 60.

5. *One third of* 1 *hundredth of a number is one* ——*th of the number.* One fourth of 1 hundredth of a number =

(f) Divide 972 by 300. (g) Divide 972 by 400.

(h) Divide 895 by 500. (i) Divide 976 by 800.

REVIEW.

6. Common multiples of 15 and 6 are ——, ——, etc. The least common multiple of 15 and 6 is ——.

7. The prime factors of 63 are ——, ——, and ——.

(j) What are the prime factors of 124? (k) Of 178?

8. 3, 3, and 5 are the prime factors of ——. Forty-five is exactly divisible by 3; by 5; by ——; by ——.

(l)

128 cu. feet.)1536 cu. ft.

(m)

12)1584 cu. ft.

(n)

160 sq. rd.)2400 sq. rd.

(o)

15)2445 sq. rd.

COMMON FRACTIONS.

1. Add $\frac{2}{15}$ and $\frac{1}{6}$. The l. c. m. of 15 and 6 is ——.

(a) Find the sum of $456\frac{7}{15}$, $341\frac{2}{5}$, $245\frac{5}{6}$, and 564.

2. From $9\frac{2}{15}$ subtract $4\frac{5}{6}$. $1\frac{2}{15} = \frac{\quad}{30}$. $\frac{5}{6} = \frac{\quad}{30}$.

(b) Find the difference of $4275\frac{2}{15}$ and $1328\frac{5}{6}$.

3. Multiply $\frac{7}{8}$ by 9. This means —— 3* ——.

(c) Find the product of $453\frac{7}{8}$ multiplied by 9.

4. Multiply $2\frac{3}{4}$ by $\frac{1}{3}$. This means —— 13* ——.

(d) Find the product of $458\frac{3}{4}$ multiplied by $\frac{1}{3}$.

5. Multiply 17 by $\frac{3}{4}$. This means —— 8* ——.

(e) Find the product of 741 multiplied by $\frac{3}{4}$.

6. Multiply 17 by $2\frac{3}{4}$. This means —— 16* ——.

(f) Find the product of 741 multiplied by $2\frac{3}{4}$.

7. Multiply $16\frac{1}{2}$ by $2\frac{1}{4}$. This means —— 22* ——.

(g) Find the product of $732\frac{1}{2}$ multiplied by $2\frac{1}{4}$.

8. Divide 8 by $\frac{3}{4}$. (4)* Change 8 to ——ths.

(h) Find the quotient of 97 divided by $\frac{3}{4}$. *Story.*

9. Divide $\frac{11}{15}$ by $\frac{1}{6}$. (9)* Change to ——ths.

(i) Find the quotient of $3\frac{11}{15}$ divided by $\frac{1}{6}$. *Story.*

10. Divide $7\frac{1}{5}$ by $2\frac{1}{5}$. (14)* Change to ——ths.

(j) Find the quotient of $55\frac{1}{5}$ divided by $2\frac{3}{5}$. *Story.*

11. Divide $\$1\frac{2}{5}$ ($\frac{7}{5}$) by 4. This means —— 15* ——.

(k) Find the quotient of $7\frac{4}{5}$ divided by 8. *Story.*

(l)	(m)	(n)	(o)	(p)
Add.	Subtract.	Multiply.	Divide.	Divide.
$375\frac{13}{15}$	$435\frac{1}{6}$	$346\frac{5}{6}$	$3\frac{1}{4}$ ft.)232 ft.	3)$365\frac{3}{4}$ ft.
$246\frac{5}{6}$	$182\frac{11}{15}$	12		

* These figures refer to notes on pages 6 and 7. See also foot-notes, page 62.

DECIMAL FRACTIONS.

1. One tenth of $6 is ——. .1 of $6.25 is ——.

2. One hundredth of $6 is ——. .01 of $6.25 is $.0625.

3. Read each of the following in two ways: $.2436,* $.0532, $.6403, $.0042, $.0002, $.6042, $.8002.

(a) Multiply $6.25 by 4.23. This means, *find* 4 *times* $6.25 + 2 *tenths of* $6.25 + 3 *hundredths of* $6.25.

Operation.	*Explanation.*
$6.25	One hundredth of $6.25 is ——.
4.23	3 hundredths of $6.25 are ——.
$.1875†	One tenth of $6.25 is ——.
$1.250	2 tenths of $6.25 are ——.
$25.00	4 times $6.25 are ——.
$26.4375	$.1875 + $1.250 + $25 =

NUMBER STORY.

If one ton of coal is worth $6.25,
1 hundredth of a ton is worth ——.
3 hundredths of a ton are worth ——.
1 tenth of a ton is worth ——.
2 tenths of a ton are worth ——.
4 tons are worth ——.
4.23 tons are worth ——.

(b) Multiply $7.35 by 3.46; (c) $4.45 by 5.24.

* (1) 24¢, 3 m., and 6 tenths of a mill. (2) 2436 ten-thousandths of a dollar.

† TO THE TEACHER.—Read the foot-note on page 133; also, the first part of page 143. If the pupil finds difficulty in "pointing off," teach him to use a *separatrix* in the multiplicand as suggested on page 133. While multiplying 6.25 by 3 hundredths, it may appear on the slate thus: $\begin{array}{r} {}^{\vee}06.25 \\ \underline{4.23} \end{array}$ Do not *at this stage of the work* allow the pupil to "point off" by counting the decimal places in the multiplicand and multiplier. Rather, lead him to *think the meaning of the problem.* See foot-notes, page 53.

DENOMINATE NUMBERS.

1. 5280 *feet are* 1 *mile.* —— rods are 1 mile.

(a) Change 320 rd. to feet. (b) Change 160 rd. to feet.

2. Six hundred feet are —— rods and —— feet.

3. Seven hundred feet are —— rods and —— feet.

4. Eight hundred feet are —— rods and —— feet.

5. Two miles are —— rods. $1\frac{1}{2}$ miles are —— rods.

(c) Change 6 mi. to rods. (d) Change 30 rd. to feet.

(e) Change 2 mi. to feet. (f) Change 75 rd. to yards.

(g) Change $2\frac{1}{2}$ mi. to feet. (h) Change $2\frac{1}{2}$ mi. to rods.

Find the value of the following:

	Commodity.	Gross Weight.	Tare.	Price.
(i)	Corn meal.	4380 lb.	1540 lb.	$15.25 per ton.
(j)	Oat straw.	3460 lb.	1620 lb.	$4.50 per ton.
(k)	Old iron.	3240 lb.	1380 lb.	$\frac{1}{2}$¢ per lb.*
(l)	Paper-rags.	3120 lb.	1260 lb.	$\frac{3}{4}$¢ per lb.
(m)	Ear corn.	3690 lb.	1230 lb.	35¢ per bu.
(n)	Oats.	2410 lb.	1250 lb.	28¢ per bu.

6. 120 inches are —— feet. 126 inches are —— feet.

(o) Change 428 in. to ft. (p) Change 580 in. to ft.

7. 72 feet are —— yards. 73 feet are —— yards.

(q) Change 853 ft. to yd. (r) Change 725 ft. to yd.

8. 33 feet are —— rods. 66 feet are —— rods.

(s) Change 627 ft. to rd.† (t) Change 594 ft. to rods.

9. 11 yards are —— rods. 22 yards are —— rods.

(u) Change 759 yd. to rd. (v) Change 638 yd. to rd.

* 1860 lbs. at 1¢ a pound would be worth $18.60; at $\frac{1}{2}$ a cent a pound the value is $\frac{1}{2}$ of $18.60, or ———.

† To divide 627 ft. by $16\frac{1}{2}$ ft., change both numbers to halves. See page 7, note (20).

MEASUREMENTS.

1. A piece of land 10 rods by 16 rods contains —— square rods. It is —— acre.

2. A piece of land 20 rods by 16 rods contains —— square rods. It is —— acres.

(a) Change 640 sq. rd. to acres. (b) Change 1280 sq. rd. to acres. (c) How many acres in a piece of land 40 rods by 64 rods?

3. Eighty square rods are —— —— of an acre.

4. Forty square rods are —— —— of an acre.

5. One hundred twenty square rods are —— —— of an acre.

6. A piece of land 20 rd. by 26 rd. contains —— square rods.

(d) How many acres in a piece of land 20 rd. by 26 rd.?

Find the number of acres in each of the following:

(e) 20 rods by 28 rods. (f) 20 rods by 30 rods.
(g) 12 rods by 60 rods. (h) 24 rods by 50 rods.
(i) 36 rods by 50 rods. (j) 48 rods by 60 rods.

7. A pile of wood 8 ft. by 4 ft. by 4 ft. contains —— cubic feet. It is —— cord.

8. Sixty-four cubic feet are —— —— of a cord.

9. Thirty-two cubic feet are —— —— of a cord.

(k) How many cubic feet in a pile of wood 12 ft. by 8 ft. by 4 ft.?

(l) How many cords in a pile 12 ft. by 8 ft. by 4 ft.?

Find the number of cords in each of the following:

(m) 6 feet by 4 feet by 8 feet. (n) 6 ft. by 8 ft. by 8 ft.
(o) 6 feet by 12 feet by 4 feet. (p) 6 ft. by 12 ft. by 8 ft.

RATIO AND PROPORTION.

1. One fourth of 3.6 is ——.* 3.6 is $\frac{1}{4}$ of ——.

2. One fifth of 3.5 is ——. 3.5 is $\frac{1}{5}$ of ——.

(a) Find 1 fourth of 78.4. (b) 78.4 is $\frac{1}{4}$ of what?

(c) Find 1 fifth of 97.5. (d) 97.5 is $\frac{1}{5}$ of what?

3. Three fourths of 3.6 are ——.† 3.6 is $\frac{3}{4}$ of ——.

4. Two thirds of 2.4 are ——. 2.4 is $\frac{2}{3}$ of ——.

(e) Find $\frac{3}{4}$ of 28.8. (f) 28.8 is $\frac{3}{4}$ of what number?

(g) Find $\frac{2}{3}$ of 37.2. (h) 37.2 is $\frac{2}{3}$ of what number?

5. 1.4 is —— —— of 2.1.‡ 2.1 is —— —— of 1.4.

6. 2.1 is —— —— of 2.8. 2.8 is —— —— of 2.1.

7. Two thirds of 2.1 are 1 half of ——.

8. One half of 2.4 is 2 thirds of ——.

(i) Two thirds of 53.7 are 1 half of what number?

(j) One half of 65.6 is 2 thirds of what number?

9. If a man can earn $12 in 2.1 days, in 2.8 days he can earn —— dollars.

(k) If a man can earn $75.45 in 2.1 months, how much can he earn in 2.8 months?

10. If 2.8 tons of coal are worth $12, 2.1 tons are worth —— dollars.

(l) If 2.8 tons of coal are worth $9.60, how much are 2.1 tons worth?

11. If a pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long is worth $5, a pile 4 ft. wide, 4 ft. high, and 32 ft. long is worth —— dollars.

* One fourth of 36 tenths is —— tenths.
† Three fourths of 36 tenths are —— tenths, or 2 and —— tenths.
‡ Fourteen tenths are —— —— of 21 tenths.

PERCENTAGE.

$37\frac{1}{2}$ per cent $= .37\frac{1}{2} = \frac{3}{8}$.
$62\frac{1}{2}$ per cent $= .62\frac{1}{2} = \frac{5}{8}$.
$87\frac{1}{2}$ per cent $= .87\frac{1}{2} = \frac{7}{8}$.

(1)	*(2)*
1. $12\frac{1}{2}\%$ of 48 =	48 is $12\frac{1}{2}\%$ of ——.
2. 25 % of 48 =	48 is 25 % of ——.
3. $37\frac{1}{2}\%$ of 48 =	48 is $37\frac{1}{2}\%$ of ——.
4. 50 % of 48 =	48 is 50 % of ——.
5. $62\frac{1}{2}\%$ of 40 =	40 is $62\frac{1}{2}\%$ of ——.
6. $87\frac{1}{2}\%$ of 56 =	56 is $87\frac{1}{2}\%$ of ——.

(3)	*(3)*
7. 3 is ——% of 24.	9 is ——% of 24.
8. 21 is ——% of 24.	15 is ——% of 24.
9. 4 is ——% of 32.	20 is ——% of 32.
10. 12 is ——% of 32.	28 is ——% of 32.
11. 5 is ——% of 40.	15 is ——% of 40.

12. Mr. A owed Mr. B $72. Mr. A had suffered many losses and it was hard for him to pay this amount. Mr. B kindly discounted the bill $12\frac{1}{2}\%$ and Mr. A paid it. He paid —— dollars.

13. Harry paid $87\frac{1}{2}\%$ of his money for books and had $2 left; before he bought the books he had —— dollars; the books cost —— dollars.

14. Mr. Hill set 40 trees in his new orchard; 8 of them died the first year; —— per cent of the trees were dead; —— per cent were alive.

15. Twenty per cent of 50 is —— per cent of 30.

16. Forty per cent of 50 is —— per cent of 80.

PERCENTAGE.

(*1*)

1. $37\frac{1}{2}$ per cent of 64 = $62\frac{1}{2}$ per cent of 64 =

(a) Find $37\frac{1}{2}\%$ of 192. (b) Find $62\frac{1}{2}\%$ of 192.

(c) Find $87\frac{1}{2}\%$ of 192. (d) Find 75% of 192.

(e) Find 5% of $192. (f) Find 7% of $192.

(g) A lawyer's commission for collecting money was 8%;* he collected $875. How much of this money should he keep? and how much should he "pay over" to the man for whom he collected the money?

(*2*)

2. 15 is $37\frac{1}{2}$ per cent of ——. 35 is $62\frac{1}{2}\%$ of ——.

(h) 165 is $37\frac{1}{2}\%$ of what? (i) 165 is $62\frac{1}{2}\%$ of what?

(j) 161 is $87\frac{1}{2}\%$ of what? (k) 168 is 75% of what?

(l) 135 is 3% of what? (m) 168 is 7% of what?

(n) A lawyer's commission for collecting money was 8%;* his commission amounted to $34.80. How much did he collect? and how much did he "pay over" to the man for whom he collected the money? †

(*3*)

3. 18 is —— per cent of 48. 30 is —— per cent of 48.

4. Forty-five dollars is what per cent of $500? This means, $45 *are how many hundredths of* $500? ‡

5. A lawyer collected $500; he retained as his commission $55 of this sum, and paid the remainder, $445, to the man for whom he collected the money; the lawyer's commission for collecting was ——%.

* 8% of what? 8% of the amount collected.

† $34.80 is 8 hundredths of what?

‡ One hundredth of $500 is $5. See page 58, problem (o).

REVIEW.

1. One fourth of 1 tenth of a number is one ——th of the number. 1 fourth of 1 tenth of 120 is ——.

(a) Divide 836 by 40. (b) Divide 75.6 by 40.

2. One fifth of 1 hundredth of a number is one ——th of the number. 1 fifth of 1 hundredth of 3500 is ——.

(c) Divide 950 by 500. (d) Divide 246.5 by 500.

(e) Divide 3660 by 600. (f) Divide 5740 by 700.

3. Multiply 19 by $3\frac{2}{3}$. (This means ——.)*

(g) Multiply 276 by $5\frac{3}{4}$. (h) Multiply 276 by 5.75.

Compare the answers to problems (g) and (h).

4. The sum of 86 hundredths and 5 tenths is ——.

(i) Add 735 ten-thousandths and 642 thousandths.

5. One mile is —— feet. 1 mile is —— rods.

(j) Change $5\frac{1}{4}$ mi. to feet. (k) Change $8\frac{3}{4}$ mi. to rods.

6. A piece of land 4 rods by 80 rods is —— acres.

(l) How many acres in a piece of land 18 rd. by 60 rd.

7. A pile of wood 4 ft. wide, 4 ft. high, and 40 ft. long contains —— cords.

(m) How many cords in a pile of wood 4 ft. wide, 4 ft. high, and 432 ft. long?

Perform the operations indicated and tell number stories:

(n) 48 cu. ft. $\times$ 6 $\times$ 8 = (o) 2304 cu. ft. $\div$ 128 cu. ft. =†

(p) 96 sq. rods $\times$ 25 = (q) 2400 sq. rd. $\div$ 160 sq. rd. =‡

(r) 960 rods $\times$ 4 = (s) 3840 rods $\div$ 320 rods =

(t) 2350 lb. $\times$ 7 = (u) 16450 lb. $\div$ 2000 lb. =

* See page 7, note 16.

† Number story for (n) and (o). In a pile of wood 48 ft. long, 6 ft. wide, and 8 ft. high there are —— cords.

‡ Number story for (p) and (q). A rectangular piece of land 96 rods by 25 rods contains —— acres.

MISCELLANEOUS PROBLEMS.

1. If I put 5 oz. of candy in each bag, 3 lb. of candy will fill —— bags, with —— ounces over.

(a) If I put 2 bushels of oats in each bag, how many bags will one ton of oats fill?

2. Henry was born on January 5, 1885; on Jan. 5, 1896, he was —— years old.

(b) Queen Victoria was born on May 24, 1819. How old was she on the 24th day of May, 1897?

3. A dealer bought 5 gal. of milk at 14¢ a gal.; he sold it at 5¢ a quart, but lost 2 quarts in over-measurement and waste; he gained —— cents.

(c) A grocer bought 3 bbl. sugar, each containing 340 lb.; he paid $4\frac{1}{2}$¢ a lb. If he sells it at the rate of 20 lb. for a dollar, but loses 40 lb. by over-weights and waste, how much does he gain?

4. Edward bought 2 lb. candy at 18¢ a pound, 2 lb. walnuts at 20¢ a pound, and 1 qt. of pop-corn for 10¢; for all he paid —— cents.

Find the amount of each of the following bills:

(d)		(e)	
3 lb. Steak	@ 15¢	5 bu. Apples	@ 35¢
$\frac{1}{2}$ bu. Potatoes	@ 40¢	$\frac{1}{2}$ bu. Beans	@ $1.50
18 lb. Sugar	@ 5¢	7 gals. Milk	@ 15c.
5 cans Corn	@ 8¢	2 cd. Wood	@ $4.50
7 heads Cabbage	@ 8¢	$1\frac{1}{2}$ tons Hay	@ $8.50

5. If the divisor is 6 and the quotient is $4\frac{1}{2}$, the dividend is ——.

(f) If the divisor is 128 and the quotient $16\frac{3}{4}$, what is the dividend?

SIMPLE NUMBERS.

1. Alfred rode his bicycle for 3 consecutive hours; the first hour he rode 10 miles; the second hour, 8 miles; the third hour, 6 miles; altogether he rode —— miles; his average speed was (24 ÷ 3) —— miles an hour.

2. A farmer bought 20 sheep; for 10 of them he paid $35, and for the other 10, $25; for all he paid —— dollars; the average cost per head was —— dollars.

(a) If 8 cows cost $204, what is the average cost per head?

(b) Five boys were examined in spelling; the marks received were 90, 95, 85, 80, and 75. What was the average standing of the boys?

(c) The temperature at noon of each day for 1 week was as follows: Sunday, 94; Monday, 91; Tuesday, 96; Wednesday, 95; Thursday, 90; Friday, 89; Saturday, 89. What was the average temperature at noon for the week?

(d) Six beef cattle weighed as follows: 1560 lb., 1430 lb., 1640 lb., 1350 lb., 1420 lb., and 1660 lb. What was the average weight per head?

REVIEW.

3. The prime factors of 70 are ——, ——, and ——.

(e) What are the prime factors of 195? (f) Of 185?

4. 2, 3, and 7 are the prime factors of ——. Forty-two is exactly divisible by 2; by 3; by ——; by——; by ——, and by ——.

(g) Multiply 864 by 50 (h) Divide 7650 by 50.

(i) Multiply 875 by 700. (j) Divide 8750 by 700.

(k) Multiply 736 by 400. (l) Divide 9384 by 400.

COMMON FRACTIONS.

1. Add $\frac{5}{16}$ and $\frac{1}{6}$. The l. c. m. of 16 and 6 is ——.

(a) Find the sum of $376\frac{7}{16}$, $254\frac{1}{2}$, $321\frac{5}{6}$, and 342.

2. From $7\frac{1}{16}$ subtract $3\frac{5}{6}$. $1\frac{1}{16} = \frac{\quad}{48}$. $\frac{5}{6} = \frac{\quad}{48}$.

(b) Find the difference of $6274\frac{1}{16}$ and $842\frac{5}{6}$.

3. Multiply $\frac{5}{16}$ by 8. This means ——————.

(c) Find the product of $357\frac{5}{16}$ multiplied by 8.

4. Multiply $1\frac{2}{5}$ by $\frac{1}{4}$. This means ——————.

(d) Find the product of $573\frac{2}{5}$ multiplied by $\frac{1}{4}$.

5. Multiply 19 by $\frac{2}{5}$. This means ——————.

(e) Find the product of 643 multiplied by $\frac{2}{5}$.

6. Multiply 19 by $2\frac{2}{5}$. This means ——————.

(f) Find the product of 643 multiplied by $4\frac{2}{5}$.

7. Multiply $15\frac{1}{2}$ by $2\frac{1}{5}$. This means ——————.

(g) Find the product of $345\frac{1}{2}$ multiplied by $4\frac{1}{5}$.

8. Divide 8 by $\frac{3}{5}$. (Change 8 to ——ths.)

(h) Find the quotient of 86 divided by $\frac{3}{5}$. *Story.*

9. Divide $\frac{7}{16}$ by $\frac{1}{6}$. (Change to ——ths.)

(i) Find the quotient of $3\frac{7}{16}$ divided by $\frac{1}{6}$. *Story.*

10. Divide $8\frac{3}{5}$ by $2\frac{2}{5}$. (Change to ——ths.)

(j) Find the quotient of $57\frac{3}{5}$ divided by $2\frac{2}{5}$. *Story.*

11. Divide \$$1\frac{4}{5}$ ($\frac{9}{5}$) by 5. This means ——————.

(k) Find the quotient of $8\frac{4}{5}$ divided by 9.

(l) Find the quotient of $453\frac{2}{3}$ divided by 4.

(m)	(n)	(o)	(p)	(q)
Add.	Subtract.	Multiply.	Divide.	Divide.
$436\frac{11}{16}$	$573\frac{1}{6}$	245	$3\frac{2}{3}$ in.)246 in.	3)$246\frac{2}{5}$ in.
$148\frac{5}{6}$	$245\frac{11}{16}$	$24\frac{2}{5}$		

DECIMAL FRACTIONS.

Division. Case I.

1. Divide .45 by .09. This means —— 4* ——.

(a) Find the quotient of 46.75 divided by .25.† *Story.*

2. Divide 9 by .6. This means —— 9* ——.

(b) Find the quotient of 874 divided by .5.‡ *Story.*

3. Divide 6 by .05. This means —— 14* ——.

(c) Find the quotient of 96 divided by .25.§ *Story.*

4. Divide 4.5 by .09. This means —— ——.

(d) Find the quotient of 46.5 divided by .25.‖ *Story.*
(e) Find the quotient of 57.5 divided by 2.5.¶ *Story.*
(f) Find the quotient of 68.5 divided by .25.** *Story.*
(g) Find the quotient of 76 divided by .25.†† *Story.*

Division. Case II.

5. Divide $.63 by 7. This means —— 5* ——.

(h) Find the quotient of $87.15 divided by 7. *Story.*
(i) Find the quotient of $375.50 divided by 25. *Story.*

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story:

(j) Divide $256 by $8.	(k) Divide $256 by 8.
(l) Divide $24.36 by $.04.	(m) Divide $24.36 by 4.
(n) Divide $53.6 by $.8	(o) Divide $53.6 by 8.

* These figures refer to notes on pages 8 and 9.
† Find how many times 25 hundredths are contained in 4675 hundredths.
‡ Find how many times 5 tenths are contained in 8740 (874.0) tenths.
§ Find how many times 25 hundredths are contained in 9600 (96.00) hundredths.
‖ Find how many times 25 hundredths are contained in 4650 (46.50) hundredths.
¶ Find how many times 25 tenths are contained in 575 (57.5) tenths.
** Find how many times 25 hundredths are contained in 6850 (68.50) hundredths.
†† Find how many times 25 hundredths are contained in 7600 (76.00) hundredths.

DENOMINATE NUMBERS.

1. 24 *sheets of paper are* 1 *quire.*

2. 20 *quires of paper are* 1 *ream.*

3. One ream of paper is —— sheets.

4. One half of a ream is —— quires, or —— sheets.

5. One fourth of a ream is —— quires, or —— sheets.

6. George bought a ream of paper for $2 and sold it at the rate of 2 sheets for a cent; he gained —— cents.

7. Arthur bought a ream of paper for $2.25 and sold it at 15¢ a quire; he gained —— cents.

REVIEW.

8. Ten thousand feet are nearly —— miles.

9. Fifteen thousand feet are nearly —— miles.

(a) Twenty thousand feet are how many feet less than four miles?

(b) Five miles are how many feet more than 25000 feet?

10. One thousamd feet are nearly one ——th of a mile.

(c) Two fifths of a mile are how many feet more than 2000 feet? (d) $\frac{3}{5}$ of a mile, than 3000 ft.?

11. Nine hundred feet are —— rods and —— feet.

12. One thousand feet are —— rods and —— feet.

13. One hundred feet are —— yards and —— foot.

14. Two hundred feet are —— yards and —— feet.

15. Three hundred feet are —— yards.

(e) Change 3 mi. to ft. (f) Change 3 mi. to rd.

(g) Change 3 A. to sq. rd. (h) Change 3 cd. to cu. ft.

(i) Change $3\frac{1}{2}$ mi. to ft. (j) Change $3\frac{1}{2}$ mi. to rd.

(k) Change $3\frac{1}{2}$ A. to sq. rd. (l) Change $3\frac{1}{2}$ cd. to cu. ft.

(m) Change $3\frac{1}{2}$ rd. to ft. (n) Change $3\frac{1}{2}$ yd. to in.

MEASUREMENTS.

1. In one cubic yard there are —— cubic feet.

(a) To dig a cellar 15 feet long, 12 ft. wide, and 6 feet deep would require the removal of how many cubic feet of earth? (b) How many cubic yards?

(c) How many square feet in the floor of the cellar described in problem (a)? (d) How many sq. yards?

2. To make an excavation 2 yards long, 2 yards wide, and 2 yards deep would require the removal of —— cubic yards.

(e) How many cubic feet in the excavation described in problem 2?

3. The area of one of the faces of a 2-yard cube is —— square yards; of all the faces, —— square yards.

(f) The area of all the faces of a 2-yard cube is how many square feet?

REVIEW.

4. A piece of land 20 rods by 8 rods contains ——.

(g) How many acres in a piece of land 20 rd. by 46 rd.?

5. A shed 16 feet long, 10 feet wide, and 6 feet high contains —— cubic feet.

(h) How many cords of wood will the shed described in problem 5 contain?

Find the number of acres in each of the following:

(i) 80 rods by $24\frac{1}{2}$ rods. (j) 40 rods by 73 rods.

(k) 20 rods by 52 rods. (l) 30 rods by 84 rods.

Find the number of cords in each of the following:

(m) 8 ft. by 12 ft. by 8 ft. (n) 16 ft. by 8 ft. by 8 ft.

(o) 16 ft. by 4 ft. by 8 ft. (p) 20 ft. by 8 ft. by 8 ft.

RATIO AND PROPORTION.

1. One fourth of .36 is ——. .36 is $\frac{1}{4}$ of ——.
2. One fifth of .45 is ——. .45 is $\frac{1}{5}$ of ——.
3. One eighth of .32 is ——. .32 is $\frac{1}{8}$ of ——.

(a) Find 1 fourth of 9.86.* (b) 9.78 is $\frac{1}{4}$ of what?
(c) Find 1 fifth of 8.24. (d) 8.24 is $\frac{1}{5}$ of what?

4. Five eighths of .32 are ——. .35 is $\frac{5}{8}$ of ——.
5. Three fifths of .45 are ——. .45 is $\frac{3}{5}$ of ——.

(e) Find $\frac{5}{8}$ of 8.40. (f) 8.40 is $\frac{5}{8}$ of what number?
(g) Find $\frac{3}{5}$ of 9.45. (h) 9.45 is $\frac{3}{5}$ of what number?

6. .32 is —— —— of .40. .40 is —— —— of .32.
7. .24 is —— —— of .40. .40 is —— —— of .24.
8. .24 is —— —— of .56. .56 is —— —— of .24.
9. .40 is —— —— of .64. .64 is —— —— of .40.

10. Three fourths of .32 are 1 half of —— hundredths.

(i) Three fourths of 9.36 are 1 half of what number?

(j) One half of 9.36 is 4 fifths of what number?

11. If .32 of an acre of land is worth $20, .40 of an acre is worth —— dollars.

(k) If .32 of an acre of land is worth $24.56, how much is .40 of an acre worth at the same rate?

12. If .40 of a ton of coal is worth $4.00, .32 of a ton is worth ——.

(l) If .40 of a ton of coal is worth $3.25, how much is .32 of a ton worth?

*Think as suggested by the following: $\frac{1}{4}$ of 9 is 2 with 1 remainder; 1 = 10 tenths; 10 tenths + 8 tenths are 18 tenths; $\frac{1}{4}$ of 18 tenths is 4 tenths with 2 tenths remainder; 2 tenths = 20 hundredths; 20 hundredths + 6 hundredths = 26 hundredths; $\frac{1}{4}$ of 26 hundredths = 6 hundredths with 2 hundredths remainder; 2 hundredths = 20 thousandths; $\frac{1}{4}$ of 20 thousandths = 5 thousandths.

$$4\overline{)9.86}$$
$$2.46$$

PERCENTAGE.

$$30 \text{ per cent} = .30 = .3 = \tfrac{3}{10}.$$
$$70 \text{ per cent} = .70 = .7 = \tfrac{7}{10}.$$
$$90 \text{ per cent} = .90 = .9 = \tfrac{9}{10}.$$

	(1)	(2)
1.	30% of 60 =	60 is 30% of ——.
2.	40% of 60 =	60 is 40% of ——.
3.	50% of 60 =	60 is 50% of ——.
4.	60% of 60 =	60 is 60% of ——.
5.	70% of 140 =	140 is 70% of ——.
6.	80% of 160 =	160 is 80% of ——.
7.	90% of 180 =	180 is 90% of ——.

	(3)	(3)
8.	4 is ——% of 40.	12 is ——% of 40.
9.	28 is ——% of 40.	36 is ——% of 40.
10.	5 is ——% of 50.	45 is ——% of 50.
11.	15 is ——% of 50.	35 is ——% of 50.
12.	6 is ——% of 60.	42 is ——% of 60.

13. In a certain orchard there were 120 trees; 10% of them were pear trees and 30% of them were cherry trees; there were —— pear trees and —— cherry trees.

14. In another orchard there were 24 pear trees; these were 30% of all the trees in the orchard; there were —— trees in the orchard.

15. There were 60 trees in an orchard; 42 of them were apple trees and the remainder pear trees; there were —— pear trees; —— per cent of the trees in the orchard were apple trees and —— per cent of them were pear trees.

16. Twenty-five % of the trees in an orchard died and were removed; there remained 36 trees; —— trees died.

PERCENTAGE.

(1)

1. 30 per cent of 42 = ($\frac{1}{10}$ of 42 is 4.2.)
2. 90 per cent of 51 = 90 per cent of 31 =
(a) Find 30% of 275. (b) Find 70% of 362.
(c) Find 90% of 436. (d) Find 30% of 475.
(e) Find 3% of 536. (f) Find 7% of 824.

(g) A lawyer's commission for collecting money was 10%; he collected $954. How much of the money should he keep? and how much should he "pay over" to the man for whom he collected the money?

(2)

3. 3.2 is 10 per cent of ——. 4.6 is 10% of ——.
4. 9.6 is 30 per cent of ——. 14.7 is 70% of ——.
(h) 372.3 is 30% of what? (i) 562.1 is 70% of what?
(j) 291.6 is 90% of what? (k) 752.4 is 30% of what?
(l) 279 is 3% of what? (m) 539 is 7% of what?

(n) A lawyer's commission for collecting money was 10%; his commission amounted to $76.20. How much did he collect? and how much should he "pay over" to the man for whom he collected the money?

(3)

5. 5.1 is —— per cent of 51. 8.3 is ——% of 83.
6. 84 is —— per cent of 840. 8.40 is ——% of 840.
7. 7.54 is —— per cent of 75.4. 7.54 is ——% of 754.

8. A lawyer collected $754; he retained as his commission $75.40 of this sum and paid the remainder to the man for whom he collected the money; the lawyer's commission for collecting was ——%.

REVIEW.

1. Six sheep cost $33; the average cost per head was ——.

(a) In a certain schoolroom, on Monday there were 35 pupils; Tuesday, 38; Wednesday, 36; Thursday, 37, and Friday, 39. What was the average daily attendance?

2. Divide $\frac{9}{16}$ by $\frac{1}{8}$. (Change to ——ths.) *Story.*

(b) Divide 323\frac{3}{4}$ by 1\frac{1}{4}$. (Change to ——ths.) *Story.*

(c) Divide $323.75 by $1.25. This means ——.

Compare the answers to problems (b) and (c). Why are they alike?

3. In $\frac{1}{4}$ of a ream of paper there are —— quires.

4. At 3 sheets for 2 cents, 1 quire of paper costs ——.

(d) At 3 sheets for 2 cents, how much will one ream of paper cost?

5. In 2 square yards there are —— square feet.

6. In 2 cubic yards there are —— cubic feet.

(e) In 18 cubic yards there are how many cu. ft.

(f) In 48 square yards there are how many square feet?

(g) How many square feet of blackboard in this room?*

(h) How many sq. yd. of blackboard in this room?*

(i) How many cubic feet of earth must be removed to make an excavation 6 ft. by 9 ft. by 15 ft.? (j) How many cubic yards?

(k) $326 + 432 + 28 + 175 + 326 + 47 + 63 + 82 + 96 =$

(l) $274 + 386 + 27 + 237 + 528 + 35 + 72 + 51 + 46 =$

(m) $438 + 275 + 43 + 324 + 136 + 32 + 85 + 37 + 91 =$

(n) $243 + 139 + 18 + 914 + 326 + 12 + 72 + 36 + 75 =$

*Require an approximate answer only.

MISCELLANEOUS REVIEW.

1. Two boys start from two cities that are 67 miles apart and ride toward each other; one rides 8 miles an hour; the other rides 12 miles an hour; in one hour they ride —— miles; in 3 hours they are —— miles apart.

(a) Two trains start from two cities that are 1000 miles apart and move toward each other; one goes 25 miles an hour; the other goes 30 miles an hour. At the end of 12 hours how far are the trains apart?

2. A man bought land for $80 and sold it for $88; he gained $2 an acre; there were —— acres.

(b) A man bought land for $1850, and sold it for $2220, thereby gaining $5 an acre. How many acres did he buy? (c) How much did he pay per acre for it?

3. A boy earned some money, spent $\frac{5}{8}$ of it, and had $6 left; he spent —— dollars.

(d) A man earned some money, spent $\frac{5}{8}$ of it, and had $25.20 left. How much did he spend?

4. The product of two numbers is 28; one of the numbers is 7; the other number is ——.

(e) The product of two numbers is 240; one of the numbers is 15. What is the other number?

5. Fifty per cent of my money is $12, and John has 3 times as much as I have; John has —— dollars.

(f) Fifty per cent of Mr. A's money is $36.45 and Mr. B has 3 times as much as Mr. A. Mr. B has ———.

6. I have $24; 50% of my money equals $33\frac{1}{3}$ per cent of Bernie's money; Bernie has —— dollars.

(g) Mr. C has $72.30; 50% of Mr. C's money equals $33\frac{1}{3}$% of Mr. D's money. How much money has Mr. D?

SIMPLE NUMBERS.

Number of pounds of milk received.

PATRONS.	M.	T.	W.	T.	F.	S.	S.	AM'T.	
L. R. Orr	236	224	259	263	248	227	236	(a)	
Levi Smith	149	167	134	158	175	136	145	(b)	
Henry Judd	375	384	363	356	325	364	356	(c)	
Peter Johnson	97	84	93	86	92	87	88	(d)	
Wm. Jones	201	196	205	208	199	215	218	(e)	
E. C. Ford..........	385	397	388	394	396	399	401	(f)	
Otto Fisk...........	87	146	175	82	185	96	133	(g)	
Fay Winslow	230	231	229	233	228	229	235	(h)	
Albert Davis	155	160	145	170	180	175	194	(i)	
Jno. Harris	435	424	412	456	428	435	457	(j)	
Total...........	(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r)	(s)

The above is a statement of the milk received at a small creamery for one week. (a) to (j) Find the amount delivered by each patron. (k) to (q) Find the amount received at the creamery each day. (r) Find the sum of (k) to (q) inclusive. (s) Find the sum of (a) to (j) inclusive.

(t) Find the average amount received daily at the creamery.

REVIEW.

1. Common multiples of 15 and 10 are ——, ——, etc. The least common multiple of 15 and 10 is ——.

2. The prime factors of 75 are ——, ——, and ——.

(u) What are the prime factors of 2750? (v) Of 280?

3. 3, 3, and 7 are the prime factors of ——. Sixty-three is exactly divisible by 3; by ——; by ——; and by ——.

(w)	(x)
27 cu. ft.)1296 cu. ft.	27)1296 cu. ft.

COMMON FRACTIONS.

1. Add $\frac{1}{15}$ and $\frac{1}{10}$. The l. c. m. of 15 and 10 is ——.

(a) Find the sum of $534\frac{7}{15}$, $257\frac{1}{10}$, $225\frac{1}{5}$, and $324\frac{1}{6}$.

2. From $8\frac{1}{15}$ subtract $5\frac{7}{10}$. $1\frac{1}{15} = \frac{}{30}$. $\frac{7}{10} = \frac{}{30}$.

(b) Find the difference of $743\frac{2}{15}$ and $181\frac{9}{10}$.

3. Multiply $\frac{7}{15}$ by 8. This means —— ——.

(c) Find the product of $235\frac{7}{15}$ multiplied by 8.

4. Multiply $1\frac{2}{5}$ by $\frac{1}{3}$. This means —— ——.

(d) Find the product of $724\frac{2}{5}$ multiplied by $\frac{1}{3}$. *Story.*

5. Multiply 16 by $\frac{3}{5}$. This means —— ——.

(e) Find the product of 721 multiplied by $\frac{3}{5}$.

6. Multiply 16 by $2\frac{3}{5}$. This means —— ——.

(f) Find the product of 721 multiplied by $7\frac{3}{5}$.

7. Multiply $18\frac{1}{2}$ by $2\frac{1}{3}$. This means —— ——.

(g) Find the product of $432\frac{1}{2}$ multiplied by $4\frac{1}{3}$.

8. Divide 7 by $\frac{4}{5}$. (Change 7 to ——ths.)

(h) Find the quotient of 94 divided by $\frac{4}{5}$. *Story.*

9. Divide $\frac{7}{15}$ by $\frac{1}{10}$. (Change to ——ths.)

(i) Find the quotient of $4\frac{11}{15}$ divided by $\frac{1}{10}$. *Story.*

10. Divide $6\frac{3}{4}$ by $2\frac{1}{2}$. (Change to ——ths.)

(j) Find the quotient of $74\frac{3}{4}$ divided by $3\frac{1}{2}$.

11. Divide $1\frac{5}{6}$ ($\frac{11}{6}$) by 3. This means —— ——.

(k) Find the quotient of $7\frac{5}{6}$ divided by 3.

(l)	(m)	(n)	(o)	(p)
Add.	Subtract.	Multiply.	Divide.	Divide.
$375\frac{11}{15}$	$436\frac{1}{6}$	$753\frac{3}{4}$	$3\frac{1}{5}$ yd.)512 yd.	5)$636\frac{2}{5}$ yd.
$286\frac{9}{10}$	$182\frac{3}{10}$	24		

DECIMAL FRACTIONS.

Division. Case I.

1. Divide .48 by .12. This means —— ——.

(a) Find the quotient of 54.24 divided by .12. *Story.*

2. Divide 6 by .4. This means —— ——.

(b) Find the quotient of 736 divided by .4. *Story.*

3. Divide 3 by .04. This means —— ——.

(c) Find the quotient of 78 divided by .12. *Story.*

4. Divide 4.8 by .12. This means —— ——.

(d) Find the quotient of 40.8 divided by .12. *Story.*
(e) Find the quotient of 43.2 divided by 1.2. *Story.*
(f) Find the quotient of 51.6 divided by .12. *Story.*
(g) Find the quotient of 75 divided by .12. *Story.*

Division. Case II.

5. Divide $.56 by 8. This means —— ——.

(h) Find the quotient of $98.40 divided by 8. *Story.*
(i) Find the quotient of $436.80 divided by 12. *Story.*

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story.

(j) Divide $924 by $7. (k) Divide $924 by 7.
(l) Divide $29.34 by $.06. (m) Divide $29.34 by 6.
(n) Divide $55.3 by $.7. (o) Divide $55.3 by 7.

(p) Multiply $265 by .3. (q) Multiply $265 by 2.3.
(r) Multiply $265 by .03. (s) Multiply $265 by .23.
(t) Multiply $265 by 4.23.

(u) At $.12 a dozen, how many dozen eggs can I buy for $63.84?

DENOMINATE NUMBERS.

(a) Copy, complete, and receipt the following Bill:

WAUKEGAN, ILL., July 30, 1896.

MR. RICHARD YATES,

Bought of C. F. BROWN & Co.

July	1	15	lb. Sugar,	@	$.05		
"	1	5	lb. Raisins,	@	.07		
"	5	$2\frac{1}{2}$	lb. Cheese,	@	.16		
"	8	1	sack Flour,	@		1	25
"	8	$\frac{1}{2}$	bu. Potatoes,	@	$.60		
"	19	5	lb. Coffee,	@	.35		
			Received Payment,			(a)	

REVIEW.

1. 500 sheets of paper are 1 ream and —— sheets.
2. 500 feet are —— rods and —— feet.
3. 500 rods are 1 mile and —— rods.
4. 500 sq. rd. are 3 acres and —— sq. rd.
5. 500 inches are —— feet and —— inches.
6. 500 feet are —— yards and —— feet.
7. 500 pounds are —— —— of a ton.

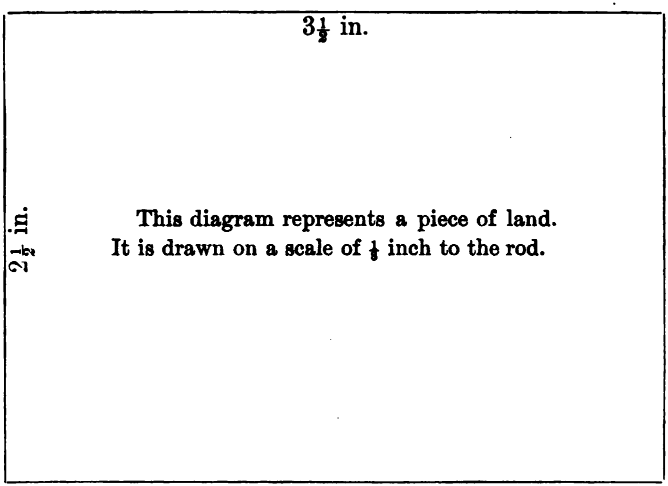
8. When hay is $8 a ton, 5000 lb. cost —— dollars.

(b) Load of straw, gross weight, 3380 lb.; tare, 1580 lb. Find the value at $5.50 a ton.

(c) Multiply 40 sq. rods by 24 and divide the product by 160 square rods. *Story.*

(d) Multiply 24 cubic feet by 8 and the product thus obtained by 6. Divide the last product by 128 cu. ft. *Story.*

MEASUREMENTS.



1. The land represented by the above diagram is —— rods long. It is —— rods wide.

(a) Find the perimeter of the piece of land represented by the diagram. (b) Find its area in square rods. (c) Find its area in acres. (d) How much is the land worth at $40 an acre?

2. Draw carefully upon your slate or paper, on a scale of $\frac{1}{8}$ in. to the rod, a diagram of a rectangular piece of land that is 30 rods by 36 rods.

(e) Find the perimeter of the land represented by the diagram you have drawn. (f) Find its area in square rods. (g) Find its area in acres. (h) How much is the land worth at $60 an acre? (i) How much will it cost to build a fence around it at 50¢ a rod? (j) Find its area in square yards.

RATIO AND PROPORTION.

1. One fifth of 2 (2.0) is ——.* $\frac{1}{5}$ of 3 is ——.

2. One fourth of 6 (6.0) is ——. $\frac{1}{5}$ of 4 is ——.

3. One eighth of 4 (4.0) is ——. $\frac{1}{8}$ of 2 is ——.

(a) Find 1 fourth of 70. (b) Find $\frac{1}{4}$ of 79.

(c) Find 1 fifth of 83. (d) Find $\frac{1}{8}$ of 66.

4. 1 is — — of 27. 2 is — — of 27.

5. 3 is — — of 27, or —.† 4 is — — of 27.

6. 5 is — — of 27. 6 is — — of 27, or —.

7. 7 is — — of 27. 8 is — — of 27.

8. 9 is — — of 27, or —. 10 is — — of 27.

9. 11 is — — of 27. 12 is — — of 27, or —.

10. 13 is — — of 27. 14 is — — of 27.

11. 15 is — — of 27, or —. 16 is — — of 27.

12. 17 is — — of 27. 18 is — — of 27, or —.

13. 1.8 is —— —— of 2.7. 2.7 is —— —— of 1.8.

14. .18 is —— —— of .27. .27 is —— —— of .18.

15. .19 is —— —— of .27. .27 is —— —— of .19.

16. 1.9 is —— —— of 2.7. 2.7 is —— —— of 1.9.

(e) 120 is what part of 140 ? ‡ (f) 140 of 120 ?

(g) 175 is what part of 185 ? (h) 185 of 175 ?

(i) 120 is what part of 128 ? (j) 128 of 120 ?

(k) 120 is what part of 160 ? (l) 160 of 120 ?

(m) 24 is what part of 144 ? (n) 48 of 144 ?

(o) 96 is what part of 144 ? (p) 100 of 144 ?

(q) 120 is what part of 150 ? (r) 125 of 150 ?

* $\frac{1}{5}$ of 20 tenths.

† 3 is three twenty-sevenths of 27, or $\frac{1}{9}$ of 27.

‡ In each such problem as (e), the number that follows the word *of* always becomes the denominator. 120 is $\frac{120}{140} = \frac{12}{14} = \frac{6}{7}$ of 140.

PERCENTAGE.

(1)

1. One % of \$435 =	(a) Find 7% of \$435.
2. One % of \$832 =	(b) Find 9% of \$832.
3. One % of \$38.4 =	(c) Find 8% of \$38.4.
4. One % of \$56.30 =	(d) Find 6% of \$56.30.
5. One % of \$8324 =	(e) Find 5% of \$8324.

(2)

6. 63 is 7% (7 hundredths) of ——.

(f) 364 is 7% of what? (g) 635 is 5% of what?

7. 72 is 9% (9 hundredths) of ——.

(h) 657 is 9% of what? (i) 584 is 8% of what?

8. 54 is 6% (6 hundredths) of ——.

(j) 456 is 6% of what? (k) 504 is 7% of what?

(3)

9. \$12 is —— % of \$200.*	**10.** \$13 is —— % of \$200.
(l) \$54 is what % of \$200?	(m) \$45 is what % of \$200?
11. \$21 is —— % of \$300.	**12.** \$22 is —— % of \$300.
(n) \$84 is what % of \$300?	(o) \$85 is what % of \$300?
13. \$36 is —— % of \$400.	**14.** \$38 is —— % of \$400.
(p) \$96 is what % of \$400?	(q) \$98 is what % of \$400?
15. \$40 is —— % of \$500.	**16.** \$42 is —— % of \$500.
(r) \$95 is what % of \$500?	(s) \$96 is what % of \$500?

*Lead the pupil to observe that he has already learned to solve "Case 3" problems in percentage in two ways. Sometimes he has found what part one number is of another and has changed the fraction thus obtained to hundredths. See problems 11 to 17, page 57. Sometimes he has found one per cent of the base and has used this number as a divisor to obtain the required result. See page 58, problem (o); page 68, problems 3 to 5; page 78, problems 4 to 6. The "Case 3" problems on this page and the greater number of those that follow in Part I., can be most readily solved by the second method. 12 is what % of 200? *One % of 200 is 2; therefore 12 is 6 per cent of 200; 13 is 6½ per cent; 54 is 27 per cent, and 45 is 22½ per cent.*

PERCENTAGE.

1. Three % of \$620 =	(a) Find 9% of \$620.
2. Eighteen is 3% of ——.	(b) \$75 is 3% of what?
3. Eighteen is ——% of 600.	(c) 138 is what % of 600?
4. Eight % of \$220 =	(d) Find 8% of \$738.
5. Forty-eight is 8% of ——.	(e) \$76 is 4% of what?
6. Thirty-six is ——% of 600.	(f) 174 is what % of 600?

(g) A farmer had 825 sheep; he sold 8% of them. How many sheep did he sell? (h) How many sheep did he have left?

(i) A dealer bought some sheep and immediately sold 72 of them; these were 8% of all he purchased. How many sheep did he purchase? (j) How many sheep did he have left?

(k) Mr. B had 400 sheep; he sold 48 of them. What % of his sheep did he sell?

(l) A man had \$485; he spent 7% of his money. How much money did he spend? (m) How much money did he have left?

(n) Peter Gray paid \$35.50 for a suit of clothes; this was 5% of all he earned in a year. How much did he earn in a year?

(o) Judge Branson had \$600 in the bank; he drew out \$198. What % of his deposit did he draw out?

7. A lawyer collected \$800; he retained as his commission \$56 of this sum and paid the remainder, —— dollars, to the man for whom he collected the money; the lawyer's commission for collecting was ——%.

(p) A lawyer collected \$850; he retained as his commission for collecting \$51 of this sum. What per cent did the lawyer charge for making the collection?

REVIEW.

(a) A creamery received milk as follows: Monday, 8250 lb.; Tuesday, 7640 lb.; Wednesday, 8375 lb.; Thursday, 7230 lb.; Friday, 6953 lb.; Saturday, 8424 lb.; Sunday, 7651 lb. Find the number of pounds received during the week. (b) Find the average daily receipts.

1. Add $\frac{11}{15}$ and $\frac{3}{10}$. (Change to ——ths.)

(c) Find the sum of $375\frac{13}{15}$, $246\frac{4}{5}$, $821\frac{7}{10}$, and 234.

(d) Find the product of 372 multiplied by $6\frac{3}{4}$.

(e) Find the product of 372 multiplied by 6.75.

Compare the answers to problems (d) and (e).

(f) Make a bill containing the following items, and find the amount:

> Aug. 2, 24 lb. sugar @ 5¢; Aug. 5, $\frac{1}{2}$ lb. tea @ 70¢; Aug. 10, 2 gal. syrup @ 35¢; Aug. 13, 5 bu. potatoes @ 55¢; Aug. 18, 5 cans peaches @ 18¢; Aug. 18, 5 gal. kerosene @ 15¢.

2. Draw a diagram of a piece of land 40 rods by 36 rods, scale, $\frac{1}{8}$ in. to a rod. The diagram is —— inches long and —— inches wide.

(g) How many square rods in the piece of land described in problem 2? (h) How many acres? (i) The perimeter of the field is how many rods?

3. 3 is — — of 25. 3 twenty-fifths are — hundredths.

4. 4 is — — of 25. 4 twenty-fifths are — hundredths.

(j) 85 is what part of 95? (k) 95 of 85?

(l) 95 is what part of 105? (m) 105 of 95?

(n) 105 is what part of 115? (o) 115 of 105?

MISCELLANEOUS PROBLEMS.

1. Horace spent $\frac{3}{5}$ of his money and had 24¢ left; he spent —— cents. Before spending any money he had —— cents.

(a) Mr. Green spent $\frac{3}{5}$ of his money and had $56.48 left. How much money did he spend? (b) Before spending any money how much did he have?

2. At $2\frac{1}{2}$¢ an ounce, $\frac{1}{4}$ of a pound of cinnamon costs —— cents.

(c) Find the cost of $5\frac{1}{2}$ pounds of cloves at $1\frac{1}{2}$¢ per ounce.

3. A farmer mixed 10 bushels of oats with 15 bushels of corn; together there were —— bushels of grain. What part of the mixture was oats? What part was corn?

(d) A dealer mixed 250 bushels of oats with 350 bushels of corn. What part of the mixture was oats? (e) What part was corn?

4. Eddie, Willie, and Fay divided a melon among themselves. Eddie had 1 fourth of the melon. Will had $\frac{1}{2}$ of what remained after Eddie had taken his share. What part of the melon did Will have? What part of the melon was left for Fay?

(f) An estate was divided as follows: The widow received 1 third of it, and each of five children received 1 fifth of what remained after the widow had received her share. What part of the whole estate did each child receive?

5. A had 3 sevenths of an estate and B had 4 sevenths of it. A's share was equal to what part of B's share?

(g) If the estate mentioned in problem **5** was worth $9660, what was the value of A's share? (h) Of B's share?

SIMLPE NUMBERS.

*Time-book, showing hours worked for one week.**

NAME.	Daily Wages	M.	T.	W.	T.	F.	S.	Hours	Days	Am't earn'd
John Miller	$2.00	10	10	8	9	12	10	(a)	(k)	(u)
T. Benjamin....	2.50	10	9	10	8	10	15	(b)	(l)	(v)
D. Gordon......	3.00	12	12	12	10	12	12	(c)	(m)	(w)
N. Hanson	2.00	9	10	12	11	10	9	(d)	(n)	(x)
A. Gillett........	2.40	10	10	10	10	10	10	(e)	(o)	(y)
Wm. Price	3.50	10	12	12	12	10	11	(f)	(p)	(z)
R. L. Wing.....	2.60	10	9	9	10	12	12	(g)	(q)	(aa)
J. Carlson......	2.60	10	8	0	10	12	12	(h)	(r)	(bb)
S. Spencer	3.00	11	10	12	10	10	12	(i)	(s)	(cc)
S. L. Judd	2.20	10	12	10	0	0	15	(j)	(t)	(dd)
Total hours.....		(ee)	(ff)	(gg)	(hh)	(ii)	jj)	(kk)	(ll)	(mm)

The above is from the time-book of a small manufactory.

(a to j) Find the number of hours worked by each employe.

(k to t) Find the number of days (10 hours) worked by each employe.

(u to dd) Find the amount earned by each employe.

(ee to jj) Find the number of hours of work done each day.

(kk) Find the sum of (a) to (j) inclusive; of (ee) to (jj) inclusive.

(ll) Find the sum of (k) to (t) inclusive.

(mm) Find the sum of (u) to (dd) inclusive.

REVIEW.

1. Common multiples of 15 and 20 are ——, ——, etc. The least common multiple of 15 and 20 is ——.

*Copy this time-book and write numbers in place of the letters; that is, instead of (a), write the number of hours worked by John Miller; in place of (k), write the number of days worked by John Miller, etc. Call 8 hours a day's work.

COMMON FRACTIONS.

1. Add $\frac{1}{8}$, $\frac{1}{10}$, and $\frac{1}{5}$. The l. c. m. of 8, 10, and 5 is ——.

(a) Find the sum of $896\frac{5}{8}$, $507\frac{3}{10}$, and $344\frac{1}{2}$.

2. From $9\frac{1}{8}$ subtract 3.2. $1\frac{1}{8} = \frac{\quad}{40}$ $.2 = \frac{\quad}{40}$.

(b) Find the difference of $1754\frac{3}{8}$ and 592.7.

3. Multiply $2\frac{7}{11}$ by 5. This means ————.

(c) Find the product of $757\frac{8}{11}$ multiplied by 5.

4. Multiply $1\frac{3}{4}$ by $\frac{1}{5}$. This means ————.

(d) Find the product of $536\frac{3}{4}$ multiplied by $\frac{1}{5}$. *Story.*

5. Multiply 21 by $\frac{3}{5}$. This means ————.

(e) Find the product of 681 multiplied by $\frac{3}{5}$.

6. Multiply 21 by $2\frac{3}{5}$. This means ————.

(f) Find the product of 681 multiplied by $8\frac{3}{5}$.

7. Multiply $11\frac{1}{2}$ by $2\frac{1}{2}$. This means ————.

(g) Find the product of $273\frac{1}{2}$ multiplied by $4\frac{1}{2}$.

8. Divide $7\frac{1}{5}$ by $\frac{3}{5}$. (Change $7\frac{1}{5}$ to ——ths.)

(h) Find the quotient of $97\frac{1}{5}$ divided by $\frac{3}{5}$. *Story.*

9. Divide $\frac{5}{8}$ by $\frac{3}{10}$. (Change to ——ths.)

(i) Find the quotient of $7\frac{5}{8}$ divided by $\frac{3}{10}$. *Story.*

10. Divide $7\frac{3}{4}$ by $2\frac{1}{2}$. (Change to ——ths.)

(j) Find the quotient of $86\frac{3}{4}$ divided by $3\frac{1}{2}$.

11. Divide $2\frac{5}{6}$ ($\frac{17}{6}$) by 3. This means ————.

(k) Find the quotient of $8\frac{5}{6}$ divided by 9.

12. Divide $21\frac{3}{4}$ by 5. This means ————.

(l) Find the quotient of $621\frac{3}{4}$ divided by 5.

(m)	(n)	(o)	(p)	(q)
Add.	Subtract.	Multiply.	Divide.	Divide.
$478\frac{7}{8}$	$539\frac{5}{8}$	$738\frac{5}{6}$	$3\frac{1}{2}$ ft.)$720\frac{1}{2}$ ft.	3)$745\frac{3}{4}$ ft.
$896\frac{3}{5}$	182.3	28		

DECIMAL FRACTIONS.

Division. Case I.

1. Divide .072 by .012. This means ———.

(a) $65.088 \div .012 =$ (b) $47.275 \div .025 =$ *Story.*

2. Divide 6 by .05. This means ———.

(c) $468 \div .05 =$ (d) $375.6 \div .05 =$ *Story.**

3. Divide 8 by .004. This means ———.

(e) $75 \div .004 =$ (f) $256.4 \div .004 =$ *Story.*

(g) $38.72 \div .004 =$ (h) $.35 \div .004 =$ *Story.*

(i) $73 \div .025 =$ (j) $.35 \div .025 =$ *Story.*

Division. Case II.

4. Divide $.056 by 8. This means ———.

(k) $\$24.368 \div 8 =$ (l) $\$35.056 \div 8 =$ *Story.*

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story.

(m) $\$336.6 \div \$6 =$ (n) $\$336.6 \div 6 =$

(o) $\$336.6 \div \$.6 =$ (p) $\$336.624 \div 6 =$

(q) $\$356 \times .7 =$ (r) $\$356 \times 2.7 =$

(s) At $.25 a dozen, how many lemons can I buy for $36.75?

(t) I paid $36.75 for 25 barrels of apples. How much did the apples cost per barrel?

(u) If one acre of land is worth $36.75, how much are 5.6 acres worth?

(v) If 2 chairs cost $37.24, how much will 9 chairs cost at the same rate?

* Pupils will most readily give a number story in such work as this, by letting the figures represent units of money, thus: At $.05 each, for $375.6 I could buy 7512 tablets. *When the pupils are well grounded in this work*, they may be encouraged to seek variety in their number stories.

DENOMINATE NUMBERS.

1. *Twelve dozen are one gross.*

2. At 48¢ a gross, 1 dozen buttons cost —— cents.

3. At 5¢ a dozen, 1 gross of buttons cost —— cents.

4. At 50¢ a gross, 6 dozen buttons cost —— cents.

5. At 36¢ a gross, 3 dozen buttons cost —— cents.

6. George bought pens at 50¢ a gross and sold them at the rate of 2 for 1 cent; on 1 gross he gained ——.

(a) Henry bought pens at 60¢ a gross and sold them at the rate of 3 pens for 2 cents. How much did he gain on one gross? (b) How much on 7 gross?

REVIEW.

(c) Make a receipted bill of the following goods sold by yourself to Mr. Frank H. Armstrong:

Aug. 2, 15 lb. shingle nails @ 4¢; Aug. 10, 6 lb. fence staples @ 5¢; Aug. 20, 130 lb. No. 9 fence wire @ $2\frac{1}{2}$¢; Aug. 25, 1 keg 20d. nails @ $3.25.*

7. One fourth of a ream of paper is —— sheets.

8. One fourth of a ton of coal is —— pounds.

9. One fourth of a mile —— rods.

10. When coal is $6 per ton, 7000 lb. cost —— dollars.

(d) Load of bran; gross weight, 2850 lb.; tare, 1050 lb. Find the value at $7.50 a ton.

(e) Multiply 36 square feet by 24, and divide the product thus obtained by 9 square feet. *Story.*

(f) Multiply 12 cubic feet by 15, and the product thus obtained by 6. Divide the last product by 27 cubic feet. *Story.*

* Ask the pupil to bring to school a shingle nail, a fence staple, a piece of No. 9 wire, and a 10d. nail; also to learn the current price of the articles named.

MEASUREMENTS.

1. Draw carefully upon your slate or paper, on a scale of $\frac{1}{8}$ inch to the rod, a diagram of a rectangular piece of land that is 30 rods by 40 rods.

(a) Find the perimeter of the land represented by the diagram you have drawn.

(b) Find its area in square rods.

(c) Find its area in acres.

(d) How much is the land worth at $72.50 an acre?

(e) How much will it cost to build a fence around it at 45¢ a rod?

(f) A strip 4 rods wide across one side of the field contains how many square rods?

(g) A strip 8 rods wide across one side of the field contains how many acres?

2. Land 5 rods by 40 rods contains ———.

3. Land 7 rods by 40 rods contains ———.

4. Land 9 rods by 40 rods contains ———.

Find the number of acres in each of the following:

(h) 32 rods by 5 rods. (i) 32 rods by 26 rods.
(j) 32 rods by 22 rods. (k) 32 rods by 34 rods.

Find the number of cords in each of the following:

(l) 12 ft. by 8 ft. by 4 ft. (m) 16 ft. by 6 ft. by 4 ft.
(n) 20 ft. by 4 ft. by 4 ft. (o) 21 ft. by 8 ft. by 4 ft.

Find the number of square yards in each of the following:

(p) 30 feet by 40 feet. (q) 25 feet by 33 feet.
(r) 18 feet by 21 feet. (s) 35 feet by 40 feet.

Find the number of cubic yards in each of the following:

(t) 12 ft. by 6 ft. by 9 ft. (u) 15 ft. by 12 ft. by 6 ft.
(v) 15 ft. by 9 ft. by 12 ft. (w) 21 ft. by 6 ft. by 18 ft.

RATIO AND PROPORTION.

1. One inch is —— —— of a yard.

2. Two inches are —— —— of a yard.

3. Three inches are —— —— of a yard.

4. Four inches are —— —— of a yard.

5. One foot and 2 inches are —— —— of a yard.

6. One foot and 8 inches are —— —— of a yard.

7. One square rod is —— —— of an acre.

8. Two square rods are —— —— of an acre.

(a) Twelve square rods are what part of an acre?

(b) Twenty-eight square rods are what part of an acre?

(c) Thirty-six square rods are what part of an acre?

9. 100 square rods are —— —— of one acre. 1 acre is —— —— of 100 square rods, or —— and —— —— times 100 square rods.

10. Two thirds of 1 ft. 6 in. are 3 fourths of —— inches.

(d) Two thirds of 6 ft. 9 in. are 3 fourths of how many in.?

(e) Two thirds of 96 sq. rd. are 1 half of how many sq. rd.?

11. If 36 sq. rd. of land is worth $60, 48 sq. rd. is worth —— dollars.

(f) If 36 sq. rd. of land is worth $75.48, how much is 48 sq. rd. worth at the same rate?

12. If 2 ft. 3 in. of silver wire is worth 24¢, 1 yard is worth —— cents.

(g) If 2 ft. 3 in. of platinum wire is worth $1.65, how much is 1 yd. worth at the same rate?

13. If 4 sq. rd. of land is worth $5, 1 acre is worth —— dollars.

(h) If 4 sq. rd. of land is worth $7.25, how much is 1 acre worth at the same rate?

PERCENTAGE.

(*1*)

1. One % of 640 =
2. $1\frac{1}{3}$% of 640 =
3. One % of 420 =
4. $2\frac{1}{2}$% of 420 =
5. One % of $972 =
(a) Find $7\frac{1}{3}$% of $972.
6. One % of $876 =
(b) Find $9\frac{1}{2}$% of $876.
7. One % of $46.80 =
(c) Find $8\frac{1}{3}$% of $46.80.

(*2*)

8. 84 is 7% (7 hundredths) of ——.
(d) 595 is 7% of what?
(e) 876 is 6% of what?
9. 63 is 9% (9 hundredths) of ——.
(f) 747 is 9% of what?
(g) 737 is 11% of what?
10. 72 is 12% (12 hundredths) of ——.
(h) 572 is 11% of what?
(i) 560 is 10% of what?*

(*3*)

11. 18 is ——% of 300.
12. 19 is ——% of 300.
(j) 87 is what % of 300?
(k) 88 is what % of 300?
13. 24 is ——% of 400.
14. 26 is ——% of 400.
(l) 196 is what % of 400?
(m) 380 is what % of 400?
15. 42 is what % of 600?
16. 46 is ——% of 600.
(n) 384 is what % of 600?
(o) 576 is what % of 600?

17. Two % of 430 =
(p) Find 9% of 430.
18. Twenty is 4% of ——.
(q) 96 is 4% of what?
19. Thirty is ——% of 600.
(r) 276 is what % of 600?

20. Twenty-five per cent of 24 is what % of 200?
21. Seventy-five per cent of 24 is what % of 300?

*The pupil should see that he may solve this problem as he solved problem (h), or he may solve it by making use of the fact that 10% = 1 tenth.

PERCENTAGE.

1. A nursery-man set out 200 trees; 8% of them died; —— trees were dead and —— trees were alive.

(a) A nursery-man set out 650 trees; 8% of them died. How many trees were dead? (b) How many lived?

2. A farmer sold 21 bushels of oats; these were 7% of all he raised; he raised —— bushels and had —— bushels left.

(c) A farmer sold 98 bushels of oats; these were 7% of all he raised. How many bushels did he raise? (d) How many bushels had he left?

3. A grocer sorted 200 bushels of apples; 12 bushels were "specked" and the remainder were sound; ——% of the apples had begun to decay and ——% of them were sound.

(e) A grocer sorted 400 bushels of apples; 56 bushels were "specked" and the remainder were sound. What % of the apples had begun to decay? (f) What % were sound?

4. Mr. Briggs earns $900 a year; he saves 8% of his wages; he saves —— dollars and spends —— dollars.

(g) Mr. Jones earns $1250 a year; he saves 8% of his wages. How much money does he save?

5. Mr. Piper paid $18 for a suit of clothes; this was 12% of his month's salary; his salary was —— dollars.

(h) Mr. Brean paid $162 for a horse, buggy, and harness; this was 12% of his year's salary. How much was his yearly salary?

6. George earned $300; he spent $15; he spent ——% of what he earned.

(i) Henry earned $350; he spent $63. What per cent of what he earned did he spend?

REVIEW.

A manufacturer employed 20 men. The daily wages of these men were as follows: Five received $3 00 each; five received $2.50 each; five received $2.00 each; and five received $1.50 each.

(a) Find the whole amount paid to these men each day.

(b) Find the average daily wages of the men.

(c) Find the amount paid per week to the men if every man worked every day except Sunday.

(d) Find the amount paid for the month of January, if the month began on Monday and every man was present every day except the Sundays of the month.

(e) Divide 336 by $4\frac{4}{5}$. (Change to ——ths.) *Story.*

(f) Divide 336 by 4.8. ($336.0 \div 4.8 = ?$) *Story.*

Compare the answers to problems (e) and (f).

(g) Bought a ream of paper for $2.00 and sold it at 18¢ per quire. How much was my gain?

(h) Bought a box (1 gross) of pens for 75¢ and sold them at the rate of 4 pens for 3 cents. How much did I gain?

(i) If 10 bushels of potatoes cost $3.50, how much will 15 bu. cost at the same rate?

1. Draw a diagram of a piece of land 32 rods by 25 rods, scale $\frac{1}{8}$ in. to a rod. The diagram is —— inches long and —— and —— —— inches wide.

(j) How many square rods in the piece of land described in problem 1? (k) How many acres? (l) The perimeter of the field is how many rods? (m) How much will it cost to build a fence around it at 35¢ a rod? (n) How much will it cost to plow it at $1.25 an acre?

2. 7 is — — of 25. 7 twenty-fifths are — hundredths.

MISCELLANEOUS PROBLEMS.

1. A woman exchanged $3\frac{1}{2}$ lb. of butter at 20¢ a pound for sugar at 5¢ a pound. She received —— pounds of sugar.

(a) A man exchanged 9 cords of wood @ $4.50 a cord for coal at $6.75 a ton. How many tons of coal did he receive? (b) How many pounds of coal did he receive? (c) How many loads of 3000 lbs. each did he receive?

2. The side of a square measures 5 ft. 8 in.; the perimeter of the square is —— feet —— inches.

(d) The side of a square measures 47 ft. 10 in. What is the measurement of the perimeter of the square?

3. An estate was divided among a widow and four children; the widow received 1 third of the estate; each child received 1 fourth of what remainded after the widow had received her share; each child received —— —— of the estate.

(e) The estate mentioned in problem 3 was worth $7140. How much did the widow receive? (f) How much did all the children together receive? (g) What each child received was equal to what part of what the widow received?

4. The width of an oblong is 8 inches; its length is twice its width; its perimeter is —— inches; its area is —— square inches.

(h) The width of a piece of land in the shape of an oblong is 32 rods; its length is twice its width. How many rods of fence required to enclose it? (i) How much will the fence cost at 35¢ a rod? (j) How many square rods in the piece? (k) How many acres in the piece? (l) What is it worth at $27.50 an acre?

5. At $1.40 a yard, $5\frac{1}{2}$ yd. of carpet cost ——.

(m) At $1.28 a yard, how much will $27\frac{1}{4}$ yd. of carpet cost?

SIMPLE NUMBERS.*

1. The sum of all the odd numbers from 1 to 7 inclusive is ——; of all the even numbers from 2 to 8 inclusive, ——.

(a) Add all the odd numbers from 1 to 37 inclusive.

(b) Add all the even numbers from 2 to 38 inclusive.

2. $5+4+6+7+3+8+6+5+4+3+5+6+3+7+5+9-50=$ ——.

(c) From the sum of 9864, 792, and 8756, subtract 4598.

(d) Multiply 8754 by 27.	(e) $9328 \times 46 =$
(f) Multiply 5964 by 38.	(g) $3896 \times 52 =$
(h) Multiply 3875 by 45.	(i) $2543 \times 69 =$
(j) Multiply 874 by 126.	(k) $963 \times 245 =$
(l) Multiply 556 by 208.	(m) $831 \times 306 =$
(n) Multiply 734 by 160	(o) $673 \times 250 =$
(p) Divide 7618 by 52.	(q) $12551 \div 49 =$
(r) Divide 9351 by 54.	(s) $11292 \div 48 =$
(t) Divide 9636 by 55.	(u) $9545 \div 46 =$
(v) Divide 7532 by 56.	(w) $10635 \div 45 =$
(x) Divide 10336 by 57.	(y) $10505 \div 44 =$
(z) Divide 3960 by 27.	(aa) $8757 \div 27 =$
(bb) Divide 6888 by 27.	(cc) $5649 \div 27 =$
(dd) Divide 6848 by 128.	(ee) $4768 \div 128 =$
(ff) Divide 5776 by 128.	(gg) $9696 \div 128 =$
(hh) Divide 2772 by 144.	(ii) $3960 \div 144 =$
(jj) Divide 4860 by 144.	(kk) $7932 \div 144 =$
(ll) Divide 6440 by 160.	(mm) $14280 \div 160 =$
(nn) Divide 11800 by 160.	(oo) $5952 \div 160 =$

*To the Teacher.—If pupils are inaccurate in their work in simple numbers, give them daily practice in exercises similar to those appearing upon this page. Lead the pupils to feel that nothing short of perfect accuracy is highly commendable. A "90% paper" in arithmetic is a *failure*. A "90% accountant" is *worthless*.

COMMON FRACTIONS.

1. Add $\frac{1}{9}$, $\frac{1}{4}$, and $\frac{1}{6}$. The l. c. m. of 9, 4, and 6 is ——.

(a) Add $346\frac{2}{9}$, 375, $486\frac{1}{4}$, 296, and $855\frac{5}{6}$.

2. From $12\frac{1}{7}$ subtract $3\frac{1}{2}$. $15\frac{1}{8} - 2\frac{1}{2} =$

(b) From 2381 subtract $154\frac{1}{3}$. (c) $596\frac{3}{8} - 148\frac{1}{2} =$

3. Multiply $3\frac{5}{9}$ by 4. **4**. Multiply $7\frac{3}{5}$ by 8.

(d) Multiply $537\frac{7}{9}$ by 7. (e) Multiply $375\frac{2}{5}$ by 9.

5. Multiply 12 by $2\frac{1}{5}$. **6**. Multiply 11 by $3\frac{1}{4}$.

(f) Multiply 874 by $2\frac{1}{5}$. (g) Multiply 954 by $3\frac{1}{4}$.

7. Multiply $11\frac{1}{2}$ by $4\frac{1}{5}$. This means ————.

(h) Multiply $373\frac{1}{2}$ by $4\frac{1}{5}$. (i) Multiply $525\frac{1}{2}$ by $4\frac{2}{5}$.

8. Divide $8\frac{1}{3}$ by $\frac{5}{6}$. (Change $8\frac{1}{3}$ to ——ths.)

(j) Divide $48\frac{1}{3}$ by $\frac{5}{6}$. (k) Divide $17\frac{2}{3}$ by $\frac{5}{6}$.

9. Divide $6\frac{3}{5}$ by $1\frac{1}{2}$. $6\frac{3}{5} = \frac{\quad}{10}$. $1\frac{1}{2} = \frac{\quad}{10}$.

(l) Divide $75\frac{3}{5}$ by $1\frac{1}{2}$. (m) Divide $82\frac{3}{5}$ by $1\frac{1}{2}$.

10. Divide $1\frac{1}{6}$ ($\frac{7}{6}$) by 3. **11**. Divide $19\frac{1}{6}$ by 3.*

(n) Divide $577\frac{1}{6}$ by 3. (o) Divide $625\frac{1}{6}$ by 3.

12. James rode his wheel at the rate of $9\frac{2}{5}$ mi. an hour: In 4 hours he rode —— and —— —— miles. In $\frac{1}{4}$ hour he rode —— and —— —— miles. In $4\frac{1}{4}$ hours he rode —— and —— —— miles.

A locomotive moved at the rate of $49\frac{3}{5}$ miles an hour:

(p) How far did it move in 4 hours? (q) In 5 hours?

(r) How far did it move in $\frac{1}{4}$ of an hour? (s) In $\frac{3}{4}$ of an hr.?

(t) How far did it move in $4\frac{1}{4}$ hours? (u) In $5\frac{3}{4}$ hours?

13. If $\frac{5}{6}$ of a yard of print is sufficient for a child's apron, $8\frac{1}{3}$ yards are sufficient for —— aprons. $8\frac{1}{3} + \frac{5}{6} =$

(v) $79\frac{1}{3}$ yards divided by $2\frac{1}{6}$ yards.

* Do not change $19\frac{1}{6}$ to sixths. Say, rather, *1 third of $19\frac{1}{6}$ is 6, with a remainder of $1\frac{1}{6}$*; 1 third of $1\frac{1}{6}$ ($\frac{7}{6}$) is $\frac{7}{18}$.

DECIMAL FRACTIONS.*

$\frac{1}{2}$ = — tenths.	$\frac{1}{2}$ = — hundredths.	$\frac{1}{2}$ = — thousandths.
$\frac{1}{3}$ = — tenths.	$\frac{1}{3}$ = — hundredths.	$\frac{1}{3}$ = — thousandths.
$\frac{2}{3}$ = — tenths.	$\frac{2}{3}$ = — hundredths.	$\frac{2}{3}$ = — thousandths.
$\frac{1}{4}$ = — tenths.	$\frac{1}{4}$ = — hundredths.	$\frac{1}{4}$ = — thousandths.
$\frac{3}{4}$ = — tenths.	$\frac{3}{4}$ = — hundredths.	$\frac{3}{4}$ = — thousandths.
$\frac{1}{5}$ = — tenths.	$\frac{1}{5}$ = — hundredths.	$\frac{1}{5}$ = — thousandths.
$\frac{2}{5}$ = — tenths.	$\frac{2}{5}$ = — hundredths.	$\frac{2}{5}$ = — thousandths.
$\frac{3}{5}$ = — tenths.	$\frac{3}{5}$ = — hundredths.	$\frac{3}{5}$ = — thousandths.
$\frac{4}{5}$ = — tenths.	$\frac{4}{5}$ = — hundredths.	$\frac{4}{5}$ = — thousandths.
$\frac{1}{6}$ = — tenths.	$\frac{1}{6}$ = — hundredths.	$\frac{1}{6}$ = — thousandths.
$\frac{5}{6}$ = — tenths.	$\frac{5}{6}$ = — hundredths.	$\frac{5}{6}$ = — thousandths.
$\frac{1}{7}$ = — tenths.	$\frac{1}{7}$ = — hundredths.	$\frac{1}{7}$ = — thousandths.
$\frac{1}{8}$ = — tenths.	$\frac{1}{8}$ = — hundredths.	$\frac{1}{8}$ = — thousandths.
$\frac{1}{9}$ = — tenths.	$\frac{1}{9}$ = — hundredths.	$\frac{1}{9}$ = — thousandths.

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story.

(a) Multiply $345 by .7.
(b) Multiply $345 by 5.7.
(c) Multiply $345 by .08.
(d) Multiply $345 by .28.
(e) Multiply $345 by 3.28.
(f) Multiply $345 by .005.
(g) Multiply $345 by .025.
(h) Multiply $345 by .325.
(i) Divide $648 by $9.
(j) Divide $648 by 9.
(k) Divide $648 by $.9.
(l) Divide $648 by $.09.
(m) Find the cost of 7.2 tons of coal at $4.75 a ton.

(n) I paid $38.25 for coal at $4.50 per ton. How many tons did I buy?

(o) I paid $663 for 156 tons of coal. What was the price per ton?

* The pupil will probably need a pencil in the solution of some of the problems in the first fourteen lines. After he is able to fill the blanks, require him to write them as suggested in the following. $\frac{1}{2} = .5$; $\frac{1}{2} = .50$; $\frac{1}{2} = .500$; $\frac{1}{3} = .3\frac{1}{3}$; $\frac{1}{3} = 33\frac{1}{3}$; $\frac{1}{3} = .333\frac{1}{3}$, etc.

DENOMINATE NUMBERS.

100 lb. = 1 **hundredweight** (cwt.).

1. At 50¢ per cwt., 1 ton of bran costs —— dollars.

2. At 40¢ per cwt., $\frac{1}{2}$ ton of feed costs —— dollars.

3. At 60¢ per cwt., $\frac{1}{4}$ ton of corn meal costs ——.

(a) At $2.50 per cwt., how much will $\frac{3}{4}$ of a ton of flour cost? (b) $\frac{9}{10}$ of a ton? (c) $\frac{4}{5}$ of a ton?

4. At $2.40 per cwt., 50 lb. of flour are worth ——.

5. At 50¢ per cwt., 1050 lb. cost ——; 850 lb. cost ——; 1250 lb. cost ——; 950 lb. cost ——; 1150 lb. cost ——.

(d) At $2.70 per cwt., how much will 1450 lb. cost? (e) 1950 lb.? (f) 1350 lb.?

6. At $2.40 per cwt., 25 lb. of flour are worth ——.

7. At 60¢ per cwt., 25 lb. are worth —— cents; 75 lb. are worth —— cents; 175 lb. are worth ——.

(g) At $2.60 per cwt., how much will 1225 lb. cost? (h) 1275 lb.? (i) 1525 lb.?

8. At $2.50 per cwt., 20 lb. of flour are worth —— cents.

9. At 60¢ per cwt., 20 lb. are worth —— cents; 40 lb. are worth —— cents; 60 lb. are worth —— cents; 80 lb. are worth —— cents.

(j) At $2.50 per cwt., how much will 1220 lb. cost; (k) 1240 lb.? (l) 1260 lb.?

10. At $2.30 per cwt., 10 lb. of flour are worth ——.

11. At 80¢ per cwt., 10 lb. of corn are worth —— cents; 20 lb. are worth —— cents; 30 lb. are worth —— cents; 40 lb. are worth —— cents; 50 lb. are worth —— cents; 60 lb. are worth —— cents; 70 lb. are worth —— cents.

(m) At $2.50 per cwt., how much will 1210 lb. cost?

MEASUREMENTS.

1. Draw carefully upon your slate or paper, on a scale of $\frac{1}{4}$ in. to the foot, a diagram of the floor of a room that is 17 ft. long and 14 ft. wide.

(a) How many square feet in the floor?

(b) How many feet of flooring will be required for the floor if, on account of the waste and the loss in matching, you must buy 1 fourth more feet of flooring than there are square feet to be covered?

(c) Making no allowance for doors, how many feet in length are the mop-boards of the room?

(d) If this floor is to be covered with a carpet that is 1 yard wide and the strips are to "run lengthwise" of the room, how many feet long will each strip be?

(e) How many strips must be purchased?

(f) How much of one strip must be turned under or cut off?

(g) How many feet (in length) of carpet must be purchased if there is no waste in matching the strips?

(h) How many yards must be purchased?

(i) How much will the carpet cost at 60¢ a yard?

(j) If the figure of the carpet is such that there is waste in matching, more carpet must be purchased than would otherwise be required. How much more if the waste on each strip (except the first) is 1 foot?

(k) How much, including the waste, will the carpet cost at 60¢ a yard?

(l) If the same floor is to be covered with carpeting that is 1 yard wide, costing 40¢ a yard, the strips to run lengthwise, and the waste in matching being 6 in. on each strip except the first, how much will the carpet cost?

RATIO AND PROPORTION.

1. Six inches are —— —— of a rod.

2. Twelve inches are —— —— of a rod.

3. One foot and 6 inches are —— —— of a rod.

4. Two feet are —— —— of a rod.

5. Two feet and six inches are —— —— of a rod.

6. Three feet and six inches are —— —— of a rod.

7. Four feet and six inches are —— —— of a rod.

8. One rod is —— —— of a mile. 2 rods are ——.

9. Three rods are —— —— of a mile. 4 rods are ——.

(a) Twelve rods are what part of a mile? (b) 10 rods? (c) 20 rods? (d) 30? (e) 40 rods? (f) 60 rods?

10. 200 rods are —— —— of a mile. 1 mile is —— —— of 200 rods, or —— and —— —— times 200 rods.

11. 12 feet are —— —— of a rod. 1 rod is —— —— of 12 feet, or —— and —— —— times 12 feet.

12. Two thirds of $7\frac{1}{2}$ feet ($\frac{15}{2}$) are 1 half of —— feet.

13. Two thirds of $4\frac{1}{2}$ ft. ($\frac{9}{2}$) are 1 half of —— feet.

(g) Two thirds of $25\frac{1}{2}$ ft. are 1 half of how many feet?

(h) Two thirds of $37\frac{1}{2}$ rd. are 1 half of how many rods?

14. If a piece of rubber hose $5\frac{1}{2}$ feet long is worth 60¢, a piece $16\frac{1}{2}$ feet is worth ———.

(i) If it is worth $1.65 to make $5\frac{1}{2}$ ft. of concrete walk, how much is it worth to make $16\frac{1}{2}$ ft. of similar walk?

15. Twenty-five rods are —— —— of $37\frac{1}{2}$ rods. $37\frac{1}{2}$ rods are —— —— of 25 rods, or —— and —— —— times 25 rods.

(j) If it costs $75.30 to make 25 rods of road, how much will it cost to make $37\frac{1}{2}$ rods at the same rate?

PERCENTAGE.

(1)

1. 50% more than 60 is ——.* 50% less than 60 =†
2. 25% more than 60 is ——. 25% less than 60 =
3. 20% more than 60 is ——. 20% less than 60 =

(a) 50% more than \$846 = (b) 50% less than \$846 =
(c) 25% more than \$576 = (d) 25% less than \$576 =
(e) 20% more than \$475 = (f) 20% less than \$475 =

(2)

4. 60 is 50% more than ——.‡ 60 is 50% less than ——.§
5. 60 is 25% more than ——. 60 is 25% less than ——.
6. 60 is 20% more than ——. 60 is 20% less than ——.

(g) \$420 is 50% more than how many dollars?
(h) \$420 is 50% less than how many dollars?
(i) \$420 is 25% more than how many dollars?
(j) \$420 is 25% less than how many dollars?
(k) \$420 is 20% more than how many dollars?
(l) \$420 is 20% less than how many dollars?

(3)

7. 50 is ——% more than 40.‖ 30 is ——% less than 40.¶
8. 60 is ——% more than 50. 35 is ——% less than 70.
9. 30 is ——% more than 20. 40 is ——% less than 50.

(m) \$660 is how many per cent more than \$440?
(n) \$630 is how many per cent less than \$840?

* 50% of 60 more than 60. 50% = $\frac{1}{2}$. $\frac{1}{2}$ of 60 = 30. 60 + 30 = 90.

† 50% of 60 less than 60. 50% = $\frac{1}{2}$. $\frac{1}{2}$ of 60 = 30. 60 − 30 = 30.

‡ Think of x as standing for the number sought (*the base*). Then 60 = x and 1 half of x, or $\frac{3}{2}$ of x. Since 60 = $\frac{3}{2}$ of x, 1 half of x equals 20, and the whole of x = 2 times 20, or 40.

§ 60 = x less 1 half of x, or $\frac{1}{2}$ of x,

‖ 50 is 10 more than 40, or $\frac{10}{40}$ *of 40 more than 40*. $\frac{10}{40} = \frac{1}{4}$ = 25%.

¶ 30 is 10 less than 40, or $\frac{10}{40}$ *of 40 less than 40*. $\frac{10}{40} = \frac{1}{4}$ = 25%.

PERCENTAGE.

1. Ira earns 80 cents a day; Ernest earns 25% more than Ira; Ernest earns —— a day. Arthur earns 25% less than Ira; Arthur earns —— a day.

(a) Mr. James earns \$860 a year; Mr. Brown earns 25% more than Mr. James. How much does Mr. Brown earn? (b) Mr. White earns 25% less than Mr. James. How much does Mr. White earn?

2. Peter earns \$60 a month; this is 25% more than Paul earns; Paul earns —— a month.*

(c) Mr. Harris earns \$1200 a year; this is 25% more than Mr. Williams earns. How much does Mr. Williams earn?

3. Richard earns \$60 a month; this is 25% less than Harry earns; Harry earns —— a month.†

(d) Mr. Smith earns \$840 a year; this is 25% less than Mr. Rice earns. How much does Mr. Rice earn?

4. George earns \$40 a month; Joseph earns \$50 a month; George earns —— per cent less than Joseph; Joseph earns —— per cent more than George.‡

(e) Mr. Dow earns \$1000 a year; Mr. Wheeler earns \$1250 a year. Mr. Wheeler earns how many per cent more than Mr. Dow? (f) Mr. Dow earns how many per cent less than Mr. Wheeler?

15 is ——% more than 10.	10 is ——% less than 15.
20 is ——% more than 15.	15 is ——% less than 20.
25 is ——% more than 20.	20 is ——% less than 25.

* Think of x as standing for what Paul earns. Then \$60 = x and 1 fourth of x, or 5 fourths of x. If \$60 is $\frac{5}{4}$ of x, then x is —— dollars.

† Let x stand for what Harry earns. Then \$60 = x less 1 fourth of x, or $\frac{3}{4}$ of x. If \$60 is $\frac{3}{4}$ of x, then x is —— dollars.

‡ In the first part of this problem Joseph's money is the base, and in the second part George's money is the base. Joseph earns \$10 more than \$40, or $\frac{1}{4}$ of \$40 more than \$40. George earns \$10 less than \$50, or $\frac{1}{5}$ of \$50 less than \$50.

REVIEW.

(a) Find the sum of all the odd numbers from 59 to 73 inclusive.

(b) Find the sum of all the even numbers from 64 to 86 inclusive.

(c) Divide 12 by $\frac{3}{4}$. (d) Multiply 12 by $\frac{4}{3}$.

Compare the answers to problems (c) and (d).

1. $\frac{1}{8}$ = —— hundredths. $\frac{1}{8}$ = —— thousandths.

(e) Change $\frac{3}{8}$ to hundredths. (f) Change $\frac{3}{8}$ to 1000ths.

(g) Change $\frac{5}{8}$ to hundredths. (h) Change $\frac{5}{8}$ to 1000ths.

(i) Change $\frac{7}{8}$ to hundredths. (j) Change $\frac{7}{8}$ to 1000ths.

2. At $3 per cwt., 500 lb. of pork cost —— dollars; 550 lb. cost ——; 510 lb. cost ——; 520 lb. cost ——.

(k) At $3.60 per cwt., what is the value of 500 lb. of pork? (l) Of 550 lb.? (m) Of 510 lb.? (n) Of 520 lb.?

3. At 60¢ a gross, 1 dozen pens cost —— cents; 3 dozen cost —— cents; 7 dozen cost —— cents.

(o) At $5.28 a gross, what is the value of 1 dozen fruit-cans? (p) Of 8 dozen?

(q) At $7.20 a gross, what is the cost of 3 dozen fruit-cans? (r) Of 3 cans?

4. A floor 15 feet by 17 feet is to be covered with carpet that is 1 yard wide. If the strips are laid lengthwise of the room, and there is no waste in matching the figure, —— strips will be required, each of which is —— feet long.

(s) How many yards of carpet will be required for the room described in problem 4? (t) How much will it cost at 90¢ a yard?

MISCELLANEOUS PROBLEMS.

1. The state of Illinois is about 380 miles long and 225 miles wide at its widest part. A map of Illinois drawn upon a scale of 10 miles to the inch must be —— inches long and —— and —— —— inches wide.

2. A certain county of Illinois appears upon the map described in problem 1 as a rectangular figure 1.8 inches by 3 inches. The county is —— miles wide and —— miles long.

(a) How many square miles in the county described in problem 2?

Tell the amount of change that should be given in each of the following instances:

3. Bought $4\frac{1}{2}$ lb. meat @ 12¢; gave the salesman $1.

4. Bought $3\frac{1}{4}$ lb. cheese @ 16¢; gave the salesman $1.

5. Bought $2\frac{3}{4}$ lb. cheese @ 16¢; gave the salesman 50¢.

6. Bought $2\frac{1}{2}$ dozen eggs @ 14¢; gave the salesman $2.

(b) Bo't 2.3 tons coal @ $6.50; gave the salesman $20.

(c) Bo't 1.4 tons hay @ $9.50; gave the salesman $15.

(d) Bo't $3\frac{1}{2}$ cords wood @ $4.50; gave the salesman $16.

7. Byron bought 8 melons for 50¢; he sold one half of them at 10¢ each and the other half at 5¢ each; he gained —— cents.

(e) A drover bought 36 sheep for $125; he sold one half of them at $4 each and the other half at $4.50 each. How much did he gain?

8. A house rents for $10 per month; the rent for one year and six months is —— dollars.

(f) A house rents for $32 per month. How much is the rent for 2 years and 4 months?

SIMPLE NUMBERS.*

TICKETS OF ADMISSION SOLD AT A COUNTY FAIR.

	Price	Wed.	Thur.	Fri.	Sat.	Total.
Children's Tickets.......	15¢	1645	1154	3561	1424	(e)
Adults' Tickets	25¢	2243	1754	3871	2124	(f)
Single Carriage Tickets..	25¢	143	174	186	75	(g)
Double Carriage Tickets.	50¢	123	175	162	137	(h)
		(a)	(b)	(c)	(d)	

(a) to (d) Find the number of tickets sold each day.

(e) Find the number of children's tickets sold.

(f) Find the number of adults' tickets sold.

(g) Find the number of single carriage tickets sold.

(h) Find the number of double carriage tickets sold.

(i) Find the sum of (a) to (d) inclusive, and (e) to (h) inclusive.

(j) Find the receipts for tickets sold Wednesday; (k) Thursday; (l) Friday; (m) Saturday; (n) total receipts for tickets sold.

(o) Find the receipts for all the children's ticket sold during the week; (p) adults' tickets; (q) single carriage tickets; (r) double carriage tickets.

(s) Find the sum of (o), (p), (q), and (r), and compare this sum with the answer to problem (n).

1. From June 28th to July 3rd it is —— days.†

2. From June 28th to Aug. 3rd it is —— days.‡

3. From June 28th to Sept. 3rd it is —— days.

(t) How many days from June 28th to Dec. 25th? (u) From April 20th to Dec. 10th?

* Insist upon accuracy: "90 per cent of accuracy" is failure.

† To June 30th is 2 days; to July 3rd, 3 more days.

‡ Two and 31 and 3.

COMMON FRACTIONS.

Reduce the following fractions to their lowest terms:

1. $\frac{24}{36} =$ $\frac{18}{27} =$ $\frac{21}{56} =$ $\frac{24}{40} =$ $\frac{15}{35} =$ $\frac{18}{48} =$

(a) $\frac{240}{360} =$ (b) $\frac{170}{275} =$ (c) $\frac{94}{224} =$ (d) $\frac{96}{256} =$

Reduce the following improper fractions to whole or mixed numbers:

2. $\frac{27}{7} =$ $\frac{34}{8} =$ $\frac{51}{6} =$ $\frac{52}{8} =$ $\frac{65}{22} =$ $\frac{37}{11} =$

(e) $\frac{375}{8} =$ (f) $\frac{476}{7} =$ (g) $\frac{348}{9} =$ (h) $\frac{278}{6} =$

Reduce the following mixed numbers to improper fractions:

3. $7\frac{3}{5} =$ $9\frac{2}{7} =$ $8\frac{5}{6} =$ $12\frac{3}{4} =$ $11\frac{2}{3} =$ $10\frac{7}{8} =$

(i) $56\frac{5}{7} =$ (j) $74\frac{3}{5} =$ (k) $86\frac{2}{9} =$ (l) $73\frac{5}{8} =$

Reduce the following fractions to equivalent fractions having a common denominator:

4. $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{6}$. The l. c. m. of 8, 4, and 6 is ——,

(m) $\frac{5}{9}$ and $\frac{7}{8}$. (n) $\frac{6}{11}$ and $\frac{2}{7}$. (o) $\frac{5}{12}$ and $\frac{3}{10}$.

5. Change 8 to a fraction whose denominator is 5.

(p) Change 47 to a fraction whose denominator is 6.

6. The sum of two numbers is 78; one of the numbers is 32; the other number is ——.

(q) The sum of two fractions is $\frac{8}{9}$; one of the fractions is $\frac{2}{5}$. What is the other fraction?

7. If it takes $\frac{3}{4}$ of a yard of cloth to make one vest, from three yards —— vests can be made.

(r) How many vests, each containing $\frac{3}{4}$ of a yard, can be made from 36 yards? (s) From 48 yards? (t) From 60 yards?

DECIMAL FRACTIONS.

Change the following common fractions to decimal fractions:

1. $\frac{1}{2} =$ $\frac{1}{4} =$ $\frac{3}{4} =$ $\frac{2}{5} =$ $\frac{3}{5} =$ $\frac{4}{5} =$

(a) $\frac{1}{3} =$ * (b) $\frac{2}{3} =$ (c) $\frac{1}{6} =$ (d) $\frac{5}{6} =$ (e) $\frac{1}{7} =$

(f) $\frac{2}{7} =$ (g) $\frac{3}{7} =$ (h) $\frac{4}{7} =$ (i) $\frac{5}{7} =$ (j) $\frac{6}{7} =$

(k) $\frac{1}{9} =$ (l) $\frac{2}{9} =$ (m) $\frac{4}{9} =$ (n) $\frac{5}{9} =$ (o) $\frac{7}{9} =$

Change the following decimal fractions to common fractions and reduce to their lowest terms:

2. $.8 =$ $.5 =$ $.4 =$ $.25 =$ $.75 =$ $.12\frac{1}{2} =$ † $.3\frac{1}{3} =$

(p) $.125 =$ (q) $.375 =$ (r) $.625 =$ (s) $.875 =$

(t) $.175 =$ (u) $.225 =$ (v) $.325 =$ (w) $.675 =$

(1) Tell the meaning of each of the following, (2) solve, and (3) tell a suggested number story: ‡

(x) Multiply \$475 by .9. (y) Multiply \$475 by 2.9.
(z) Multiply \$534 by .07. (aa) Multiply \$534 by .37.
(bb) Multiply \$534 by 2.37. (cc) Multiply \$534 by .003.
(dd) Multiply \$534 by .043. (ee) Multiply \$534 by .243.

(ff) Divide \$724 by \$8. (gg) Divide \$724 by 8.
(hh) Divide \$724 by \$.8. (ii) Divide \$724 by \$.08.

(jj) At \$12.50 per ton, find the cost of 3.7 tons of hay.
(kk) At \$9.75 per ton, find the cost of 3.4 tons of hay.

* Change these to thousandths. There are 1000 thousandths in a whole (a unit); in $\frac{1}{3}$ of a unit there are $\frac{1}{3}$ of 1000 thousandths. This answer may be written, $.333\frac{1}{3}$ or 333+.

† Observe that you can divide both the numerator and denominator of $12\frac{1}{2}$ hundredths by $12\frac{1}{2}$.

‡ Do not omit the number story, or business problem. The number story for problem (x) might be: *Mr. Hoyt bought .9 of an acre of land at \$475 an acre; the land cost him —— dollars.*

DENOMINATE NUMBERS.

Find the cost:*

1. 3000 lb. hay at $6 per ton.	**2**. 500 lb. hay @ $6.
3. 1500 lath at $3 per M.	**4**. 250 lath @ $3.
5. 1250 lb. pork at $4 per cwt.	**6**. 125 lb. pork @ $4.
7. 2500 lb. coal at $5 per ton.	**8**. 400 lb. coal @ $5.
9. 650 lb. beef @ $5 per cwt.	**10**. 25 lb. beef at $5.
11. 2500 brick @ $8 per M.	**12**. 250 brick @ $8.

Find the value:

13. 1 brick at $8 per M.	**14**. 100 brick @ $8.
15. 1 lb. hay at $10 per ton.	**16**. 100 lb. hay @ $10.
17. 1 lb. of beef @ $4 per cwt.	**18**. 10 lb. beef @ $4.
(a) 2150 brick at $8 per M.	(b) 625 brick @ $8.
(c) 2150 lb. pork at $4 per cwt.	(d) 625 lb. pork @ $4.
(e) 2150 lb. hay at $10 per ton.	(f) 650 lb. hay @ $10.
(g) 3240 lath at $3 per M.	(h) 750 lath @ $3.
(i) 3240 lb. coal @ $5.50 per ton.	(j) 760 lb. coal @ $5.50.
(k) 3240 lb. beef @ $6.50 per cwt.	(l) 86 lb. beef @ $6.50.

19. At 60¢ a gross, 2 doz. buttons cost —— cents.

(m) At $1.60 per gross, 75 doz. buttons cost how much?

(n) Make a receipted bill of the following goods sold by yourself to Christopher Columbus:

Sept. 1, 1230 lb. pork @ $4.60 per cwt., and 1080 lb. beef @ $6.75 per cwt.

20. At $2.40 per ream, 1 quire of paper costs —— cents; 12 sheets cost —— cents; 6 sheets cost —— cents.

(o) Bought paper @ $2.40 per ream and sold it at 15¢ a quire. What was the gain on 3 reams?

(p) At 7¢ a quire, how much will 1440 sheets of paper cost?

* Pork and beef are usually bought and sold by the hundredweight (cwt.), hay and coal by the ton, and lath and brick by the thousand (M.).

MEASUREMENTS.

1. Draw carefully upon your slate or paper, on a scale of $\frac{1}{8}$ in. to the foot, a diagram of a rectangular grass plot 50 ft. by 40 ft. Represent a gravel walk 5 feet wide just outside the grass plot and extending entirely around it. The diagram of the grass plot is —— and —— —— inches long and —— inches wide. The width of the walk as shown in the diagram is —— —— of an inch. The entire length of the figure including the diagram of the grass plot and walk is —— and —— —— inches.

(a) How many square feet in the grass plot described in problem 1? (b) How many square yards?

(c) The perimeter of the grass plot described in problem 1 is how many feet? (d) How many yards?

(e) The perimeter of the outside of the gravel walk described in problem 1, is how many feet? (f) How many yards?

(g) The outside perimeter of the gravel walk described in problem 1 is how much greater than the inside perimeter? (h) How many square feet in the walk? (i) How many square yards in the walk? (j) How much did it cost to make the walk at 54¢ per square yard?

2. A floor 14 ft. by 17 ft. is to be covered with a carpet that is one yard wide. If the breadths are 17 feet long, —— breadths will be required. If the breadths are 14 ft. long, —— breadths will be required.

(k) If there is no waste in matching, how many yards of carpet will be required for the room described in problem 2 if 17-foot strips are used? (l) How many yards if 14-foot strips are used?

RATIO AND PROPORTION.

1. One cwt. is —— —— of a ton. 2 cwt. =

2. Three cwt. are —— —— of a ton. 4 cwt. =

3. Five cwt. are —— —— of a ton. 6 cwt. =

4. Seven cwt. are —— —— of a ton. 8 cwt. =

5. Fifteen cwt. are —— —— of a ton. One ton is —— —— of 15 cwt., or —— and —— —— times 15 cwt.

6. Sixteen cwt. are —— —— of a ton. One ton is —— —— of 16 cwt., or —— and —— —— times 16 cwt.

7. Eight cwt., are —— —— of a ton. One ton is —— —— of 8 cwt., or —— and —— —— times 8 cwt.

8. Twelve cwt. of hay is —— —— of a ton of hay.

(a) If one ton of hay is worth $12.75, how much is 12 cwt. of hay worth at the same rate?

9. Eighteen cwt. of coal is —— —— of a ton of coal.

(b) If one ton of coal is worth $6.50, how much is 18 cwt. of coal worth at the same rate?

10. Six cwt. of flour is —— —— of a ton of flour. One ton of flour is —— —— of 6 cwt. of flour, or —— and —— —— times cwt.

(c) If 6 cwt. of flour is worth $12.30, how much is 1 ton of flour worth at the same rate?

11. Forty pounds are —— —— of 1 cwt.

(d) If 1 cwt. of sugar is worth $5.25, how much is 40 lb. of sugar worth?

12. Eighty pounds are —— —— of 1 cwt. 1 cwt. is —— —— of 80 lb., or —— —— times 80 lb.

(e) If 80 lb. of beef is worth $5.60, how much is 1 cwt. of beef worth at the same rate?

PERCENTAGE.

1. 40% more than 25 is ——.* 40% less than 25 is ——.

2. $16\frac{2}{3}$% more than 30 is ——. $16\frac{2}{3}$% less than 30 is ——.

3. $33\frac{1}{3}$% more than 36 is ——. $33\frac{1}{3}$% less than 36 is ——.

(a) 40% more than \$570 = (b) 40% less than \$570 =
(c) $16\frac{2}{3}$% more than \$834 = (d) $16\frac{2}{3}$% less than \$834 =
(e) $33\frac{1}{3}$% more than \$726 = (f) $33\frac{1}{3}$% less than \$726 =

(2)

4. 49 is 40% more than ——.† 36 is 40% less than ——.‡

5. 42 is $16\frac{2}{3}$% more than ——. 40 is $16\frac{2}{3}$% less than ——.

6. 48 is $33\frac{1}{3}$% more than ——. 30 is $33\frac{1}{3}$% less than ——.

(g) \$245 is 40 per cent more than how many dollars?
(h) \$831 is 40 per cent less than how many dollars?
(i) \$658 is $16\frac{2}{3}$ per cent more than how many dollars?
(j) \$645 is $16\frac{2}{3}$ per cent less than how many dollars?

(3)

7. 70 is ——% more than 50.§ 50 is ——% less than 60 ‖

8. 40 is ——% more than 30. 30 is ——% less than 50.

(k) \$450 is how many per cent less than \$750?
(l) \$400 is how many per cent less than \$480?
(m) \$480 is how many per cent more than \$360?

* 40% of what? 40% = $\frac{2}{5}$. $\frac{2}{5}$ of 25 = 10. 25 + 10 = 35.

† Think of x as standing for the number sought (the base). Then 49 = x and 2 fifths of x, or $\frac{7}{5}$ of x. Since 49 is $\frac{7}{5}$ of x, 1 fifth of x is 7, and the whole of x = 5 times 7, or 35.

‡ 36 = x less 2 fifths of x, or $\frac{3}{5}$ of x.

§ 70 is 20 more than 50, or $\frac{20}{50}$ of 50 more than 50. $\frac{20}{50} = \frac{2}{5}$. $\frac{2}{5}$ = 40%.

‖ 50 is 10 less than 60, or $\frac{10}{60}$ of 60 less than 60. $\frac{10}{60} = \frac{1}{6}$, or $16\frac{2}{3}$%.

PERCENTAGE.

1. Hiram had \$50; Samuel had \$40; Hiram had ——% more than Samuel; Samuel had ——% less than Hiram.

(a) Mr. Cooper has 880 bushels of corn; Mr. Judd has 660 bu. Mr. Judd has how many per cent less than Mr. Cooper? (b) Mr. Cooper has how many per cent more than Mr. Judd?

2. In 1896 a farmer raised 60 bu. of oats to the acre; this was a 50% better yield than he had in 1895; in 1895 he raised —— bushels to the acre.*

(c) In 1896 a farmer sold 726 bushels of wheat; this was 50% more than he sold in 1895. How many bushels did he sell in 1895?

3. William earns \$30 a month; Benjamin earns 40% more than William; Benjamin earns —— dollars a month. Joseph earns 50% more than Benjamin; Joseph earns —— dollars a month.

(d) Isaac earns \$27.50 a month; Ralph earns 40% more than Isaac. How much does Ralph earn per month? (e) Clinton earns 50% more than Ralph. How much does Clinton earn per month?

4. Good cheese is worth 18¢ a pound; this is 50% higher than it was 1 year ago; a year ago it was worth —— cents.

(f) A certain brand of flour is worth \$5.55 per barrel. If this is 50% more than it was worth a year ago, what was it then worth?

18 is ——% more than 12. 12 is ——% less than 18.
24 is ——% more than 18. 18 is ——% less than 24.

* Think of x as standing for the yield of 1895. Then 60 bu. = x and 1 half of x, or $\frac{3}{2}$ of x. Since 60 bu. = $\frac{3}{2}$ of x, x = —— bushels.

REVIEW.

1. From April 25th to May 5th it is —— days.

(a) How many days from April 25th to August 5th?

2. From Feb. 20th of a leap-year to March 10th, it is —— days.

(b) How many days from Feb. 20th of a leap-year to July 4th?

(c) Divide 64 by $\frac{4}{5}$. (d) Multiply 64 by $\frac{5}{4}$.

Compare the answers to problems (c) and (d).

3. $\frac{1}{9}$ = —— hundredths. $\frac{1}{9}$ = —— thousandths.

(e) Change $\frac{2}{9}$ to hundredths. (f) Change $\frac{2}{9}$ to 1000ths.

(g) Change $\frac{4}{9}$ to hundredths. (h) Change $\frac{4}{9}$ to 1000ths.

Change the following decimal fractions to common fractions and reduce them to their lowest terms:

4. $.37\frac{1}{2}$ = * $.62\frac{1}{2}$ = $.87\frac{1}{2}$ = $.16\frac{2}{3}$ = $.33\frac{1}{3}$ =

(i) .15 = (j) .85 = (k) .65 = (l) .75 = (m) .55 =

Find the cost:

5. 5500 brick at \$8 per M. **6.** 250 brick @ \$8.

(n) 7600 brick at \$6 per M. (o) 520 brick @ \$6.

(p) There is a 4-foot gravel walk around a rectangular grass plot that is 30 ft. by 40 ft. How many feet in the perimeter of the grass plot? (q) How many feet in the perimeter of the outside of the walk? (r) How many square feet in the grass plot? (s) How many square feet in the gravel walk?

(t) If one ton of coal is worth \$7.50, how much is 14 cwt. of coal worth?

(u) If one hundredweight of corned beef is worth \$4.50, 90 lb. of beef is worth how much?

* Divide the numerator and the denominator of $\frac{37\frac{1}{2}}{100}$ by $12\frac{1}{2}$.

MISCELLANEOUS PROBLEMS.

1. I am thinking of the surface of a box that is 6 in. long, 4 in. wide, and 3 in. high. The surface of the bottom is —— square inches. The surface of one side is —— square inches. The surface of one end is —— square inches. The surface of the entire box—top, bottom, sides, and ends—is —— square inches.

(a) Find the entire surface of a box that is 15 in. long, 12 inches wide, and 8 inches high.

(b) Make a receipted bill of the following goods sold by yourself to your teacher:

March 6, 150 lb. sugar @ $4\frac{1}{2}$¢; 7 lb. coffee @ 35¢;
March 18, 4 lb. cheese @ 15¢; 1 vinegar barrel, $1.00;
March 25, 6 gal. kerosene @ 12¢; 3 gal. syrup @ 45¢.

(c) A farmer sold 15 head of cattle weighing 19650 lb. at $5.40 cwt. How much did he receive for them? (d) What was the average weight per head?

(e) If there is 1 half a ream in a package of paper, how many sheets in 6 such packages?

(f) A merchant had $275 in the bank Monday morning. His deposits for the week were as follows: Monday, $86; Tuesday, $55; Wednesday, $72; Thursday, $64; Friday, $83; Saturday, $124. He drew from the bank during the week $354.24. How much remained on deposit?

2. I gave 3 doz. eggs, worth 15¢ a dozen, in part payment for 20 lb. sugar at 4¢ a pound. There remains unpaid —— cents.

(g) I gave 2250 lb. hay at $10 per ton in part payment for 2 tons of coal @ $6.50 per ton. How much remains unpaid?

SIMPLE NUMBERS.

1. The sum of two numbers is 38; one of the numbers is 12; the other number is ——.

(a) The sum of two numbers is 12346; one of the numbers is 4734. What is the other number?

2. The difference of two numbers is 17; the less number is 45; the greater number is ——.

(b) The difference of two numbers is 547; the less number is 3476. What is the greater number?

3. The difference of two numbers is 14; the greater number is 45; the less number is ——.

(c) The difference of two numbers is 607; the greater number is 4045. What is the less number?

4. In a problem the multiplier is 6 and the product is \$42; the multiplicand is ——.

$$\begin{array}{rl} ? & \text{multiplicand.} \\ 6 & \text{multiplier.} \\ \hline \$42 & \text{product.} \end{array}$$

(d) In a problem the multiplier is fifteen and the product is nine hundred forty-five. What is the multiplicand?

5. In a problem the multiplicand is \$25 and the product is \$125; the multiplier is ——.

$$\begin{array}{rl} \$25 & \text{multiplicand.} \\ ? & \text{multiplier.} \\ \hline \$125 & \text{product.} \end{array}$$

(e) In a problem the multiplicand is two hundred thirty-five dollars and the product is five thousand four hundred five dollars. What is the multiplier?

6. In a problem the divisor is \$12 and the quotient is 8; the dividend is —— ——.

$$\begin{array}{r} \$12)\,? \\ \hline 8 \end{array}$$

(f) In a problem the divisor is \$146 and the quotient is 27. What is the dividend?

7. In a problem the quotient is 12 and the divisor is \$9; the dividend is —— ——.

$$\begin{array}{r} \$9)\quad \\ \hline 12 \end{array}$$

COMMON FRACTIONS.

1. Three fourths of 36 feet are —— feet. $\frac{3}{4}$ of 37 =

(a) Three fourths of 984 feet are how many feet?

(b) Three fourths of 985 = (c) Three fourths 986 =

2. 36 feet are three fourths of —— feet. 37 ft. are three fourths of —— and —— —— feet. 38 ft. are $\frac{3}{4}$ of ——.

(d) 576 ft. are three fourths of how many feet?

(e) 577 is three fourths of what? (f) 578 is $\frac{3}{4}$ of what?

3. Paul spent $\frac{1}{2}$ of the money his father gave him for a book and $\frac{1}{3}$ of it for a knife, and had 12¢ left. Before he spent any money he had —— cents.*

(g) A man spent $\frac{1}{2}$ of his month's wages for fuel and $\frac{1}{3}$ of it for groceries, and had $17.25 left. How much was his wages? (h) How much did he spend for fuel? (i) How much did he spend for groceries?

4. Harris paid $4 for chickens at $\frac{2}{5}$ of a dollar each. He bought —— chickens. $4 \div \frac{2}{5}$ means, *find how many times 2 fifths are contained in* 4 (20 *fifths*).

(j) A man paid $375 for wheat at $\$\frac{3}{5}$ a bushel. How many bushels did he buy?

(k) A man paid $368 for apples at $\$1\frac{3}{5}$ a barrel. How many barrels did he buy?

5. Ellis sold $2\frac{1}{2}$ cords of wood at $\$4\frac{1}{2}$ a cord; he should receive for the wood —— and —— —— dollars. ($2\frac{1}{2}$ times $4\frac{1}{2}$ means, 2 *times* $4\frac{1}{2}$ *and* 1 *half of* $4\frac{1}{2}$.)

(l) A man sold $12\frac{1}{2}$ acres of land at $\$56\frac{1}{2}$ an acre. How much should he receive for the land?

* Think of x as standing for the money his father gave him. Then he spent 1 half of x and 1 third of x, or 5 sixths of x, and had —— —— of x remaining.

DECIMAL FRACTIONS.

1. At $60 an acre, 3.2 acres of land are worth —— dollars. 3.2 times 60 means, 3 *times* 60, *plus* .2 *of* 60.

(a) At $85 an acre, how much are 6.7 acres of land worth? (b) 6.2 acres at $75 an acre?

2. At $70 an acre, 3.02 acres of land are worth ——. 3.02 times 70 means, 3 *times* 70, *plus* 2 *hundredths of* 70.

(c) At $65 an acre, how much are 5.03 acres of land worth? (d) 8.06 acres at $95 an acre?

3. At $40 an acre, 2.24 acres of land are worth —— dollars. 2.24 times 40 means, 2 *times* 40, *plus* 2 *tenths of* 40, *plus* 4 *hundredths of* 40.

(e) At $45 an acre, how much are 4.35 acres of land worth? (f) 7.26 acres at $64 an acre?

Find the cost:

(g) 5.34 acres @ $265.
(h) 7.34 acres @ $465.
(i) 6.23 acres @ $52.5.
(j) 7.03 acres @ $325.
(k) 3.27 acres @ $43.5.
(l) 5.37 acres @ $54.6.
(m) 1.56 acres @ $276.
(n) 24.3 acres @ $342.
(o) 32.6 acres @ $41.6.

$265	Price per acre.
5.34	Number of acres.
$10.60	Value of .04 of an acre.
$79.5	Value of .3 of an acre.
$1325.	Value of 5 acres.
$1415.10	Value of 5.34 acres.

NOTE.—While the pupil is multiplying by 4, a *separatrix* may stand between the 2 and 6 of the multiplicand; thus, $\$2^{\vee}65$. This will help him to remember that he is really multiplying $2.65, the value of 1 hundredth of an acre, by 4. When he is ready to multiply by 3, the separatrix should be erased and written thus: $\$26^{\vee}5$. This will help him to remember that he is really multiplying $26.5, the value of 1 tenth of an acre, by 3. After a time he can simply imagine the separatrix in its place. Require the pupil to write the decimal point in each partial product when, in the process of multiplication, he reaches the place where it belongs. The pupil may now be taught that when he has solved a problem in multiplication of decimals, if he has "pointed off" correctly, *the decimal places in the product will be equal to those in the multiplicand and multiplier counted together.*

DENOMINATE NUMBERS.

1. From March 26th to April 2d, it is —— days. If March 26th is Monday, April 2d is ——.

2. From April 20th to May 5th, it is —— days, or —— weeks and —— day. If April 20th is Friday, May 5th is ——.

3. From April 20th to May 12th it is —— days, or —— weeks and —— day. If April 20th is Friday, May 12th is ——.

(a) How many days from April 20th to June 9th?

(b) If April 20th is Friday, what day of the week is June 9th? (c) June 12th?

4. In a year in which there is a Feb. 29th, there are —— days, or —— weeks and —— days.

5. In a year in which there is no Feb. 29th, there are —— days, or —— weeks and —— day.

6. If the first day of February of a common year is Monday, the first day of February of the next year is ——.

7. If the first day of February of a leap-year is Monday, the first day of February of the next year is ——.

8. If the tenth of February of a leap-year is Saturday, the tenth day of February of the next year is ——.

9. If the 17th day of April of a leap-year is Wednesday, the 17th day of April of the next year is ——.

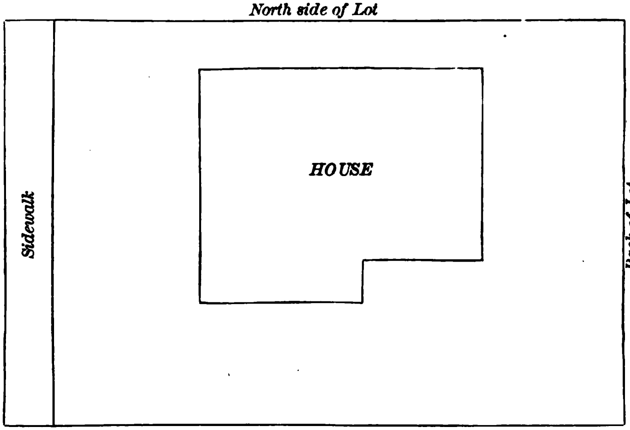
Find how many weeks and days:

(d) Apr. 10 to July 4.* (e) May 5 to July 10.
(f) May 18 to Aug. 5. (g) June 15 to Oct. 4.
(h) July 12 to Sept. 1. (i) Aug. 1 to Sept. 25.

* Think as suggested in the following; April 10 to Apr. 30, 20 days; Apr. 30 to May 31, 31 days, May 31 to June 30, 30 days; June 30 to July 4, 4 days. 20 days + 31 days + 30 days + 4 days = ?

MEASUREMENTS.

The following diagram of a house and lot is drawn on a scale of 24 feet to an inch.



(a) How many feet long is the lot, not including the walk?

(b) How many feet wide is the lot?

(c) How many feet from the sidewalk to the house?

(d) How far from the house to the back of the lot?

(e) How far from the house to the north side of the lot?

(f) How many feet long is the house?

(g) How many feet wide is the front of the house?

(h) How many feet wide is the rear of the house?

(i) How far from the south side of the lot to the house?

(j) How wide is the sidewalk? (k) How much will the sidewalk cost at 12¢ per square foot? *

(l) How much is the lot worth at $25 a foot front? †

* There are 306 square feet in the walk. In finding the cost the pupil may think that at 1¢ a foot it would cost $3.06, and at 12¢ a foot, 12 times $3.06; or he may think that if 1 foot costs 12¢, 306 feet would cost 306 times 12¢.

† The expression "*foot front*" stands for a strip 1 foot wide and as long as the lot is "deep."

RATIO AND PROPORTION.

1. One ton is —— and —— —— times 800 lb. If 800 lb. of hay is worth $3.00, 1 ton is worth —— dollars.

(a) If 800 lb. of flour is worth $18.50, how much is 1 ton worth at the same rate?

2. One mile is —— times 80 rd. If it costs $30 to build 80 rd. of fence, to build one mile of similar fence will cost —— dollars.

(b) If it costs $375 to make 80 rd. of road, how much will it cost to make 1 mile of road at the same rate?

3. One hour is —— and —— —— times 24 minutes. Harry rode 5 miles in 24 minutes; this was at the rate of —— an hour.

(c) A locomotive moved 19 miles in 24 minutes. This was equal to what rate per hour?

4. One minute is —— times 20 seconds. A pump rod made 18 strokes in 20 seconds; this was at the rate of —— strokes a minute.

(d) A certain pulley revolves 106 times in 20 seconds. What is its rate of revolution per minute? (e) How many times does it revolve in one hour?

(f) Find how many times your pulse beats in 20 seconds. This is at what rate per minute? Per hour?

5. One pound is —— and —— —— times 12 ounces. If 12 oz. cheese is worth 15¢, 1 lb. is worth ——.

(g) If 12 ounces of onion seed is worth 72¢, how much is 1 pound worth?

6. If 50 sq. rods of land is worth $20, at the same rate 1 acre is worth —— dollars.

(h) If 50 square rods of land is worth $345, how much is 1 acre worth at the same rate?

PERCENTAGE.

(1)

1. $12\frac{1}{2}\%$ more than 56 is ——. $12\frac{1}{2}\%$ less than 56 =
2. 10% more than 60 is ——. 10% less than 60 =
3. 30% more than 50 is ——. 30% less than 50 =

(a) $12\frac{1}{2}\%$ more than \$972 = (b) $12\frac{1}{2}\%$ less than \$972 =
(c) 10% more than \$845 = (d) 10% less than \$845 =
(e) 30% more than \$650 = (f) 30% less than \$650 =

(2)

4. 45 is $12\frac{1}{2}\%$ more than ——.* 49 is $12\frac{1}{2}\%$ less than ——.†
5. 77 is 10% more than ——. 54 is 10% less than ——.
6. 39 is 30% more than ——. 28 is 30% less than ——.

(g) \$747 is $12\frac{1}{2}$ per cent more than how many dollars?
(h) \$868 is $12\frac{1}{2}$ per cent less than how many dollars?
(i) \$572 is 10 per cent more than how many dollars?
(j) \$729 is 10 per cent less than how many dollars?
(k) \$299 is 30 per cent more than how many dollars?

(3)

7. 88 is ——% more than 80. 56 is ——% less than 64.
8. 63 is ——% more than 56. 35 is ——% less than 50.

(l) \$350 is how many per cent less than \$500?
(m) \$450 is how many per cent more than \$400?

* Think of x as standing for the number sought. Then $45 = x$ and 1 eighth of x, or $\frac{9}{8}$ of x. Since 45 is 9 eighths of x, 1 eighth of x is 5, and the whole of x is 8 times $5 = 40$.

† $49 = x$ less 1 eighth of x, or $\frac{7}{8}$ of x. Since 49 is $\frac{7}{8}$ of x, 1 eighth of x is 7, and the whole of x is 8 times $7 = 56$.

PERCENTAGE.

1. Five years ago the population of a certain city was 6000; it has increased $33\frac{1}{3}$ %; its present population is ——.

(a) Eight years ago the population of a certain city was 9750; it has increased $33\frac{1}{3}$%. What is its present population?

2. A man sold a horse for $100 which was 25% more than he gave for it. He gave —— dollars for the horse.

(b) A man sold a farm for $6825 which was 25% more than he gave for it. How much did he give for the farm?

3. Alice has 50¢; Jane has 60¢; Jane has ——% more than Alice; Alice has ——% less than Jane.*

(c) Mr. Lyon has 600 acres of land; Mr. Whitney has 400 acres. Mr. Lyon has how many per cent more than Mr. Whitney? (d) Mr. Whitney has how many per cent less than Mr. Lyon?

4. By selling a horse for $60, the owner would lose 25%; the horse cost —— dollars.

(e) By selling a farm for $4320, the owner would lose 25%. How much did the farm cost him?

5. By selling a horse for $60, the owner would gain 25%; the horse cost —— dollars.

(f) By selling a farm for $5300, the owner would gain 25%. How much did the farm cost him?

6. 25% of the vinegar in a cask leaked out and 36 gal. remained; before the leakage there were —— gallons.

(g) 25% of the water in a tank leaked out and 465 gallons remained. How many gallons in the tank before the leakage?

* In one part of this problem 50 is the base; in the other part, 60 is the base.

REVIEW.

1. The sum of three numbers is 27; two of the numbers are 8 and 10; the other number is ——.

(a) The sum of three numbers is 2756; two of the numbers are 784 and 975. What is the other number?

2. Arthur rode on his bicycle three consecutive hours; the first hour he rode 12 miles; the second hour, 10 miles, and the third hour 8 miles; his average speed per hour was —— miles.

(b) The attendance at a certain school for one week was as follows: Monday, 35; Tuesday, 38; Wednesday, 37; Thursday, 36; Friday, 34. What was the average daily attendance?

3. At $\$7\frac{1}{2}$ per ton, $2\frac{1}{2}$ tons of coal cost ——.*

(c) At $\$348\frac{1}{2}$ an acre, how much will $5\frac{1}{2}$ acres cost?

(d) At $348.50 an acre, how much will 6.5 acres cost?

(e) Compare the answers to (c) and (d). How much is their difference?

4. The first day of January, 1897, was Friday. Tell the day of the week of the first day of January of each of the following years: 1898, ——; 1899, ——; 1900, ——; 1901;† ——; 1902, ——; 1903, ——; 1904, ——; 1905, ——; 1906, ——; 1907, ——; 1908, ——; 1909, ——; 1910, ——.

(f) Upon what day of the week will the first day of January, 1925, occur?

5. A plot of a certain garden is drawn on a scale of 20 feet to an inch. A line $3\frac{3}{4}$ inches long represents —— feet.

(g) A certain map is drawn on a scale of 25 mi. to an inch. A line $15\frac{1}{2}$ in. long represents how many miles?

* 2 times $7\frac{1}{2}$ and 1 half of $7\frac{1}{2}$.

† Remember that the year 1900 is not a leap year.

MISCELLANEOUS PROBLEMS.

1. In a pane of glass 9 in. by 12 in. there are —— sq. inches.

(a) How many square inches in 36 panes of glass each 9 in. by 12 in.? (b) How many square feet?

2. Mr. Black received $30 per month as rent for a house. In one year he received —— dollars.

(c) At $35 per month, how much is the rent of a house for 2 years and 6 months?

3. I paid $2.00 for coffee at 25¢ a pound; I purchased —— pounds.

(d) Paid $34.75 for coffee at 25¢ a pound. How many pounds were purchased?

4. In a floor 12 ft. by 12 ft. there are —— square feet; there are —— square yards.

(e) In a lot 24 feet by 96 feet, there are how many square feet? (f) How many square yards?

5. If a train moves at the rate of 20 miles an hour, to move 110 miles will require —— hours.

(g) If a train moves at the rate of 35 miles an hour, how long will it take to go 1000 miles?

6. A boy bought 10 chickens for 25¢ each, and 10 for 35¢ each; the average price paid was —— cents.

(h) A man bought 10 horses at $135 each and 10 at $124.50 each. What was the average price?

7. A man sold a horse at $\frac{4}{5}$ of what it cost him, thereby losing $10; the horse cost him —— dollars; he sold it for —— dollars.

(i) A man sold a farm for $\frac{4}{5}$ of what it cost him, thereby losing $1275. How much did the farm cost him? (j) For how much did he sell it?

SIMPLE NUMBERS.

Review page 11.

1. Name five odd numbers; five even numbers.

2. Name three exact divisors of 36; of 48; of 75.

Review page 21.

3. Name five integral numbers; five fractional numbers; five mixed numbers.

4. $\frac{13}{20}$ and .7 are —— numbers. $4\frac{1}{2}$ and 7.2 are ———.

Review page 31.

5. Name five prime numbers; five composite numbers.

6. Which of the following are prime and which are composite? 2, 22, 5, 37, 45, 49, 53, 72, 87.

Review page 41.

7. What are the prime factors of 36? Of 35? Of 34? Of 33?

8. Of what number are 2, 2, 3, and 5 the prime factors?

Review page 51.

9. Name three common multiples of 4 and 6.

10. What is the least common multiple of 4 and 6?

Review page 61.

(a) Multiply 746 by 20. (b) Multiply 394 by 80.
(c) Multiply 547 by 300. (d) Multiply 834 by 700.

Review page 71.

(e) Divide 891 by 30. (f) Divide 1265 by 50.
(g) Divide 728 by 40. (h) Divide 2478 by 70.

Review page 81.

(i) A farmer bought 30 sheep; for 5 of them he paid \$6 per head; for 10 he paid \$5 per head; for the remainder he paid \$70. How much did the 30 sheep cost him? (j) What was the average price per head?

COMMON FRACTIONS.

Review page 12.

1. Name three fractions that have a common denominator; three that do not have a common denominator.

2. Change the following to equivalent fractions having a common denominator: $\frac{2}{7}$ and $\frac{4}{5}$.

Review page 32.

3. Tell the terms of each of the following: $\frac{3}{7}$, $\frac{5}{9}$, $\frac{7}{11}$.

4. Reduce each of the following to its lowest terms: $\frac{12}{24}$, $\frac{18}{12}$, $\frac{15}{20}$, $\frac{45}{81}$, $\frac{27}{33}$. (a) $\frac{240}{380}$. (b) $\frac{575}{625}$.

5. Reduce each of the following to a whole or mixed number: $\frac{16}{3}$, $\frac{25}{5}$, $\frac{24}{7}$, $\frac{32}{10}$, $\frac{97}{12}$. (c) $\frac{335}{8}$. (d) $\frac{476}{9}$.

Review page 42.

6. Reduce each of the following to an improper fraction: $9\frac{2}{5}$, $7\frac{5}{8}$, $11\frac{2}{3}$, $5\frac{7}{9}$. (e) $28\frac{5}{9}$. (f) $47\frac{5}{6}$. (g) $94\frac{3}{7}$. (h) $86\frac{5}{12}$.

Review page 52.

(i) $\frac{7}{8} + \frac{5}{12} =$	(j) $\frac{4}{15} + \frac{7}{10} =$	(k) $\frac{7}{10} + \frac{5}{12} =$
(l) $\frac{5}{6} - \frac{7}{9} =$	(m) $\frac{8}{15} - \frac{3}{10} =$	(n) $\frac{9}{10} - \frac{7}{12} \doteq$
(o) $\frac{5}{8} \times \frac{2}{3} =$	(p) $\frac{5}{11} \times \frac{3}{4} =$	(q) $\frac{3}{7} \times \frac{4}{5} =$
(r) $\frac{11}{12} \div \frac{3}{8} =$	(s) $\frac{8}{15} \div \frac{1}{10} =$	(t) $\frac{9}{10} \div \frac{5}{12} =$

Review pages 62 and 72.

(u) Find the product of $794\frac{1}{2}$ multiplied by $6\frac{1}{2}$.*

(v) Find the quotient of $835\frac{1}{2}$ bu. divided by $2\frac{1}{4}$ bu.*

(w) Find the quotient of $654\frac{1}{3}$ bu. divided by 9.*

* Solve, and tell a suggested number story.

DECIMAL FRACTIONS.

$$\begin{array}{r} {}^{\vee}6.40 \\ 3.7 \\ \hline 4.480 \\ 19.20 \\ \hline 23.680 \end{array}$$

$$\begin{array}{r} {}^{\vee}73.42 \\ 3.56 \\ \hline 4.4052 \\ 36.710 \\ 220.26 \\ \hline 261.3752 \end{array}$$

$.005\overline{)38.455^{\vee}}$ *
$7691.$

$.05\overline{)38.47^{\vee}5}$ †
769.5

$.5\overline{)38.4^{\vee}75}$ ‡
76.95

$5\overline{)38.^{\vee}475}$ §
7.695

$.5\overline{)78.0^{\vee}}$ ‖
$156.$

$.05\overline{)78.00^{\vee}}$ ¶
$1560.$

Review pages 13, 23, 33, 43, 53, 63, 73, and 133.

Observe again the fact that when a problem in multiplication of decimals has been solved accurately, the number of decimal places in the product is equal to the number of decimal places in the multiplicand and multiplier counted together. This fact should be used as a test of the accuracy of the work rather than as a rule for "pointing off."

Observe that when the decimal point in the first partial product has been located, the remainder of the "pointing off" may be done mechanically by placing the point in each of the other partial products and in the complete product, directly under the one in the first partial product.

Review pages 83, 93, and 103.

All abstract work in division of decimals may be regarded as belonging to Case I.; that is, the pupil may consider that he is to find how many times the divisor is contained in the dividend.

Before beginning to divide, place a separatrix (˅) *in the dividend immediately after that figure in the dividend that is of the same denomination as the right hand figure of the divisor. When in the process of division this separatrix is reached, the decimal point must be written in the quotient.*

* Find how many times 5 thousandths are contained in 38455 thousandths.

† Find how many times 5 hundredths are contained in 3847 hundredths.

‡ Find how many times 5 tenths are contained in 384 tenths.

§ Find how many times 5 units are contained in 38 units.

‖ Find how many times 5 tenths are contained in 780 tenths.

¶ Find how many times 5 hundredths are contained in 7800 hundredths.

DENOMINATE NUMBERS.

Review page 14.

1. One half a ton is —— lb. 1 tenth of a ton = —— lb. 1 hundredth of a ton = —— lb. 1 thousandth of a ton = —— lb.

Review page 24.

2. A bushel of wheat weighs —— pounds.

(a) 72 bu. of wheat weigh how much more than 2 tons?

Review page 34.

(b) Change 3.26 tons to pounds.

(c) 4.7 tons are how many pounds?

(d) Change 3264 lb. to tons. (e) 5624 lb. to tons.

Review page 44.

(f) Find the cost of 7360 lb. coal at $7.25 per ton.

(g) Find the cost of 5360 lb. hay at $9.50 per ton.

Review page 54.

(h) Change 28 rods to feet. (i) 7 miles to rods.

(j) Change 506 yd. to rods. (k) 2880 rods to miles.

Review page 64.

(l) The gross weight of a load of bran was 2850 lb.; tare 1275 lb. Find the cost at $8 per ton?

(m) The gross weight of a load of oats was 2970 lb.; tare 1050 lb. How many bushels? (n) Find the cost at 25¢ a bushel.

Review page 74.

(o) A mountain is 11000 ft. high. How many feet more than 2 miles high is it?

(p) A mountain is 5 mi. high. How many feet high is it?

MEASUREMENTS.

Review pages 15 and 25.

1. Which is the larger, a five foot square or an oblong 4 feet by 6 feet?

(a) Which is the larger, a 25 ft. square or an oblong 26 ft. by 24 ft.?

(b) Find the area of a 15 ft. square.

(c) Find the solid content of a 15 ft. cube.

Review page 35.

2. Every rectangular figure has —— sides and —— right angles. If the sides are equal, the figure is a ——. If two of the sides are longer than the other two, the figure is an ——.

3. Draw a 4-sided figure that is neither a square nor an oblong. Is the figure you have drawn rectangular?

4. All the angles of a square are —— angles.

5. All the angles of an oblong are —— angles.

6. Angles that are not right angles are either —— angles or —— angles.

Review page 45.

7. Every rectangular solid has —— faces. These faces may be either squares or ——. If they are all squares the solid is a ——.

8. Cut from a potato or a turnip a solid with six faces, some of which are not rectangular. Observe the acute angles and the obtuse angles.

Review pages 55, 65, 75, and 85.

(d) In $2\frac{1}{2}$ cords there are how many cubic feet?

(e) In $2\frac{1}{2}$ acres there are how many square rods?

RATIO AND PROPORTION.

Review pages 16, 26, 36, 46, 56, 66, 76, 86, and 96.

1. A 1-ft. square equals what part of a 2-ft. square?

2. A 1-ft. cube equals what part of a 2-ft. cube?

3. A 2-ft. square equals what part of a 3-ft. square?

4. A 2-ft. cube equals what part of a 3-ft. cube?

5. A 2-yd. square equals how many times a 1-yd. square?

6. A 2-yd. cube equals how many times a 1-yd. cube?

7. A 3-yd. sq're equals what part of a 4-yd. square?

(a) A 10-rod square equals what part of a 12-rd. sq're?

(b) A 12-rd. sq're equals what part of a 16-rd. sq're?

(c) A 12-rd. sq're equals what part of a 24-rd. sq're?

(d) A 12-rd. sq're equals what part of a 36-rd. sq're?

8. A 3-ft. cube equals what part of a 4-ft. cube?

(e) A 3-ft. cube equals what part of a 5-ft. cube?

(f) A 3-ft. cube equals what part of a 6-ft. cube?

(g) A 3-ft. cube equals what part of a 9-ft. cube?

9. A $\frac{1}{2}$-ft. square equals what part of a 1-ft. square?

(h) A $\frac{1}{2}$-ft. cube equals what part of a 1-ft. cube?

10. A 1-ft. sq're equals how many times a $\frac{1}{4}$-ft. sq're?

(i) A 1-ft. cube equals how many times a $\frac{1}{4}$-ft. cube?

11. The surface of a 1-foot cube equals what part of the surface of a 2-foot cube?

(j) The surface of a 2-ft. cube equals what part of the surface of a 3-ft. cube?

(k) The surface of a 1-ft. cube equals what part of the surface of a 1-yd. cube?

(l) If a 1-inch cube of silver is worth $3.60, how much is a 3-inch cube of the same metal worth?

PERCENTAGE.

Review pages 17 and 18.

1. 25% of 24 is ——.	(a) Find 25% of $3479.
2. 24 is 25% of ——.	(b) $3479 is 25% of what?
3. 8 is ——% of 24.	(c) 75 is what % of 375?

Review pages 27 and 28.

4. $12\frac{1}{2}$% of 72 is ——.	(d) Find $12\frac{1}{2}$% of $650.
5. 72 is 12% of ——.	(e) $650 is $12\frac{1}{2}$% of what?
6. 12 is ——% of 72.	(f) 35 is what % of 245?

Review pages 37 and 38.

7. 10% of 45 is ——.	(g) Find 10% of $725.
8. 45 is 10% of ——.	(h) $725 is 10% of what?
9. 5 is ——% of 45.	(i) 55 is what % of 440?

Review pages 47 and 48.

10. $66\frac{2}{3}$% of 48 is ——.	(j) Find $66\frac{2}{3}$ of $756.
11. 48 is $66\frac{2}{3}$% of ——.	(k) $756 is $66\frac{2}{3}$% of what?
12. 36 is ——% of 48.	(l) $450 is what % of $600?

Review pages 57 and 58.

13. 60% of 75 is ——.	(m) Find 60% of $810.
14. 75 is 60% of ——.	(n) $810 is 60% of what?
15. 20 is ——% of 40.	(o) $84 is what % of $210?

Review pages 67 and 68.

16. 80% of 60 is ——.	(p) Find 80% of $640.
17. 60 is 80% of ——.	(q) $640 is 80% of what?
18. 50 is ——% of 60.	(r) $550 is what % of $660?

Review pages 77, 78, 87, and 88.

19. $37\frac{1}{2}$% of 24 is ——.	(s) Find $37\frac{1}{2}$% of $576.
20. 24 is $37\frac{1}{2}$% of ——.	(t) $576 is $37\frac{1}{2}$% of what?
21. 24 is ——% of 80.	(u) $675 is what % of $750?

PERCENTAGE.

Review pages 97 and 98.

1. One % of 357 = (a) Find 13% of 357.
2. 39 is 3% of ——. (b) 264 is 8% of what?
3. 15 is ——% of 300.* (c) 60 is what % of 750?†

Review pages 107 and 108.

4. One % of 736 = (d) Find $8\frac{1}{2}$% of 736.
5. 108 is 9% of ——. (e) 375 is 5% of what?
6. 57 is ——% of 300. (f) 41.5 is what % of 830?

Review pages 117 and 118.

7. 25% more than 80 = 25% less than 80 =
8. 40 is 25% more than ——.‡ 45 is 25% less than ——.
9. 75 is ——% more than 60.§ 60 is ——% less than 75.

10. Alice has $40; Jane has $50; Mary has $60.

(g) Jane has what per cent more than Alice?
(h) Mary has what per cent more than Jane?
(i) Mary has what per cent more than Alice?
(j) Jane has what per cent less than Mary?
(k) Alice has what per cent less than Jane?
(l) Alice has what per cent less than Mary?
(m) Alice's money equals what % of Jane's money?
(n) Alice's money equals what % of Mary's money?
(o) Jane's money equals what % of Alice's money?
(p) Jane's money equals what % of Mary's money?
(q) Mary's money equals what % of Alice's money?
(r) Mary's money equals what % of Jane's money?

* First find 1% of 300.
† First find 1% of 750.
‡ Let x = the number sought, the base; then 40 = x and 1 fourth of x, or $\frac{5}{4}$ of x.
§ 75 is how many more than 60? 15 is what % of 60?

Review pages 113 and 123.

1. $\frac{7}{8}$ = —— hundredths. (a) $\frac{7}{8}$ = —— thousandths.

(b) Change .275 to a common fraction and reduce it to its lowest terms. (c) .375. (d) .425. (e) .575. (f) .625.

Review pages 133 and 143.

(g) Find the cost of 6.28 acres of land at $2.75 an A.*

(h) Find the cost of 3.46 tons of coal @ $6.75 a ton.

(i) Divide 6.25 by 5. (6.ᵛ25 ÷ 5 units.)

(j) Divide 6.25 by .5 (6.2ᵛ5 ÷ 5 tenths.)

(k) Divide 6.25 by .05. (6.25ᵛ ÷ 5 hundredths.)

(l) Divide 36 by 5. (36ᵛ divided by 5 units.)

(m) Divide 36 by .5. (36.0ᵛ ÷ 5 tenths.)

(n) Divide 36 by .05. (36.00ᵛ ÷ 5 hundredths.)

(o) Divide 36 by .005. (36.000ᵛ ÷ 5 thousandths.)

(p) Divide 57.26 by 7. (57.ᵛ26 divided by 7 units.)

(q) Divide 57.26 by .7. (57.2ᵛ6 ÷ 7 tenths.)

(r) Divide 57.26 by .07. (57.26ᵛ ÷ 7 hundredths.)

(s) Divide 57.26 by .007. (57.260ᵛ ÷ 7 thousandths.)

(t) Divide 67.5 by 25. (67.ᵛ5 divided by 25 units.)

(u) Divide 67.5 by 2.5. (67.5ᵛ ÷ 25 tenths.)

(v) Divide 67.5 by .25. (67.50ᵛ ÷ 25 hundredths.)

(w) Divide 67.5 by .025. (67.500ᵛ ÷ 25 thousandths.)

(x) Divide 6.75 by 25. (25 units in 6 units = 0., etc.)

(y) Divide 6.75 by 2.5. (6.7ᵛ5 ÷ 25 tenths.)

* Require the pupil to put the work on the blackboard and to explain by telling (1) the cost of 1 hundredth of an acre; (2) of 8 hundredths; (3) of 1 tenth; (4) of 2 tenths; (5) of 1 acre; (6) of 6 acres; (7) of 6.28 acres. How many decimal places in the product? How many in the multiplicand? How many in the multiplier?

CONTENTS—PART II.

PART II.

NOTATION.

1. The expression of numbers by symbols is called **notation**.

2. In mathematics two sets of symbols are employed to represent numbers; namely, ten characters—1, 2, 3, 4, 5, 6, 7, 8, 9, 0—called **figures**; and the **letters** *a*, *b*, *c*, *d*, . . . *x*, *y*, *z*.

NOTE.—The figures from 1 to 9 are called *digits*. The term *significant figures* is sometimes applied to the digits. The tenth character (0) is called a cipher, zero, or naught.

THE ARABIC NOTATION.

3. The method of representing numbers by figures and places is called the **Arabic Notation**. It is the *principle of position* in writing numbers that gives to the system its great value.

Fourth place.	Third place.	Second place.	First place.	First decimal place.	Second decimal place.	Third decimal place.
2	4	3	8.	5	9	6

Units of the fourth order.	Units of the third order.	Units of the second order.	**Primary units.**	Units of the first decimal order.	Units of the second decimal order.	Units of the third decimal order.
2	4	3	8.	5	9	6

4. A figure standing alone or in the first place represents primary units, or units of the first order; a figure standing in the second place represents units of the second order; a figure standing in the third place represents units of the third order; a figure standing in the first decimal place represents units of the first decimal order, etc.

5. The following are the names of the units of eight orders:

Fourth decimal order . . .	ten-thousandths.
Third decimal order . . .	thousandths.
Second decimal order . . .	hundredths.
First decimal order	tenths.

DECIMAL POINT.

First order	primary units.
Second order	tens.
Third order	hundreds.
Fourth order	thousands.

6. In a row of figures representing a number (342.65), the figure on the right represents the lowest order given; the figure on the left, the highest order given. In general, any figure represents an order of units higher than the figure on its right (if there be one), and lower than the figure on its left (if there be one).

7. Ten units of any order equal one unit of the next higher order; thus, ten hundredths equal one tenth; ten tenths equal one primary unit, etc.

8. The naught, or zero, is used to mark vacant places; thus, the figures 205 represent 2 hundred, no tens, and 5 primary units.

NOTE 1.—Observe that a figure ***always stands for units.*** If it occupies the first place, it stands for primary units; if it occupies the second place, it stands for tens (that is, units of tens); the third place, for hundreds; the first decimal place, for tenths; the second decimal place, for hundredths, etc. Thus, a figure 5 always stands for five—*five* primary units, *five* thousand, *five* hundredths, *five* tenths, according to the place it occupies.

NOTE 2.—In reading integral numbers, the primary unit should be, and usually is, most prominent in consciousness. Thus, the number 275 is made up of 2 hundreds, 7 tens, and 5 primary units; but 2 hundreds equal two hundred (200) primary units, and seven tens equal seventy (70) primary units; these (200 + 70 + 5) we almost unconsciously combine in our thought, and that which is present in consciousness is 275 ***primary units.*** So in the number 125,246, there are units of six orders, which we reduce in thought to primary units, and say, one hundred twenty-five thousand two hundred forty-six ***primary units.***

NOTE 3.—In reading decimals, too, the primary unit should be prominent in consciousness. Thus, .256 is made up of 2 tenths, 5 hundredths, and 6 thousandths; but 2 tenths equal 200 thousandths, and 5 hundredths equal 50 thousandths; these (200 + 50 + 6) we combine in our thought, and that which should be present in consciousness is 256 thousandths *of a primary unit.*

9. EXERCISE.

Write in figures:

1. Two hundred fifty-four thousand one hundred.
2. One hundred seventy-five and two hundred six thousandths.
3. Eighty-four and three hundred five thousandths.
4. Three hundred seven and eighty-seven hundredths.
5. Seven thousand four hundred twenty-four.
6. Twenty-four thousand six hundred fifty-one.
7. One hundred thirty-five thousand two hundred.

(a) *Find the sum of the seven numbers.*

10. Exercise.

Read in two ways as suggested in the following:

324.61. (1) 3 hundreds, 2 tens, 4 primary units, 6 tenths, 1 hundredth. (2) Three hundred twenty-four and sixty-one hundredths.

Use the word *and* in place of the decimal point only.

1. 2746.2.	5. 2651.4.
2. 546.85.	6. 80.062.
3. 24.006.	7. 2085.7.
4. 1.6285.	8. 120.08.

11. Exercise.

Observe that any number may be read by giving the name of the units denoted by the right-hand figure to the entire number; thus, 146 is 146 primary units; 21.8 is 218 tenths; 3.25 is 325 hundredths.

1. 27 = 2 tens + 7 primary units = 27 primary units.
2. 2.7 = 2 primary units + 7 tenths = —— tenths.
3. .27 = 2 tenths + 7 hundredths = —— hundredths.
4. .027 = 2 hundredths + 7 thousandths = —— thousandths.
5. .436 = 4 tenths + 3 hundredths + 6 thousandths = —— thousandths.
6. 5.247 = 5 primary units + 2 tenths + hundredths + 7 thousandths = 5247 ——ths.
7. 3.24 = —— hundredths.
8. 5.206 = —— thousandths.
9. 25.13 = —— hundredths.
10. 14.157 = —— thousandths.
11. 275.4 = —— tenths.

Note.—Exercise 11 and Exercise 12 are important as a preparation for the clear understanding of division of decimals.

12. EXERCISE.

Observe that any part of a number may be read by giving the name of the units denoted by the last figure of the part to the entire part; thus, 24.65 is 246 tenths and 5 hundredths; 14.275 is 1427 hundredths and 5 thousandths. In a similar manner read each of the following:

1. 2.75 = —— tenths and —— hnndredths.
2. 32.46 = —— tenths and —— hundredths.
3. 1.425 = —— hundredths and —— thousandths.
4. 24.596 = —— tenths and —— thousandths.
5. 321.45 = —— tenths and —— hundredths.
6. 14.627 = —— hundredths and —— thousandths.
7. 2.6548 = —— hundredths and —— ten-thousandths.

13. EXERCISE.

Observe that in reading a mixed decimal in the usual way, we divide it into two parts and give the name of the units denoted by the last figure of each part to each part; thus, 2346.158 is read 2346 (primary units) and 158 thousandths.

Read the following in the usual manner. Do not use the word *and* in reading the numbers in the second column:

1. 200.006.	.206.	6. 800 and 24.	824.
2. 400.0005.	.0405.	7. 9000 and 6.	9006.
3. 500.025.	.525.	8. 2400 and 8.	2408.
4. 200 and 40.	240.	9. 17000 and 4.	17004.
5. 700 and 35.	735.	10. 46500 and 40.	46540.

14. EXERCISE.

Write in figures:

1. Two hundred and eight thousandths.
2. Two hundred eight thousandths.
3. Six hundred and twelve thousandths.
4. Six hundred twelve thousandths.

15. Reference Table.

Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Primary Units.
157,	896,	275,	832,	456,	297,	143,	215,	367,	291,	326,	415.

16. Note the number of decimal places in each of the following expressions:

1. .4 = 4 tenths. (1 decimal place.)
2. .27 = 27 hundredths. (2 decimal places.)
3. .346 = 346 thousandths. (3 decimal places.)
4. .2758 = 2758 ten-thousandths.
5. .07286 = 7286 hundred thousandths.
6. .000896 = 896 millionths. (6 decimal places.)
7. .000,468,275 = —— billionths. (9 decimal places.)
8. .000,000,000,462 = —— trillionths.
9. .000,000,000,000,527 = —— quadrillionths.
10. In any number of thousandths there are —— decimal places.
11. In any number of millionths there are —— decimal places.
12. In any number of billionths there are —— decimal places.
13. In any number of hundredths there are —— decimal places.
14. In any number of ten-thousandths there are —— decimal places.
15. In any number of hundred thousandths there are —— decimal places.

Algebra—Notation.

17. Letters are used to represent numbers; thus, the letter a, b, or c may represent a number to which any value may be given.

18. Known numbers, or those that may be known without solving a problem, when not expressed by figures, are usually represented by the first letters of the alphabet; as, a, b, c, d.

ILLUSTRATIONS.

(a) To find the perimeter of a square when its side is given.

Let a = one side.*

Then $4\,a$ = the perimeter.

Hence the rule: *To find the perimeter of a square, multiply the number denoting the length of its side by 4.*

(b) To find the perimeter of an oblong when its length and breadth are given.

Let a = the length.

Let b = the breadth.

Then $2a + 2b$, or $(a + b) \times 2$ = the perimeter.

Hence the rule: *To find the perimeter of an oblong, multiply the sum of the numbers denoting its length and breadth by 2.*

19. Unknown numbers, or those which are to be found by the solution of a problem, are usually represented by the last letters of the alphabet; as x, y, z.

ILLUSTRATION.

(a) There are two numbers whose sum is 48, and the second is three times the first. What are the numbers?

$$
\begin{array}{lrl}
\text{Let} & x = & \text{the first number.} \\
\text{Then} & 3\,x = & \text{the second number,} \\
\text{and} & x + 3\,x = & 48. \\
 & 4\,x = & 48. \\
 & x = & 12. \quad 3\,x = 36.
\end{array}
$$

* That is, the *number* of units in one side. The letter stands for the number.

20. The sign of multiplication is usually omitted between two letters representing numbers, and between figures and letters; thus, $a \times b$, is usually written ab; $b \times 4$, is written $4\,b$. $6\,ab$, means, 6 times a times b, or $6 \times a \times b$.

21. Exercise.

Find the numerical value of each of the following expressions, if $a = 8$, $b = 5$, and $c = 2$:

1. $a + b + c =$
2. $a + b - c =$
3. $2a + b + c =$
4. $a + b - 2c =$
5. $2ab =$
6. $3abc =$
7. $2ab + 5c =$
8. $ab + bc =$

(a) Find the sum of the eight results.

22. Exercise.

Find the numerical value of each of the following expressions if $a = 20$, $b = 5$, and $c = 2$:

1. $3(a + b) =$ *
2. $2(a - b) =$
3. $4(a + b + c) =$
4. $2(a + b - c) =$
5. $a \div b =$
6. $(a + b) \div c =$ †
7. $(a + b) \div 3c =$
8. $(a + 2b) \div 2c =$

(b) Find the sum of the eight results.

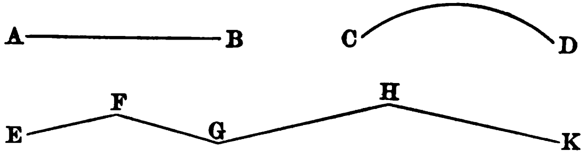
23. Exercise.

1. If $2x = 20$, $x = ?$
2. If $5x = 40$, $x = ?$
3. If $6x = 72$, $x = ?$
4. If $2x + 3x = 60$, $x = ?$
5. If $3x + 4x = 56$, $x = ?$

* This means, 3 times the sum of a and b.

† This means, the sum of a and b, divided by c.

Geometry.



24. A geometrical line has length, but neither breadth nor thickness.

NOTE.—Lines drawn upon paper or upon the blackboard are not geometrical lines, since they have breadth and thickness. They *represent* geometrical lines.

25. A straight line is the shortest distance from one point to another point.

26. A curved line changes its direction at every point.

27. A broken line is not straight, but is made up of straight lines.

1. The line *AB* is a ——— ———.
2. The line *CD* is a ——— ———.
3. The line *EF* is a ——— ———.
4. The line *FG* is a ——— ———.
5. The line *EK* is a ——— ———.

6. The perimeter of a square is a —— line made up of —— equal —— lines.

7. The perimeter of a regular pentagon is a —— line made up of —— equal —— lines.

8. The circumference of a circle is a —— line, every point in which is equally distant from a point called the center of the circle.

9. Imagine a straight line drawn upon the surface of a stovepipe. Can you draw a straight line upon the surface of a sphere?

28. Miscellaneous Review.

1. If a equals one side* of a regular pentagon, the perimeter of the pentagon is —— ——.

2. If b equals the perimeter of a square, the side of the square equals $b \div$ ——.

3. If a equals a straight line connecting two points and b equals a curved line connecting the same points, then a is —— than b.

4. Find the difference between two hundred seven thousandths, and two hundred and seven thousandths.

5. How many zeros in 1 million expressed by figures? 1 billion? 1 trillion?

6. How many decimal places in any number of millionths? billionths? trillionths?

7. How many decimal places in 25 thousandths? in 275 thousandths? in 4346 thousandths?

8. A figure in the second integral place represents units how many times as great as those represented by a figure in the second decimal place?

9. If $a = 6$, $b = 2$, and $d = 8$, what is the numerical value of the following? $12a + 3b - 5d$.

10. John had a certain amount of money and James had 5 times as much; together they had 354 dollars. How many dollars had each?

Let x = the number of dollars John had.
Then $5x$ = the number of dollars James had,
and $x + 5x$ = 354 dollars.
$6x$ = 354 dollars.
x = ? $5x$ = ?

* The expression "a equals one side" means that a equals the number of units in one side. Remember that in this kind of notation *the letters employed stand for numbers.*

ADDITION.

29. Addition (in arithmetic) is the process of combining two or more numbers into one number.

NOTE 1.—The word *number*, as here used, stands for *measured magnitude*, or *number of things*.

30. The **sum** is the number obtained by adding.

31. The **addends** are the numbers to be added..

32. The sign, +, which is read *plus*, indicates that the numbers between which it is placed are to be added; thus, $6+4$, means that 4 is to be added to 6.

33. The sign, =, which is read *equal* or *equals*, indicates that that which is on the left of the sign, equals that which is on the right of the sign; thus, $3+4=7$. $5+4+2=6+5$.

34. PRINCIPLES.

1. *Only like numbers can be added.*
2. *The denomination of the sum is the same as that of the addends.*

35. PRIMARY FACTS OF ADDITION.

There are forty-five primary facts of addition. See Elementary Book, pp. 32 and 82. The nine which many pupils fail to memorize perfectly are given below. (Note 1, p. 443.)

7	8	9	8	9	8	9	9	9
6	5	4	6	5	7	6	7	8
13	13	13	14	14	15	15	16	17

36. Examples of Addition.

1.	2.	3.
3754	37.426	$24.305
2862	1.48	$6.752
1457	375.062	$375.08
8073	413.968	$406.137

4.	5.	6.
275 acres.	43 gal. 2 qt. 1 pt.	$5a + 2b$
146 acres.	24 gal. 1 qt. 1 pt.	$27a + 3b$
27 acres.	63 gal. 3 qt. 1 pt.	$46a + 4b$
448 acres.	131 gal. 3 qt. 1 pt.	$78a + 9b$

37. *Observe* that in written problems in addition the figures that stand for units of the same order are usually written in the same column.

1. In example 2, what figures represent units of the second decimal order? Of the third decimal order?

2. In example 5, what figures represent units of the first integral order?

38. *Observe* that in written problems in addition of denominate numbers, the figures that stand for units of the same denomination and order are usually written in the same column.

1. In example 5, what figures represent units of gallons? Of quarts?

2. In example 5, what figures represent tens of gallons?

3. In example 4, what figures represent hundreds of acres?

Addition—Simple Numbers.

39. Find the sum of 275, 436, and 821.

Operation. *Explanation.*

$$\begin{array}{r} 275 \\ 436 \\ 821 \\ \hline 1532 \end{array}$$

The sum of the units of the first order is 12; this is equal to one unit of the second order and 2 units of the first order. Write the 2 units of the first order, and add the 1 unit of the second order to the other units of the second order.

The sum of the units of the second order is 13; this is equal to 1 unit of the third order and 3 units of the second order. Write the 3 units of the second order, and add the 1 unit of the third order to the other units of the third order.

The sum of the units of the third order is 15; this is equal to 1 unit of the fourth order and 5 units of the third order, each of which is written in its place.

The sum of 275, 436, and 821 is 1532.

40. Problems.

1. Add 3465, 4268, 3279, 6534, 5731.
2. Add 5732, 6721, 3466, 4269, 6535.
3. Add 2768, 5329, 4685, 3752, 8467.
4. Add 4671, 5315, 6248, 1533, 7232.

5. Add 375, 506, 258, 327, 580, 648, 846.
6. Add 436, 307, 449, 498, 736, 274, 888.
7. Add 625, 494, 742, 673, 574, 654, 638.
8. Add 564, 693, 684, 502, 376, 726, 877.

(a) Find the sum of the eight sums.

To the Teacher.—Impress upon the pupil the fact that in arithmetic nothing short of accuracy is commendable. One figure wrong in one problem in ten is failure. The young man or the young woman who cannot solve ten problems like those on this page without an error, is worthless as an accountant.

Addition—Decimals.

41. Find the sum of 4.327, 8.29 and .836.

Operation.	*Explanation.*
4.327 8.29 .836 13.453	The sum of the units of the third decimal order is 13; this is equal to 1 unit of the second decimal order and 3 units of the third decimal order. Write the 3 units of the third decimal order and add the 1 unit of the second decimal order to the other units of that order.

The sum of the units of the second decimal order is 15; this is equal to 1 unit of the first decimal order and 5 units of the second decimal order. Write the 5 units of the second decimal order and add the 1 unit of the first decimal order to the other units of that order.

The sum of the units of the first decimal order is 14; this is equal to 1 unit of the first integral order and 4 units of the first decimal order. Write the 4 units of the first decimal order and add the 1 unit of the first integral order to the other units of that order.

The sum of the units of the first integral order is 13; this is equal to 1 unit of the second integral order and 3 units of the first integral order, each of which is written in its place.

The sum of 4.327, 8.29, and .836 is 13.453.

42. Problems.

1. Add 474.36, 21.37, 38.007, and 487.
2. Add 78.63, 61.993, .725, and 724.64.
3. Add .7, .84, .375, .0275, and .25326.
4. Add 85.997, 47.9994, 72, and 53.93.
5. Find the sum of seven hundred ninety-eight and nine hundred ninety-four thousandths, and seven hundred ninety-four thousandths.

(a) Find the sum of the five sums.

To the Pupil.—Can you solve these five problems and find the sum of the five sums on first trial without an error?

Addition—United States Money.

43. Find the sum of the money represented in the following columns:

$324.45
28.47
375.28
6.94
175.89
27.56
475.39
802.21
354.48
916.37
144.50
75.34
8.88
246.25
$3962.01

Explanation.

Sums of the units of each order:

Second decimal order (cents)	81
First decimal order (dimes) .	70
First integral order	82
Second integral order . . .	56
Third integral order . . .	39

Observe that the 8 of the first sum is included in the 70 of the second sum; that the 7 of the second sum is included in the 82 of the third sum; that the 8 of the third sum is included in the 56 of the fourth sum, and that the 5 of the fourth sum is included in the 39 of the fifth sum. Hence the sum of the five sums is represented by the figures 3962.01.

44. PROBLEMS.*

1.	2.	3.	4.
$256.35	$275	$725	$743.65
145.24	146	854	854.76
321.75	281	719	678.25
286.44	675	697	713.56
308.92	284	716	843.18
244.31	552	448	769.45
986.24	496	504	724.54
275.46	628	715	616.73
383.27	682	603	847.90

(a) Find the sum of the four sums.

*TO THE PUPIL.—Remember that nothing short of absolute accuracy is of any value in such work as this.

Addition—Denominate Numbers.

45. Find the sum of 7 bu. 2 pk. 5 qt., 3 bu. 3 pk. 3 qt., 6 bu. 1 pk. 7 qt., and 9 bu. 3 pk. 5 qt.

Operation.

7	bu.	2	pk.	5	qt.
3	bu.	3	pk.	3	qt.
6	bu.	1	pk.	7	qt.
9	bu.	3	pk.	5	qt.
27	bu.	3	pk.	4	qt.

Explanation.

The sum of the number of quarts is 20; this is equal to 2 pecks and 4 quarts. Write the 4 quarts and add the 2 pecks to the pecks given in the second column.

The sum of the number of pecks is 11; this is equal to 2 bushels and 3 pecks. Write the 3 pecks, and add the 2 bushels to the bushels given in the third column.

The sum of the number of bushels is 27, which is written in its place.

The sum is 27 bu. 3 pk. 4 qt.

46. PROBLEMS.

1. Add.	2. Add.
6 bu. 2 pk. 6 qt.	5 bu. 0 pk. 7 qt.
4 bu. 2 pk. 2 qt.	4 bu. 1 pk. 6 qt.
5 bu. 2 pk. 2 qt.	3 bu. 0 pk. 1 qt.
6 bu. 3 pk. 7 qt.	5 bu. 0 pk. 5 qt.
4 bu. 3 pk. 3 qt.	1 bu. 1 pk. 2 qt.
8 bu. 2 pk. 6 qt.	2 bu. 3 pk. 3 qt.
7 bu. 0 pk. 5 qt.	4 bu. 2 pk. 4 qt.
5 bu. 1 pk. 4 qt.	2 bu. 0 pk. 6 qt.
8 bu. 0 pk. 6 qt.	6 bu. 0 pk. 5 qt.
7 bu. 3 pk. 2 qt.	4 bu. 1 pk. 0 qt.
6 bu. 3 pk. 1 qt.	7 bu. 0 pk. 2 qt.
6 bu. 1 pk. 7 qt.	3 bu. 2 pk. 2 qt.
7 bu. 2 pk. 0 qt.	3 bu. 1 pk. 2 qt.
6 bu. 1 pk. 6 qt.	3 bu. 2 pk. 1 qt.

(a) Find the sum of the two sums.

Algebraic Addition.

47. A coefficient is a number that indicates how many times a literal quantity* is to be taken; thus, in the expression $4ab$, 4 is the coefficient of ab.†

When no coefficient is expressed, it is understood that 1 is the coefficient; thus, in the expression $4a + b$, the coefficient of b is 1.

48. The terms of an algebraic expression are the parts that are separated by the sign + or −. There are three terms in the following: $ab + 3c + 4abc$. There are only two terms in the following: $8a \times 4b + 5a \div 6b$.

49. *Positive terms* are usually preceded by the plus sign.

50. *Negative terms* are preceded by the minus sign.

If no sign is expressed, the term is understood to be positive.

51. When the literal part of two or more terms is the same, the terms are said to be *similar*.

52. Problems.

Unite the terms in each of the following algebraic expressions into one equivalent term:

1. $5x + 3x + 2x =$	5. $4ab + 2ab + 3ab =$
2. $4x + 5x - 3x =$	6. $2ab + 5ab - 4ab =$
3. $6a - 2a + 4a =$	7. $3bc - 5bc + 6bc =$
4. $3b + 4b - 2b =$	8. $bx + 2bx + 3bx =$

53. Language Exercise.

Referring to the problems given above, use the following words in complete sentences: Coefficient, terms, positive, negative, similar, literal.

* The word *quantity* in algebra means *number*. The expression literal quantity means number expressed by letters.

† The term coefficient is sometimes applied to the literal part of an expression: thus, in the expression abc, ab is the coefficient of c. Usually, however, the term coefficient has reference to the *numerical coefficient*.

Algebraic Addition.

54. Regarding the following *positive* numbers as representing *gains* and the *negative* numbers as representing *losses*, find the total gain (or loss) in each case; that is, find the algebraic sum of the numbers in each group:

1.	2.	3.	4.	5.	6.
70	85	45	$8a$	$9b$	$2ab$
25	-35	-65	$3a$	$-3b$	$-6ab$
95	50	-20			

NOTE.—The positive sums of Nos. 1, 2, 4, and 5 indicate actual gain; the negative sums of Nos. 3 and 6 indicate actual loss.

55. Regarding each of the following *positive* numbers as representing a *rise* and each of the *negative* numbers as representing a *fall* of the mercury in a thermometer, find the total rise (or fall) in each case; that is, find the algebraic sum of the numbers in each group:

1.	2.	3.	4.	5.	6.
16	12	8	$8a$	$7b$	$5c$
10	-4	4	$2a$	$-3b$	$3c$
5	6	-16	$3a$	$4b$	$-12c$
31	14	-4			

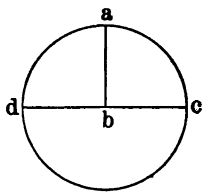
NOTE.—The positive sums of Nos. 1, 2, 4, and 5 indicate actual rise; the negative sums of Nos. 3 and 6 indicate actual fall.

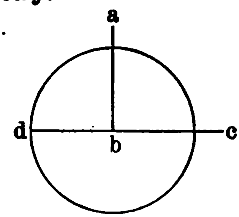
56. PROBLEMS.

Find the sum:

1.	2.	3.
$12+4-6$	$3a+2b+c$	$2ab+6bc$
$4+6-3$	$2a-b+3c$	$3ab+bc$
$5-2+8$	$a+5b-6c$	$-ab+3bc$

Geometry.



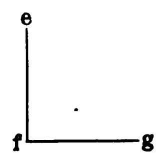


57. A **circle** is a plane figure bounded by a curved line, every point in which is equally distant from a point within, called the *center.*

58. The line that bounds a circle is a **circumference.**

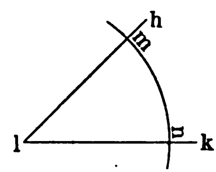
59. A straight line passing through the center of a circle and ending in the circumference is a **diameter.**

60. A straight line from the center of a circle to the circumference is a **radius.**



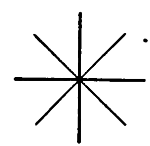
61. Any part of a circumference is an **arc.**

62. For the purpose of measurement, every circumference is considered as divided into 360 equal parts, called degrees.



63. Two lines meeting at a point form an **angle.** The point in which the two lines meet is the **vertex** of the angle.

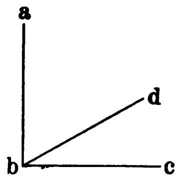
64. Every angle may be regarded as having its vertex at the center of a circle, and the angle is measured by the part of the arc intercepted; thus, the angle *hlk* is measured by the arc *mn.*



65. The angle *efg* is an angle of 90 degrees, called also a right angle. The angle *abc* is an angle of 90 degrees.

66. All the angles about a point together equal four right angles.

67. Miscellaneous Review.

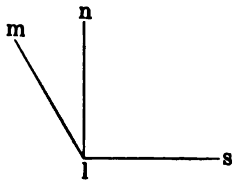


1. The angle *abd* is an angle of about — degrees.

The angle *dbc* is an angle of — degrees.

The angle *abd* + the angle *dbc* = the angle *abc*.

The angle *abc* is an angle of — degrees.



2. The angle *mln* is an angle of about — degrees.

The angle *mln* + the angle *nls* = the angle *mls*.

The angle *mls* is an angle of about —.

3. Copy the following figures and add by column and by line. Prove by comparing the sum of the sums of the columns with the sum of the sums of the lines. That pupil who can solve this problem without an error, on first trial, has taken an important step toward making himself valuable as an accountant.

	Mon.	Tues.	Wed.	Thur.	Frid.	Sat.	Total.
A.......	2.70	2.95	2.80	3.00	2.65	2.45	
B.......	3.43	3.12	3.26	3.62	3.28	3.39	
C.......	3.00	2.90	3.15	3.20	2.95	3.05	
D.......	2.00	1.76	2.22	1.93	1.98	1.87	
E.......	4.15	4.25	4.15	4.35	4.45	4.25	
F.......	3.00	3.30	3.12	3.18	3.24	3.15	
G.......	5.10	4.90	4.95	5.05	5.15	4.95	
H.......	2.10	2.12	2.20	2.04	2.06	2.25	
I.......	3.50	3.60	3.40	3.30	3.50	3.60	
K.......	3.05	2.90	3.15	2.95	3.15	2.00	
Total..							

SUBTRACTION.

68. Subtraction (in arithmetic) is the process of taking one number from (out of) another.

NOTE 1.—The word *number*, as here used, stands for *measured magnitude*, or *number of things*.

69. The **minuend** is the number from which another number is taken.

70. The **subtrahend** is the number taken from another number.

71. The **difference** is the number obtained by subtracting.

72. The sign –, which is read *minus*, indicates that the number that follows the sign is to be taken from (out of) the number that precedes it; thus, 8 – 3, means, that 3 is to be taken from (out of) 8.

73. PRINCIPLES.

1. *Only like numbers can be subtracted.*
2. *The denomination of the difference is the same as that of the minuend and the subtrahend.*

74. PRIMARY FACTS OF SUBTRACTION.

There are eighty-one primary facts of subtraction which should be learned while learning the facts of addition. (Note 2, p. 443.)

75. Examples of Subtraction.

1.	2.	3.
2687	57.38	\$675.46
1298	28.146	\$282.75
1389	29.234	\$392.71

4.	5.	6.
576 pounds	25 bu. 3 pk. 5 qt.	$25a + 6b$
288 pounds	12 bu. 1 pk. 7 qt.	$10a + 2b$
288 pounds	13 bu. 1 pk. 6 qt.	$15a + 4b$

76. *Observe* that in written problems in subtraction the subtrahend is usually placed under the minuend and the difference under the subtrahend; and that, as in addition, the units of the same order are written in the same column.

1. In example 2, what figures represent units of the third decimal order? of the second integral order? of the first decimal order? of the first integral order?

2. In example 5, what figures represent units of the first integral order? of the second integral order?

77. *Observe* that in subtraction of denominate numbers the figures that stand for units of the same denomination and order are usually written in the same column.

1. In example 4, what figures represent tens of pounds? hundreds of pounds?

2. In example 5, what figures represent bushels and units of the first order?

78. *Observe* that in both addition and subtraction the decimal points, if there are any, usually appear in column.

Subtraction—Simple Numbers.

79. Find the difference of 8274 and 5638.

Operation.	*Explanation.*
8274 5638 2636	Eight is greater than 4. In the minuend, take one unit of the second order from the 7 units of the second order. This unit of the second order, combined with the 4 units of the first, makes 14 units of the first order.

Eight units of the first order from 14 units of the first order leave 6 units of the first order. Three units of the second order from 6 (7—1) units of the second order leave 3 units of the second order. Six is greater than 2. In the minuend take one unit of the fourth order from the 8 units of the fourth order. This unit of the fourth order, combined with the 2 units of the third order, makes 12 units of the third order. Six units of the third order from 12 units of the third order leave 6 units of the third order. Five units of the fourth order from 7 (8 – 1) units of the fourth order leave 2 units of the fourth order.

The difference of 8274 and 5638 is 2636.

80. Problems.

1. From 35642 subtract 12456.
2. From 87544 subtract 64358.
3. From 90070 subtract 13256.
4. From 8164 subtract 3275.
5. From the sum of 8539, 2647, 3984, 1461, 7353, 6016, and 2364, subtract 22364.
6. From the sum of 1352, 3425, 2640, 3724, 6575, 7360, and 6276, subtract 21352.
7. From 6 thousand 7 hundred 25, subtract 1 thousand 8 hundred 36.
8. From seven thousand four hundred sixty-five, subtract two thousand three hundred fifty-four.

(a) Find the sum of the eight differences.

Subtraction—Decimals.

81. Find the difference of 28.36 and 15.432.

Operation.	*Explanation.*
28.36 15.432 12.928	One unit of the second decimal order (1 from 6) equals 10 units of the third decimal order. Two from 10 leaves 8. Three from 5 (6−1) leaves 2.

Four is greater than 3. One unit of the first integral order (1 from 8) equals 10 units of the first decimal order. $10 + 3 = 13$. Four from 13 leaves 9.

Five from 7 (8 − 1) leaves 2. One from 2 leaves 1.

The difference of 28.36 and 15.432 is 12.928.

82. Problems.

1. From 100 take .3456.
2. From 100 take 5.246.
3. From 100 take 44.236.
4. From 100 take .6544.
5. From 100 take 4.754.
6. 100 − 44.764.
7. 250 − 159.63.
8. 250 − 36.75.
9. 250 − 140.37.
10. 250 − 163.25.

(a) Find the sum of the ten differences.

83. Miscellaneous.

1. The sum of two numbers is 3.7464; one of the numbers is 1.3521. What is the other number?

2. The difference of two numbers is 2.3254; the less number is 7.6746. What is the greater number?

3. The difference of two numbers is 2.3943; the greater number is 10. What is the less number?

4. From 10 subtract 7.6744.

(b) Find the sum of the four results.

To the Pupil.—Work with care. Make no errors. The sum of the results should be correct *on first trial.*

Subtraction—United States Money.

84. Find the difference of $27.25 and $14.51.

Operation.	*Explanation.*
$27.25	One cent from 5 cents = 4 cents.
$14.51	Five dimes from 12 dimes = 7 dimes.
$12.74	Four dollars from 6 dollars (7−1) = 2 dollars.
	One ten-dollars from 2 ten-dollars = 1 ten-dollars.
	The difference of $27.25 and $14.51 is $12.74.

85. PROBLEMS—ADDITION AND SUBTRACTION.*

Find the sum that each depositor has to his credit:

A.			B.		
Deposit		$254.20	Deposit		$175.30
Deposit		38.60	Check	$30.50	
Check	$12.50		Check	21.75	
Check	10.80		Deposit		54.20
Check	3.60		Check	18.34	
Check	5.40		Check	6.24	
Balance	——		Balance	——	

C.			D.		
Deposit		$745.80	Deposit		$824.70
Check	$87.50		Check	$69.50	
Check	89.20		Check	78.25	
Check	96.40		Check	81.66	
Check	94.60		Deposit		61.40
Deposit		45.80	Check	93.76	
Balance	——		Balance	——	

E. Deposit, $1000. Checks, $275, $324, $400.

F. Deposit, $864. Checks, $375, $146, $279.

(a) Find the amount of the six balances.

* That bank clerk who makes one error a day in carrying out his balances, which he does not himself discover and correct, will not retain his position.

Subtraction—Denominate Numbers.

86. Find the difference of 15 yd. 2 ft. 4 in. and 8 yd. 1 ft. 10 in.

Operation.
15 yd. 2 ft. 4 in.
8 yd. 1 ft. 10 in.
7 yd. 0 ft. 6 in.

Explanation.

Ten inches are more than 4 in.; 1 ft. (from the 2 ft.) equals 12 in.; 12 in. and 4 in. are 16 in.; 10 in. from 16 in. leave 6 in.

One ft. from 1 ft. (2—1) leaves 0 ft.

Eight yd. from 15 yd. leave 7 yd.

The difference of 15 yd. 2 ft. 4 in. and 8 yd. 1 ft. 10 in. is 7 yd. 6 in.

87. Problems.

1. From 12 yd. 1 ft. 8 in.	subtract 5 yd. 2 ft. 3 in.
2. From 8 yd. 2 ft. 6 in.	subtract 5 yd. 1 ft. 10 in.
3. From 9 yd. 1 ft.	subtract 2 yd. 2 ft. 7 in.
4. From 6 yd. 2 ft. 5 in.	subtract 4 yd. 8 in.
5. From 7 yd.	subtract 5 yd. 1 ft. 1 in.
6. From 7 yd. 1 ft. 4 in.	subtract 4 yd. 9 in.
7. From 11 yd. 6 in.	subtract 4 yd. 1 ft. 2 in.
8. From 10 yd. 2 ft.	subtract 7 yd. 5 in.

(a) Find the sum of the eight differences.

88. Problems.

1. How many days from April 25 to May 1?
2. How many days from April 25 to May 10?
3. How many days from April 25 to May 20?
4. How many days from April 25 to June 1?
5. How many days from April 25 to June 10?
6. How many days from April 25 to June 30?
7. How many days from April 25 to July 5?

(b) Find the sum of the seven answers.

Algebraic Subtraction.

89. Regarding the following minuends as representing A's gain (or loss), and the subtrahends as representing B's gain (or loss), subtract B's from A's.*

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
A,	80	60	20	$7a$	$-4b$	$-12c$
B,	30	-20	50	$2a$	$-6b$	$-6c$
	50	80	-30	$5a$	$2b$	$-6c$

NOTE.—The positive differences for Monday, Tuesday, Thursday, and Friday indicate that A's gain was greater (or his loss less) than B's. The negative differences for Wednesday and Saturday indicate that A's gain was less (or his loss greater) than B's.

90. Regard the following minuends as representing distances one boat sails from a given point, and the subtrahends as representing distances another boat sails from the same point. Distances sailed north are here represented by positive numbers, and distances sailed south by negative numbers. Find how far the first boat is from the second.

	1.	2.	3.	4.	5.	6.
1st B.,	75	85	15	$8a$	$-4b$	$-15c$
2nd B.,	25	-35	35	$2a$	$-9b$	$-5c$
	50	120	-20			

NOTE.—The positive differences in Nos. 1, 2, 4, and 5 indicate that the first boat is north of the second boat. The negative differences in Nos. 3 and 6 indicate that the first boat is south of the second boat.

91. Rule for algebraic subtraction: *Conceive the sign* (*or signs*) *of the subtrahend to be changed* ($-$ to $+$ and $+$ to $-$), *then proceed as in addition.*

* See page 168, Art. 54.

Algebraic Subtraction.

1. A gained \$1200 and lost \$250; B gained \$500 and lost \$350. How much more was A's wealth increased by the two transactions than B's?

$$\begin{array}{ccc} \$1200 - \$250 & 12a - 5b & 14a - 3b \\ \underline{\$500 - \$350} & \underline{5a - 7b} & \underline{6a - 5b} \\ \$700 + \$100 & & \end{array}$$

2. C gained \$1500 and \$650; D gained \$600 and lost \$250. How much more was C's wealth increased by the two transactions than D's?

$$\begin{array}{ccc} \$1500 + \$650 & 15a + 13b & 13a + 5b \\ \underline{\$600 - \$250} & \underline{6a - 5b} & \underline{4a - 3b} \\ \$900 + \$900 & & \end{array}$$

3. E gained \$1300 and lost \$450; F gained \$400 and \$250. How much more was E's wealth increased by the two transactions than F's?

$$\begin{array}{ccc} \$1300 - \$450 & 13a - 9b & 17a - 8b \\ \underline{\$400 + \$250} & \underline{4a + 5b} & \underline{4a + 6b} \\ \$900 - \$700 & & \end{array}$$

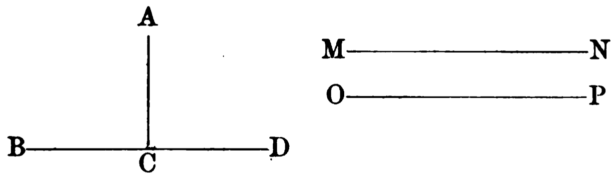
4. G gained \$1200 and lost \$500; H gained \$900 and lost \$100. How much more was G's wealth increased by the two transactions than H's?

$$\begin{array}{ccc} \$1200 - \$500 & 12a - 5b & 16a - 8b \\ \underline{\$900 - \$100} & \underline{9a - b} & \underline{4a - 2b} \\ \$300 - \$400 & & \end{array}$$

5. Review the foregoing and observe that in every instance subtracting a positive number is equivalent to adding an equal negative number, and subtracting a negative number is equivalent to adding an equal positive number.

Geometry.

92. Direction of Lines.



1. When one straight line meets another straight line in such a manner that two right angles are formed by the lines, the two lines are said to be *perpendicular* to each other.

2. Two lines side by side extending in the same direction are said to be *parallel.*

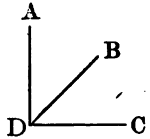
3. Of the lines given above:

AC is ———— to *BD.*
BD is ———— to *AC.*
MN is ———— to *OP.*

4. A line extending in the direction of the *horizon* is said to be *horizon*tal. A line on the floor of the room is horizontal; a line on the ceiling is horizontal; a line on the blackboard, every point in which is equally distant from the floor, is horizontal. For convenience, lines drawn upon paper, that are parallel with the top and bottom of the paper, may be regarded as representing horizontal lines.

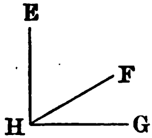
5. A line suspending a piece of lead (plumbum) is called a plumb-line. A line in the direction of a plumb-line is said to be vertical. A vertical line is perpendicular to a horizontal line. Lines on the blackboard may or may not be vertical or horizontal. For convenience, lines drawn upon paper that are parallel with the sides of the paper may be regarded as vertical.

93. Miscellaneous Reviews.



1. An angle that is equal to one half of a right angle, is an angle of —— degrees.

2. The angle ADB is an angle of —— degrees.



3. If from a right angle, an angle of 30 degrees be taken, the remaining angle is an angle of —— degrees.

4. The angle FHG is an angle of —— degrees.

5. During the month of November, 1897, there were consumed at the Illinois Institution for the Education of the Blind, 64 loads of coal. The weight of each load in pounds is given below. Find the total weight.

6100	8020	5490	5190
8380	6860	6800	7130
4850	6230	6560	7090
8010	6780	6690	7790
7080	6980	5780	6810
6620	6240	6980	8600
6450	6420	5990	9100
6570	6310	4740	6740
7950	6300	5520	5380
4750	6530	3630	7640
8840	6950	4930	5650
7290	6980	5150	5900
4960	4920	6420	6200
8330	5880	6770	6620
6300	7030	6220	7170
7080	5160	6020	9210

MULTIPLICATION.

94. Multiplication is the process of taking a number (of things) a number of times.

The word ***number*** as first used in the above statement stands for measured magnitude. The second word ***number*** does not stand for measured magnitude, but rather for pure number, representing simply the ***times*** the number (of things) is to be repeated.

95. The **multiplicand** is the number (of things) taken or repeated.*

96. The **multiplier** is the number that shows how many times the multiplicand is to be repeated.

97. The **product** is the number (of things) obtained by multiplying.

98. The sign, ×, which is read *multiplied by*, indicates that the number preceding the sign is a multiplicand, and the number following it a multiplier.

For other uses of this sign, see notes 3, 4, and 5, page 443.

99. PRINCIPLES.

1. *The multiplier is always an abstract number.*

2. *The denomination of the product is always the same as that of the multiplicand.*

100. PRIMARY FACTS OF MULTIPLICATION.

There are sixty-four primary facts of multiplication. See note 6, page 444.

* "The multiplicand, however written, must always be understood to express measured quantity; it is always concrete."—*Psychology of Number, McClellan & Dewey*, page 76.

101. Examples of Multiplication.

1.	2.	3.
4865	37.258	\$375.42
3	3	3
14595	111.774	\$1126.26

4.	5.	6.
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
5	5	5
1820 tons	309 yd. 0 ft. 1 in.	$40a - 30b$

102. *Observe* that in each of the above examples the multiplier is a pure number.

103. *Observe* that in each of the above examples the denomination of the product is the same as the denomination of the multiplicand.

104. Multiplication and Addition Compared.

Find the sum of each of the following groups of numbers and compare the result with the product in the corresponding problem in article 101:

1.	2.	3.
4865	37.258	\$375.42
4865	37.258	\$375.42
4865	37.258	\$375.42

4.	5.	6.
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$

Multiplication—Simple Numbers.

105. Find the product of 563 and 7.

Operation.

```
  563
    7
 ----
 3941
```

Explanation.

Seven times 3 units of the first order are 21 units of the first order; they are equal to 1 unit of the first order and 2 units of the second order. Write the 1 unit of the first order and add the 2 units of the second order to the next partial product.

Seven times 6 units of the second order are 42 units of the second order; $42 + 2 = 44$; 44 units of the second order equal 4 units of the second order and 4 units of the third order. Write the 4 units of the second order and add the 4 units of the third order to the next partial product.

Seven times 5 units of the third order are 35 units of the third order; $35 + 4 = 39$; 39 units of the third order equal 9 units of the third order and three units of the fourth order. Write the 9 units of the third order and the 3 units of the fourth order.

The product of 563 and 7 is 3941.

106. Find the product of 3426 and 57.

Operation.

```
  3426
    57
 ------
 23982
17130
 ------
195282
```

Explanation.

Seven times 3426 equals 23982. Fifty times 3426 equals 171300. Fifty times the number plus 7 times the number equals 57 times the number. Therefore adding 23982 and 171300 gives 57 times 3426.

The product of 3426 and 57 is 195282.

107. PROBLEMS.

1. Multiply 3241 by 27.
2. Multiply 6759 by 27.
3. Multiply 4328 by 36.
4. Multiply 5672 by 36.
5. Multiply 3479 by 54.
6. $6521 \times 54 =$
7. $3572 \times 74 =$
8. $6428 \times 74 =$
9. $3521 \times 29 =$
10. $6479 \times 29 =$

(a) Find the sum of the ten products.

Multiplication—Decimals.

108. Find the product of 728.37 and .6.

Operation.

$$\begin{array}{r} 72^{\vee}8.37 \\ .6 \\ \hline 437.022 \end{array}$$

Explanation.

To multiply by .6 means to take 6 times 1 tenth of the number. One tenth of 728.37 is 72.837. Six times 72.837 equals 437.022.

NOTE 1.—The separatrix is used to indicate the place of the decimal point in the number that is one tenth of the multiplicand.

NOTE 2.—The decimal point should be written in the product when, in the process of multiplication, the place is reached where it belongs. Do not multiply all the figures and then attempt to determine the place of the point.

109. Find the product of 746.2 and .25.

Operation.

$$\begin{array}{r} 7^{\vee}46.2 \\ .25 \\ \hline 37.310 \\ 149.24 \\ \hline 186.550 \end{array}$$

Explanation.

To multiply by .25 means to take 25 times 1 hundredth of the number.

One hundredth of 746.2 is 7.462. Five times 7.462 equals 37.310. Twenty times 7.462 equals 149.24. $37.310 + 149.24 = 186.550$.

110. *Observe* that when a multiplication of decimals is complete, the number of decimal places in the product is equal to the number of decimal places in the multiplicand and multiplier.

111. PROBLEMS.

1. Multiply 324.6 by .7.
2. Multiply 324.6 by .27.
3. Multiply 324.6 by 2.7.
4. Multiply 324.6 by 27.
5. Multiply 324.6 by 5.4.
6. $324.6 \times 54 =$
7. $324.6 \times .48 =$
8. $324.6 \times 6.3 =$
9. $324.6 \times 3.07 =$
10. $324.6 \times .08 =$

(a) Find the sum of the ten products.

Multiplication—United States Money.

112. Find the product of $34.50 and 53.4.

Operation.	*Explanation.*
$34.50	To multiply by 53.4, means to take 53 times the
53.4	multiplicand plus 4 tenths of the multiplicand.
$13.800	One tenth of $34.50 is $3.45.
$103.50	Four tenths of $34.50 is $13.80.
$1725.0	Three times $34.50 is $103.50.
$1842.300	Fifty times $34.50 is $1725.
	The sum of the partial products is $1842.30.

Practical application of the foregoing.

If one acre of land costs $34.50, how much will 53.4 acres cost?

One tenth of an acre costs $3.45.

Four tenths of an acre cost $13.80.

Three acres cost $103.50.

Fifty acres cost $1725.

53.4 acres cost $1842.30.

113. Complete the following bill and find the amount:

INST. FOR THE BLIND,

1897. *To* GEO. E. SYBRANT, *Dr.*

Dec.	4	27 bbl. Apples	@	$2.25		
"	10	56 bush. Potatoes	@	.52		
"	12	13 bush. Beans	@	1.75		
"	16	34 bush. Turnips	@	.35		
"	18	50 bbl. Flour	@	4.90		
"	21	74 lb. Butter	@	.19		
"	22	53 lb. Tea	@	.42		
"	24	37 bush. Onions	@	.55		
"	31	58 lb. Ham	@	.14		

TO THE PUPIL.—Remember that any inaccuracy in solving business problems makes the work valueless. Accuracy ranks next in importance to integrity in the selection of an accountant.

Multiplication—Denominate Numbers.

114. Find the product of 3 tons 850 lb. and 8.

Operation.	*Explanation.*
3 tons 850 lb.	Eight times 850 lb. equals 6800 lb.
8	6800 lb. equals 3 tons 800 lb.
27 tons 800 lb.	Write the 800 lb. and add the 3 tons to the next partial product.

Eight times 3 tons equals 24 tons; 24 tons plus 3 tons equals 27 tons. 3 tons 800 lb. multiplied by 8 equals 27 tons 800 lb.

115. Problems.

1. If one side of a square garden measures 6 rd. 8 ft., what is the perimeter of the garden?

2. The circumference of a certain bicycle track is 13 rd. 12 ft. How far does the rider travel who goes around it 12 times?

3. The length of a rectangular field is 15 rd. 10 ft. and the width 9 rd. 8 ft. What is the perimeter of the field?

4. There is a walk 5 feet wide around a rectangular grass plat 3 rd. 6 ft. by 2 rd. 10 ft. What is the outside perimeter of the walk?

5. How far does the person travel who walks once around the grass plat described in problem 4, if he keeps his track in the center of the walk?

(a) Find the sum of the five answers.

116. Problems.

1. If a train moves at the rate of a mile in 1 min. 25 sec., in how long a time will it move 325 miles?

2. If the circumference of a wagon wheel is 15 feet 6 inches, how far will the wagon move while the wheel revolves 1000 times?

Algebraic Multiplication.

117. Examples.

No. 1.

$$\begin{array}{l} 8 + 4 - 3 \\ 4 \\ \hline 32 + 16 - 12 \end{array}$$

No. 2.

$$\begin{array}{l} a + 3b - 4c \\ 4 \\ \hline 4a + 12b - 16c \end{array}$$

No. 3.

$$\begin{array}{l} 3a - 2b + c \\ d \\ \hline 3ad - 2bd + cd \end{array}$$

No. 4.

$$\begin{array}{l} 2a + 3b - 5c \\ 2d \\ \hline 4ad + 6bd - 10cd \end{array}$$

1. *Observe* that in the above examples we multiply each term of the multiplicand by the multiplier.

2. Prove example No. 1 by uniting the terms of the multiplicand and comparing 4 times the number thus obtained with the number obtained by uniting the terms of the product.

3. Verify example No. 2 by letting $a = 5, b = 3$, and $c = 2$.

4. Verify example No. 3 by giving the following values to the letters: $a = 7, b = 4, c = 3, d = 5$.

5. Verify example No. 4 by giving any values you may choose to each letter.

118. Problems.

1. Multiply $3ab - 2bc + 5c$ by $2d$.
2. Multiply $2ax + 4bx - y$ by 5.
3. Multiply $3bc + ab - bc$ by $3d$.
4. Multiply $x - y + z$ by $3ab$.
5. Multiply $ax + bx - cx$ by $2y$.
6. Verify each of the above problems by giving the following values to the letters: $a = 3, b = 2, c = 4, d = 5, x = 7, y = 6, z = 8$.

Algebraic Multiplication.

119. EXPONENT.

1. $a \times a$, or aa which means a multiplied by a, is usually written a^2. This is read a square or a second power.

2. b^3 (to be read b cube or b third power) means b taken three times as a factor. It is $b \times b \times b$.

3. a^4 (to be read a fourth power, or simply a fourth) means that a is taken four times as a factor. It is $a \times a \times a \times a$.

4. The small figure at the right of a letter tells the number of times the letter is to be used as a factor. The figure so used is called an exponent. When the exponent is 1, it is not usually expressed; thus, a means a^1.

120. PROBLEMS.

On the supposition that $a = 2$, $b = 3$, and $c = 4$, find the numerical value of each of the following expressions:

1. $a^2 + 2ab + b^2$	6. $5a^2b - 2bc$
2. $3ab^2 + 5bc^2$	7. $a^3b^3 - c^3$
3. $4a^2b^2 + 3b^2c^2$	8. $a^2b^2 + c^2$
4. $2a^2b + 2ab^2$	9. $a^2b^2c^2 - ab^3$
5. $3b^2c^2 + 5ab$	10. $2a^2b^2c^2$

(a) Find the sum of the numerical values of the above.

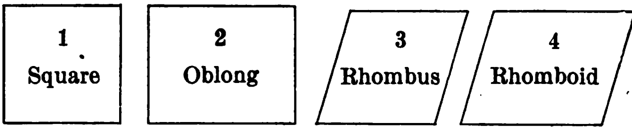
121. EXAMPLES.

No. 1.	No. 2.
$4ax + 2by + c$	$3b^2x + 2by - c$
a^2	$2b^2$
$4a^3x + 2a^2by + a^2c$	$6b^4x + 4b^3y - 2b^2c$

Verify each of the above examples by letting $a = 2$, $b = 3$, $c = 4$, $x = 5$, $y = 6$.

Geometry.

122. Parallelograms.



1. Any side of any one of the above figures is parallel to the opposite side of the same figure. Hence the figures are called *parallelograms.*

2. Each of the above figures has four sides. Hence the figures are called *quadrilaterals.*

3. If all the sides of a figure are equal, the figure is said to be *equilateral.*

4. If all the angles of a parallelogram are right angles (angles of 90°) the figure is said to be *rectangular.*

5. Which of the above figures are equilateral?

6. Which of the above figures are rectangular?

7. Which of the above figures are not equilateral?

8. Which of the above figures are not rectangular?

9. Which of the above figures are parallelograms?

10. Which of the above figures are quadrilaterals?

11. Can you draw a quadrilateral that is not a parallelogram?

12. Is any one of the above figures an equilateral rectangular parallelogram?

13. In a rhomboid or rhombus two of the angles are less than right angles and two of them are greater than right angles. Convince yourself by cutting a rhomboid from paper and comparing it with rectangular figures that two of the angles of a rhomboid are as much less than two right angles as the other two are greater than two right angles.

123. Miscellaneous Reviews.

1. If one of the angles of a rhombus is an angle of 80 degrees, what is the number of degrees in each of the other angles?

2. Draw a rhomboid one of whose angles is an angle of 70; give the number of degrees in each of the other angles.

3. An oblong has four right angles. The angles of a rhomboid are together equal to how many right angles?

4. If an oblong is a feet long and b feet wide, the number of square feet in the area is ab.* If the side of a square is a feet, the number of square feet in its area is ——.

5. If a rectangular solid is a feet long, b feet wide, and c feet thick, the number of cubic feet in its solid contents is abc. If the side of a cube is a feet, the number of cubic feet in its solid contents is ———.

6. If a man earns b dollars each week and spends c dollars, in one week he will save ——— dollars; in 7 weeks he will save ——— dollars.

7. A framed picture, on the inside of the frame, is 18 in. by 22 in.; the frame is 4 inches wide. How many inches in the outside perimeter of the frame?

8. Think of two fields: one is 9 rd. by 16 rd.; the other is 12 rd. by 12 rd. How do the square rods of the two fields compare? How much more fence would be required to enclose one field than the other?

* This means *the product* of a and b. Observe that it is the number a (not a feet) that we multiply by the number b (not b feet). While it is probably true (see footnote, p. 181) that the multiplicand always expresses measured quantity, it is also true that we often find the product of two factors mechanically. Indeed this is what we usually do in all multiplication of abstract numbers. In this case we find the product of a and b and know from former observations that this number equals the number of square feet in the oblong.

DIVISION.

124. Division is (1) the process of finding how many times one number is contained in another number; or (2), it is finding one of the equal parts of a number.

NOTE.—The word *number* as used above stands for measured magnitude.

125 The **dividend** is the number (of things) to be divided.

NOTE.—Since in multiplication the multiplicand and product must always be considered concrete (see foot-note, p. 181), then in division, the dividend, and either the divisor or the quotient, must be so regarded.

126. The **divisor** is the number by which we divide.

NOTE.—The word *number* as used in Art. 126 may stand for measured magnitude or for pure number, according to the aspect of the division problem. In the problem 324 ÷ 6, if we desire to find how many times 6 is contained in 324, the 6 stands for measured magnitude—a number of things. But if we desire to find one sixth of 324, then the 6 is pure number, and is the ratio of the dividend to the required quotient.

127. The **quotient** is the number obtained by dividing.

NOTE.—If the divisor is pure number, the quotient represents measured magnitude. If the divisor represents measured magnitude, the quotient is pure number.

128. The sign ÷, which is read *divided by*, indicates that the number before the sign is a dividend and the number following the sign a divisor. See notes 7 and 8, page 445.

129. Examples in Division.

No. 1.	No. 2.
$\$5)\underline{\$1565}$	$5)\underline{\$1565}$
313	\$313

No. 3.	No. 4.
2 bush.)$\underline{\text{246 bush.}}$	2)$\underline{\text{246 bush.}}$
123	123 bush.

No. 5.	No. 6.
$2a)\underline{6ab+8ac-12a}$	$2)\underline{6ab+8ac-12a}$
$3b+4c-6$	$3ab+4ac-6a$

1. In example No. 1, we are required to find ———.*
2. In example No. 2, we are required to find ———.†
3. In example No. 3, we are required to find ———.
4. In example No. 4, we are required to find ———.
5. In example No. 5, we are required to find ———.
6. In example No. 6, we are required to find ———.

NOTE.—Let it be observed that all the examples given on this page, indeed all division problems, *may* be regarded as requirements to find how many times one number of things is contained in another number of like things. Referring to example No. 2 given above: If one were required to find one fifth of 1565 silver dollars, he might first take 5 dollars from the 1565 dollars, and put one of the dollars taken in each of five places. He might then take another five dollars from the number of dollars to be divided, and put one dollar with each of the dollars first taken. In this manner he would continue to distribute fives of dollars until all the dollars had been placed in the five piles. He would then count the dollars in each pile. Observe, then, that one fifth of 1565 dollars is as many dollars as \$5 is contained times in \$1565. It is contained 313 times; hence one fifth of 1565 dollars is 313 dollars.

It is not deemed advisable to attempt such an explanation as the foregoing with young pupils; but the more mature and thoughtful pupils may now learn that it is possible to solve all division problems by one thought process—finding how many times one number of things is contained in another number of like things.

Fill the blank with the words, *how many times five dollars are contained in \$1565.
†Fill the blank with the words, *one fifth of \$1565.*

Division—Simple Numbers.

130. Find the quotient of 576 divided by 4.

"Short Division." *Explanation No. 1.*

```
4)576
  144
```

One fourth of 5 hundred is 1 hundred with a remainder of 1 hundred; 1 hundred equals 10 tens; 10 tens plus 7 tens are 17 tens. One fourth of 17 tens is 4 tens with a remainder of 1 ten; 1 ten equals 10 units; 10 units plus 6 units are 16 units. One fourth of 16 units is 4 units. Hence one fourth of 576 is 144.

Explanation No. 2.

Four is contained in 5 hundred, 1 hundred times, with a remainder of 1 hundred; 1 hundred equals 10 tens; 10 tens and 7 tens are 17 tens. Four is contained in 17 tens, 4 tens (40) times with a remainder of 1 ten; 1 ten equals 10 units; 10 units and 6 units are 16 units. Four is contained in 16 units 4 times.

Hence 4 is contained in 576, 144 times.

131. Find the quotient of 8675 divided by 25.

"Long Division." *Explanation.*

```
25)8675(347
   75
   ---
   117
   100
   ----
    175
    175
    ---
```

Twenty-five is contained in 86 hundred, 3 hundred times with a remainder of 11 hundred; 11 hundred equal 110 tens; 110 tens plus 7 tens equal 117 tens. Twenty-five is contained in 117 tens 4 tens (40) times with a remainder of 17 tens; 17 tens equal 170 units; 170 units plus 5 units equal 175 units. Twenty-five is contained in 175 units 7 times.

Hence 25 is contained in 8675, 347 times.

132. PROBLEMS.

1. 93492 ÷ 49
2. 92169 ÷ 77
3. 72855 ÷ 45
4. 34694 ÷ 38
5. 5904 ÷ 328
6. 7693 ÷ 157
7. 8190 ÷ 546
8. 12960 ÷ 864

(a) Find the sum of the eight quotients.

Division—Decimals.

133. Find the quotient of 785.65 divided by .5.

Operation.

$$\begin{array}{r} .5)\overline{785.6^{\vee}5} \\ 1571.3 \end{array}$$

Explanation.

First place a separatrix (v) after that figure in the dividend that is of the same denomination as the right-hand figure of the divisor—in this case after the figure 6. Then divide, writing the decimal point in the quotient when, in the process of division, the separatrix is reached—in this case after the figure 1.

It was required to find how many times 5 tenths are contained in 7856 tenths. 5 tenths are contained in 7856 tenths 1571 times. There are yet 15 hundredths to be divided. 5 tenths are contained in 15 tenths 3 times; in 15 hundredths 3 tenths of a time.

NOTE.—By holding the thought for a moment upon that part of the dividend which corresponds in denomination to the divisor, the place of the decimal point becomes apparent.

5 apples are contained in 7856 apples 1571 times.
5 tenths are contained in 7856 tenths 1571 times.

134. Solve and explain the following problems with special reference to the placing of the decimal point:

1. Divide 340 by .8 — $.8)\overline{340.0^{\vee}}$
2. Divide 468.5 by .25 — $.25)468.50^{\vee}$
3. Divide 38.250 by 12.5 — $12.5)38.2^{\vee}50$
4. Divide 87 by 2.5 — $2.5)87.0^{\vee}$
5. Divide 546 by .75 — $.75)546.00^{\vee}$
6. Divide .576 by 2.4 — $2.4).5^{\vee}76$

7. $86 \div .375 =$ 8. $94.5 \div .8 =$
9. $75 \div .15 =$ 10. $125 \div .5 =$
11. $12.5 \div .05 =$ 12. $1.25 \div .5 =$

(a) Find the sum of the twelve quotients.

Division—United States Money.

135. Divide \$754.65 by \$.27.

Operation.

```
$.27)$754.65‸(2795
     54
     ---
     214
     189
     ---
      256
      243
      ---
       135
       135
       ---
```

Explanation.

This means, find how many times 27 cents are contained in 75465 cents. 27 cents are contained in 75465 cents, 2795 times.

PROBLEM.

At 27¢ a bushel, how many bushels of oats can be bought for \$754.65? As many bushels can be bought as \$.27 is contained times in \$754.65. It is contained 2795 times.

136. Divide \$754.65 by 27.

Operation.

```
27)$754.‸65($27.‸95
   54
   ---
   214
   189
   ---
    256
    243
    ---
     135
     135
     ---
```

Explanation.

This means, find one 27th of \$754.65. One 27th of \$754.65 is \$27.95.

NOTE.—One might find 1 27th of \$754.65 by finding how many times \$27 is contained in \$754.65.

PROBLEM.

If 27 acres of land are worth \$754,65, how much is one acre worth?

137. Divide \$754.65 by .27.

Operation.

```
.27)$754.65‸($2795
     54
     ---
     214
     189
     ---
      256
      243
      ---
       135
       135
       ---
```

Explanation.

This means, find 100 27ths of \$754.65. One 27th of \$754.65 is \$27.95. 100 27ths of \$754.65 is \$2795.

NOTE.—In practice, we find one 27th of 100 times \$754.65.

PROBLEM.

If .27 of an acre of land is worth \$754.65, how much is 1 acre worth at the same rate?

Division—Denominate Numbers.

138. Divide 46 rd. 12 ft. 8 in. by 4.

Operation.

$$4\overline{)46 \text{ rd. } 12 \text{ ft. } 8 \text{ in.}}$$
$$11 \text{ rd. } 11 \text{ ft. } 5 \text{ in.}$$

Explanation.

This means, find 1 fourth of 46 rd. 12 ft. 8 in.

One fourth of 46 rd. is 11 rd. with a remainder of 2 rd.; 2 rd. equal 33 ft.; 33 ft. plus 12 ft. equal 45 ft.

One fourth of 45 ft. equals 11 ft. with a remainder of 1 ft.; 1 ft. equals 12 in.; 12 in. plus 8 in. equals 20 in.

One fourth of 20 in. equals 5 in.

One fourth of 46 rd. 12 ft. 8 in. equals 11 rd. 11 ft. 5 in.

PROBLEM.

The perimeter of a square garden is 46 rd. 12 ft. 8 in. How far across one side of it?

139. MISCELLANEOUS.

Tell the meaning of each of the following, solve, explain, and state in the form of a problem the conditions that would give rise to each number process:

1. Multiply 64 rd. 14 ft. 6 in. by 8.
2. Divide 37 rd. 15 ft. 4 in. by 5.
3. Divide \$675.36 by \$48.
4. Divide \$675.36 by 48.
5. Divide \$675.36 by .48.
6. Divide \$675.36 by \$4.8.
7. Divide \$675.36 by 4.8.
8. Divide \$675.36 by \$.48.
9. Multiply \$356.54 by .36.
10. Multiply \$356.54 by 3.6.
11. Multiply \$356.54 by 36.
12. Can you multiply by a number of dollars?
13. Can you divide by a number of dollars?

Algebraic Division.

140. Examples.

No. 1.

$$4\overline{)12 + 5 \times 4 - 8}$$
$$3 \;+\; 5 \;-\; 2$$

No. 2.

$$a\overline{)12a + 2ab + 4a}$$
$$12 \;+\; 2b \;+\; 4$$

No. 3.

$$2\overline{)2^3 + 3 \times 2^2 - 2}$$
$$2^2 + 3 \times 2 - 1$$

No. 4.

$$b\overline{)ab^3 + cb^2 + 3b}$$
$$ab^2 + cb + 3$$

1. Prove Nos. 1 and 3, by (1) reducing each dividend to its simplest form, (2) dividing it so reduced, by the divisor, and (3) comparing the result with the quotient reduced to its simplest form.
2. Verify No. 2 by letting $a = 3$, and $b = 5$.
3. Verify No. 4 by letting $a = 3$, $b = 5$, and $c = 7$.

141. $(6 \times a \times a \times a \times a \times a) \div (2 \times a \times a) = 6a^5 \div 2a^2 = 3a^3$.

Observe that to divide one algebraic term by another we must find the *quotient of the coefficients* and the *difference of the exponents.*

142. Problems.

1. $6a^5b \div 2a =$
2. $4a^4b^2 \div 2a =$
3. $8a^3b^3 \div 2a =$
4. $10a^2b^4 \div 2a =$

5.

$$2a\overline{)6a^5b + 4a^4b^2 - 8a^3b^3 + 10a^2b^4}$$

6. Verify problem 5 by letting $a = 3$ and $b = 5$.

Algebraic Division.

143. PROBLEMS.

1. Divide $4a^3x + 8a^2x^2 + 6ax^3$ by $2ax$.
2. Multiply the quotient of problem 1 by $2ax$.
3. Verify problems 1 and 2 by letting $a = 2$ and $x = 3$.

4. Divide $3ab^3 + 6a^2b^2 + 9a^3b$ by $3ab$.
5. Multiply the quotient of problem 4 by $3ab$.
6. Verify problems 4 and 5 by letting $a = 3$ and $b = 5$.

7. Divide $2x^3y + x^2y^2 - xy^3$ by xy.
8. Multiply the quotient of problem 7 by xy.
9. Verify problems 7 and 8 by letting $x = 2$ and $y = 3$.

10. Divide $5a^3y^2 - 2a^2y^3 + a^2y^4$ by a^2y.
11. Multiply the quotient of problem 10 by a^2y.
12. Verify problems 10 and 11 by letting $a = 1$ and $y = 2$.

13. Divide $3b^4x + b^3x^2 - 3b^2x^3$ by bx.
14. Multiply the quotient of problem 13 by bx.
15. Verify problems 13 and 14 by letting $b = 3$ and $x = 4$.

Observe that when the divisor is a positive number, each term of the quotient has the same sign as the term in the dividend from which it is derived.

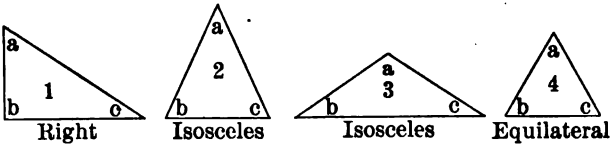
$$2\overline{)8 - 6} \atop \quad 4 - 3$$

One half of $+ 8$ is $+ 4$; one half of $- 6$ is $- 3$.

16. $2x\overline{)4x^5 - 6x^4 + 8x^3 - 2x^2 + 6x.}$

Geometry.

144. Triangles.



1. A triangle has —— sides and —— angles.

2. A right triangle has one right angle; that is, one angle of —— degrees.

3. An isosceles triangle has two angles that are equal and two sides that are equal.

4. An equilateral triangle has equal sides and equal angles.

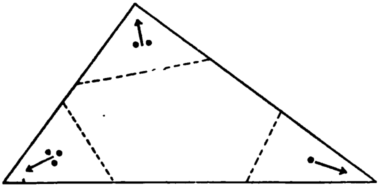


Fig. 5

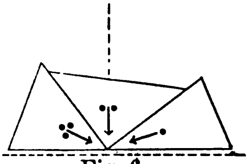
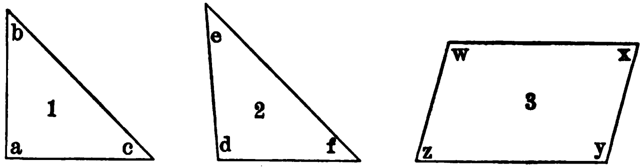


Fig. 6

5. Cut from paper a triangle similar to the one shown in Fig. 5. Then cut it into parts as shown by the dotted lines. Re-arrange the 3 angles of the triangle as shown in Fig. 6. Compare the sum of the 3 angles with two right angles as shown in Fig. 6. Convince yourself that the three angles of this triangle are together equal to two right angles.

6. Cut other triangles and make similar comparisons, until you are convinced that *the sum of the angles of any triangle is equal to two right angles.*

145. Miscellaneous Review.



1. If in figure 1, the angle a is a right angle, and the angle b is equal to the angle c, the angle b is an angle of how many degrees?

2. If in figure 2, the angle d is an angle of 95° and the angle e is an angle of 40°, the angle f is an angle of how many degrees?

3. If in figure 3, the angle x is an angle of 75°, the angle w is an angle of how many degrees?

4. If in an oblong there are ab square feet, and the oblong is a feet long, it is —— feet wide. $ab \div a =$

5. If in a rectangular solid there are abc cubic feet, and the solid is a feet long and b feet wide, it is ——— feet thick. $abc \div ab =$

6. Verify problems 4 and 5 by letting $a = 3$, $b = 4$, and $c = 2$.

7. There is a field that contains 1736 square rods; it is 28 rods long. How wide is the field?

8. There is a solid that contains 4320 cubic inches; it is 24 inches long and 15 inches wide. How thick is the solid?

9. How many square inches of surface in the solid described in problem 8?

PROPERTIES OF NUMBERS.

TO THE TEACHER.—Under this head, *number in the abstract* is discussed with little or no distinction between *numbers of things* and *pure number*. It is dissociation and generalization, without which there could be little progress in the "science of number" or in the "art of computation."

146. Every number is *fractional, integral,* or *mixed.*

1. A **fractional number** is a number of the equal parts of some quantity considered as a unit; as, $\frac{3}{5}$, .9, 5 sixths.

2. An **integral number** is a number that is not, either wholly or in part, a fractional number; as, 15, 46, ninety-five.

3. A **mixed number** is a number one part of which is integral and the other part fractional; as, $5\frac{1}{2}$, 27.6, $274\frac{5}{8}$.

147. An **exact divisor** of a number is a number that is contained in the number an integral number of times.

5 is an exact divisor of 15.
5 is not an exact divisor of 1.5.
$16\frac{2}{3}$ is an exact divisor of 100.

148. Every integral number is *odd* or *even.*

1. An **odd number** is a number of which two is not an exact divisor; as, 7, 23, 141.

2. An **even number** is a number of which two is an exact divisor; as, 8, 24, 142.

Properties of Numbers.

149. Every integral number is *prime* or *composite.*

1. A **prime number** is an integral number that has no exact integral divisors except itself and 1; as, 23, 29, 31, etc.

2. Is *two* a prime number? *three? nine?*

3. Name the prime numbers from 1 to 97 inclusive. Find their sum.

4. A **composite number** is an integral number that has one or more integral divisors besides itself and 1; as, 6, 8, 9, 10, 12, 14, 15, etc.

5. Name the composite numbers from 4 to 100 inclusive. Find their sum.

6. Is *eight* a composite number? *eleven? fifteen?*

(*a*) Find the sum of the results of problems No. 3 and No. 5.

150. To find whether an integral number is *prime* or *composite.*

1. Is the number 371 prime or composite?

Operation.

2)371	5)371
185+	74+
3)371	7)371
123+	53

Explanation.

Beginning with 2 (the smallest prime number except the number 1), it is found by trial not to be an exact divisor of 371.

3 is not an exact divisor of 371.

5 is not an exact divisor of 371.

7 is an exact divisor of 371. Therefore 371 is a composite number, being composed of 53 sevens, or of 7 fifty-threes.

Observe that we use as trial divisors only prime numbers. If 2 is not an exact divisor of a number, neither 4 nor 6 can be. Do you see why?

Properties of Numbers.

2. Is the number 397 prime or composite?

Operation.

2)397	3)397
198+	132+
5)397	7)397
79+	56+
11)397	13)397
36+	30+
17)397	19)397
23+	20+

Explanation.

By trial it is found that neither 2, 3, 5, 7, 11, 13, 17, nor 19 is an exact divisor of 397.

No composite number between 2 and 19 can be an exact divisor of 397; for since *one* 2 is not an exact divisor of the number, several 2's, as 4, 6, 8, 12, etc., cannot be; since *one* 3 is not an exact divisor of the number, several 3's, as 6, 9, 12, etc., cannot be; since *one* 5 is not an exact divisor of the number, several 5's, as 10 and 15, cannot be; since *one* 7 is not an exact divisor of the number, two 7's (14) cannot be.

No number greater than 19 can be an exact divisor of the number; for if a number greater than 19 were an exact divisor of the number, the quotient (which also must be an exact divisor) would be less than 20. But it has already been proved that no integral number less than 20 is an exact divisor of 397. Therefore 397 is a prime number.

Observe that in testing a number to determine whether it is prime or composite, we take as trial divisors prime numbers only, beginning with the number two.

Observe that as the divisors become greater, the quotients become less, and that *we need make no trial by which a quotient will be produced that is less than the divisor.*

3. Determine by a process similar to the foregoing whether each of the following is prime or composite: 127, 249, 257, 371.

151. Any divisor of a number may be regarded as *a factor* of the number. An exact integral divisor of a number is an *integral factor* of the number.

Properties of Numbers.

152. Prime Factors.

1. An integral factor that is a prime number is a **prime factor.**

5 is a prime factor of 30.
7 is a prime factor of —— and ——.
3 is a prime factor of —— and ——.
2 and 3 are prime factors of —— and —— and ——.
3 and 5 are prime factors of —— and —— and ——

2. Resolve 105 into its prime factors.

Operation.	*Explanation.*
5)105 3)21 7	Since the prime number 5 is an exact divisor of 105, it is a prime factor of 105. Since the prime number 3 is an exact divisor of the quotient (21), it is a prime factor of 21 and 105.

Since 3 is contained in 21 exactly 7 times, and since 7 is a prime number, 7 is a prime factor of 21 and of 105. Therefore the prime factors of 105 are 5, 3, and 7.

Observe that if 7 and 3 are prime factors of 21 they must be prime factors of 105, for 105 is made up of 5 21's. 7 is contained 5 times as many times in 105 as it is in 21.

Observe that every composite number is equal to the product of its prime factors.

$$105 = 5 \times 3 \times 7. \quad 18 = 3 \times 3 \times 2.$$

Observe that 2 times 3 times a number equals 6 times the number; 3 times 5 times a number equals 15 times the number, etc.

Observe that instead of multiplying a number by 21, it may be multiplied by 3 and the product thus obtained by 7, and the same result be obtained as would be obtained by multiplying the number by 21. Why?

Properties of Numbers.

153. MULTIPLES, COMMON MULTIPLES, AND LEAST COMMON MULTIPLES.

1. A **multiple** of a number is an integral number of times the number.

30 and 35 and 40 are multiples of 5.
16 and 20 and 32 are multiples of 4.

2. A **common multiple** of two or more numbers is an integral number of times each of the numbers.

30 is a common multiple of 5 and 3.
40 is a common multiple of 8 and 10.
—— is a common multiple of 9 and 6.
—— is a common multiple of 8 and 12.

3. A common multiple of two or more integral numbers contains *all the prime factors found in every one of the numbers, and may contain other prime factors.*

$48 = 2 \times 2 \times 2 \times 2 \times 3$. $150 = 2 \times 3 \times 5 \times 5$. A common multiple of 48 and 150 must contain four 2's, one 3, and two 5's. It may contain other factors.

$$2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 1200.$$
$$2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 2 = 2400.$$

1200 and 2400 are common multiples of 48 and 150.

4. The **least common multiple** (l. c. m.) of two or more numbers is the *least* number that is an integral number of times each of the numbers.

40 and 80 and 120 are common multiples of 8 and 10; but 40 is the least common multiple of 8 and 10.

5. The least common multiple of two or more numbers contains *all the prime factors found in every one of the numbers, and no other prime factors.*

Properties of Numbers.

$36 = 2 \times 2 \times 3 \times 3$. $120 = 2 \times 2 \times 2 \times 3 \times 5$. The l. c. m. of 36 and 120 must contain three 2's, two 3's, and one 5. $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$, l. c. m. of 36 and 120.

6. To find the l. c. m. of two or more numbers: *Resolve each number into its prime factors. Take as factors of the l. c. m. the greatest number of 2's, 3's, 5's, 7's, etc., found in any one of the numbers.*

EXAMPLE.

Find the l. c. m. of 24, 35, 36, and 50.

OPERATION.

$$24 = 2 \times 2 \times 2 \times 3.$$
$$35 = 5 \times 7.$$
$$36 = 2 \times 2 \times 3 \times 3.$$
$$50 = 2 \times 5 \times 5.$$
$$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 12600, \text{ l. c. m.}$$

EXPLANATION.

24 has the greatest number of 2's as factors.
36 has the greatest number of 3's as factors.
50 has the greatest number of 5's as factors.
35 is the only number in which the factor 7 occurs.

There must be as many 2's among the factors of the l. c. m. as there are 2's among the factors of 24; as many 3's as there are 3's among the factors of 36; as many 5's as there are 5's among the factors of 50; as many 7's as there are 7's among the factors of 35; that is, three 2's, two 3's, two 5's, and one 7.

Find the l. c. m.:

7. Of 48 and 60.
8. Of 60 and 75.
9. Of 50 and 60.
10. Of 30 and 40.
11. Of 20, 30, and 40.
12. Of 40, 50, and 60.
13. Of 24, 48, and 36.
14. Of 25, 35, and 40.

Algebra—Parentheses.

154. When an expression consisting of two or more terms is to be treated as a whole, it may be enclosed in a parenthesis.

$$\begin{cases} 12+(5+3)=? \\ 12+5+3=? \end{cases} \qquad \begin{cases} 7a+(3a+2a)=? \\ 7a+3a+2a=? \end{cases}$$

Observe that removing the parenthesis makes no change in the results.

$$\begin{cases} 12-(5+3)=? \\ 12-5-3=? \end{cases} \qquad \begin{cases} 7a-(3a+2a)=? \\ 7a-3a-2a=? \end{cases}$$

Observe the change in signs made necessary by the removal of the parenthesis.

$$\begin{cases} 12-(5-3)=? \\ 12-5+3=? \end{cases} \qquad \begin{cases} 7a-(3a-2a)=? \\ 7a-3a+2a=? \end{cases}$$

Observe the change in signs made necessary by the removal of the parenthesis.

A careful study and comparison of the foregoing problems will make the reasons for the following apparent:

I. *If an expression within a parenthesis is preceded by the plus sign, the parenthesis may be removed without making any changes in the signs of the terms.*

II. *If an expression within a parenthesis is preceded by a minus sign, the parenthesis may be removed; but the sign of each term in the parenthesis must be changed; the sign* + *to* −, *and the sign* − *to* +.

155. Remove the parenthesis, change the signs if necessary, and combine the terms:

1. $15-(6+4)=$	5. $15b-(12b-4b)=$
2. $18+(4-3)=$	6. $18c+(9c-3c)=$
3. $27-(8+3)=$	7. $24d-(5d+3d)=$
4. $45+(12-3)=$	8. $36x-(5x+4x)=$

Algebra—Parentheses.

156. Multiplying an Expression Enclosed in a Parenthesis.

1. $6(7+4)=?$* $\qquad 6(7a+4b)=?$

$6(a+b)=?$ Ans. $6a+6b$.

$a(b+c)=?$ Ans. $ab+ac$.

Observe that in multiplying the sum of two numbers by a third number, the sum may be found and multiplied; or each number may be multiplied and the sum of the products found.

In the last three examples given above, substitute 5 for a, 3 for b, and 2 for c; then perform again the operations indicated, and compare the results with those obtained when the letters were employed.

2. $6(7-4)=?$ $\qquad 6(7a-4b)=?$

$6(a-b)=?$ Ans. $6a-6b$.

$a(b-c)=?$ Ans. $ab-ac$.

Observe that in multiplying the difference of two numbers by a third number, the difference may be found and multiplied; or each number may be multiplied and the difference of the products found.

In the last three problems given above, substitute 5 for a, 3 for b, and 2 for c; then perform again the operations indicated, and compare the results with those obtained when the letters were employed.

157. Problems.

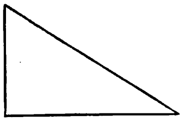
If $a=5$, $b=3$, and $c=2$, find the value of the following:

1. $3(a+b)-2(b+c)$.
2. $4(a+2b)-3(b-c)$.
3. $2(2a-b)+2(2b-c)$.

* This means, that the sum of 7 and 4 is to be multiplied by six; or that the sum of six 7's and six 4's is to be found.

Geometry.

158. TRIANGLES—*Continued.*



Right Triangle.

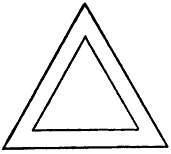
1. The sum of the angles of any triangle is equal to —— right angles or —— degrees.

2. In a right triangle there is one right angle. The other two angles are together equal to ———.

3. In a certain right triangle one of the angles is an angle of 40°. How many degrees in each of the other two angles? Draw such a triangle.

4. Convince yourself by drawings and measurements that every equilateral triangle is equiangular.

Equilateral Triangles.

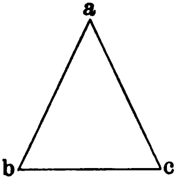


Equiangular Triangles.

5. Note that in every equiangular triangle each angle is one third of 2 right angles. So each angle is an angle of —— degrees.

6. If any one of the angles of a triangle is greater or less than 60, can the triangle be equiangular? Can it be equilateral?

7. If angle *a* of an isosceles triangle measures 50°, how many degrees in angle *b*? In angle *c*?



159. Miscellaneous Review.

1. I am thinking of a right triangle one of whose angles measures 32°. Give the measurements of the other two angles. Draw such a triangle.

2. I am thinking of an isosceles triangle; the sum of its two equal angles is 100°. Give the measurement of its third angle. Draw such a triangle.

3. Let a equal the number of degrees in one angle of a triangle and b equal the number of degrees in another angle of the same triangle; then the number of degrees in the third angle is $180° - (a + b)$. If a equals 30, and b equals 45, how many degrees in the third angle?

4. Name three common multiples of 16 and 12.

5. Name the *least* common multiple of 16 and 12.

6. Find the sum of all the prime numbers from 101 to 127 inclusive.

7. Find the prime factors of 836.

8. With the prime factors of 836 in mind or represented on the blackboard, tell the following:

(*a*) How many times is 19 contained in 836?

(*b*) How many times is 11×19 contained in 836?

(*c*) How many times is $19 \times 11 \times 2$ contained in 836?

160. Problems.

Find the l. c. m.

1. Of 18 and 20.	6. Of 36, 72, and 24.
2. Of 13 and 11.	7. Of 45, 81, and 27.
3. Of 24 and 32.	8. Of 33, 55, and 88.
4. Of 16 and 38.	9. Of 45, 65, and 85.
5. Of 46 and 86.	10. Of 3, 5, 7, and 11.

(*a*) Find the sum of the ten results.

DIVISIBILITY OF NUMBERS.

161. NUMBERS EXACTLY DIVISIBLE BY 2; BY $2\frac{1}{2}$; BY $3\frac{1}{3}$; BY 5; BY 10.

1. An integral number is exactly divisible by 2 if the right-hand figure is 0, or if the number expressed by its right-hand figure is exactly divisible by 2.

EXPLANATORY NOTE.—Every integral number that may be expressed by two or more figures may be regarded as made up of a certain number of tens and a certain number (0 to 9) of primary units; thus, 485 is made up of 48 tens and 5 units; 4260 is made up of 426 tens and 0 units; 27562 is made up of 2756 tens and 2 units. But ten is exactly divisible by 2; so any number of tens, or *any number of tens plus any number of twos*, is exactly divisible by 2.

2. Tell which of the following are exactly divisible by 2, and why: 387, 5846, 2750, 2834.

3. Any number, integral or mixed, is exactly divisible by $2\frac{1}{2}$ if the part of the number expressed by figures to the right of the tens' figure, is exactly divisible by $2\frac{1}{2}$.

4. Show why the statement made in No. 3 is correct, employing the thought process given in the "Explanatory Note" above.

5. Tell which of the following are exactly divisible by $2\frac{1}{2}$, and why: 485, 470, 365, $472\frac{1}{2}$, $3847\frac{1}{2}$.

6. Any number, integral or mixed, is exactly divisible by $3\frac{1}{3}$ if the part of the number expressed by figures to the right of the tens' figure is exactly divisible by $3\frac{1}{3}$.

7. Tell which of the following are exactly divisible by $3\frac{1}{3}$, and why: 780, $283\frac{1}{3}$, $576\frac{2}{3}$, 742, 80.

Divisibility of Numbers.

8. Any integral number is exactly divisible by 5 if its right-hand figure is 0 or 5. Show why.

9. Any integral number is exactly divisible by 10 if its right-hand figure is ——.

162. PROBLEMS.

1. How many times is $2\frac{1}{2}$ contained in $582\frac{1}{2}$?*
2. How many times is $2\frac{1}{2}$ contained in 375?
3. How many times is $2\frac{1}{2}$ contained in $467\frac{1}{2}$?
4. How many times is $2\frac{1}{2}$ contained in 4680?
5. How many times is $3\frac{1}{3}$ contained in $786\frac{2}{3}$?†
6. How many times is $3\frac{1}{3}$ contained $543\frac{1}{3}$?
7. How many times is $3\frac{1}{3}$ contained in 8640?
8. How many times is 5 contained in 3885?
9. How many times is 5 contained in 1260?

163. NUMBERS EXACTLY DIVISIBLE BY 25; BY $33\frac{1}{3}$; BY $12\frac{1}{2}$; BY $16\frac{2}{3}$; BY 20; BY 50.

1. Any integral number is exactly divisible by 25 if its two right-hand figures are zeros, or if the part of the number expressed by its two right-hand figures is exactly divisible by 25.

EXPLANATORY NOTE.—Every integral number expressed by three or more figures may be regarded as made up of a certain number of hundreds and a certain number (0 to 99) of primary units; thus 4624 is made up of 46 hundreds and 24 units; 38425 is made up of 384 hundreds and 25 units; 8400 is made up of 84 hundreds and 0 units. But a hundred is exactly divisible by 25; so any number of hundreds, or *any number of hundreds plus any number of 25's* is exactly divisible by 25.

* $2\frac{1}{2}$ is contained in $582\frac{1}{2}$ $(4 \times 58) + 1$ times. Why?
† $3\frac{1}{3}$ is contained in $786\frac{2}{3}$ $(3 \times 78) + 2$ times. Why?

Divisibility of Numbers.

2. Tell which of the following are exactly divisible by 25, and why: 37625, 34836, 27950, 38575.

3. Every number, integral or mixed, is exactly divisible by $33\frac{1}{3}$, if that part of the number expressed by the figures to the right of the hundreds' figure is exactly divisible by $33\frac{1}{3}$.

4. Show why the statement made in No. 3 is correct, employing the thought process given in the "Explanatory Note" under No. 1 on the preceding page.

5. Tell which of the following are exactly divisible by $33\frac{1}{3}$, and why: $36466\frac{2}{3}$, 2375, $46833\frac{1}{3}$, 38900, 46820.

6. Any number, integral or mixed, is exactly divisible by $12\frac{1}{2}$, if the part of the number expressed by the figures to the right of the hundreds' figure, is exactly divisible by $12\frac{1}{2}$. Show why.

7. Tell which of the following are exactly divisible by $12\frac{1}{2}$, and why: 375, $837\frac{1}{2}$, 6450, 4329, $7467\frac{1}{2}$.

8. Any number, integral or mixed, is exactly divisible by $16\frac{2}{3}$, if________________________________

9. Tell which of the following are exactly divisible by $16\frac{2}{3}$: $4633\frac{1}{3}$, 5460, 2350, 37400, $27583\frac{1}{3}$, $25416\frac{2}{3}$.

10. Any integral number is exactly divisible by 20 if the number expressed by its two right-hand figures is exactly divisible by 20. Show why.

11. Tell which of the following are exactly divisible by 20, and why: 3740, 2650, 3860, 29480, 3470.

12. Tell which of the following are exactly divisible by 50, and why: 2460, 3450, 6800, 27380, 25450.

Divisibility of Numbers.

164. Problems.

1. How many times is 25 contained in 2450 ?*
2. How many times is 25 contained in 3775 ?
3. How many times is $33\frac{1}{3}$ contained in $4666\frac{2}{3}$?†
4. How many times is $33\frac{1}{3}$ contained in $3433\frac{1}{3}$?
5. How many times is $12\frac{1}{2}$ contained in $4737\frac{1}{2}$?
6. How many times is $12\frac{1}{2}$ contained in $3662\frac{1}{2}$?
7. How many times is $16\frac{2}{3}$ contained in $2533\frac{1}{3}$?
8. How many times is $16\frac{2}{3}$ contained in 4550 ?

165. Numbers Exactly Divisible by 9.

1. Any number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

Explanatory Note.—Any number more than nine is a certain number of nines and as many over as the number indicated by the sum of its digits. Thus, 20 is two nines and 2 over; 41 is four nines and $4 + 1$ over; 42 is four nines and $4 + 2$ over; 200 is twenty-two nines and 2 over; 300 is thirty-three nines and 3 over; 320 is a certain number of nines and $3 + 2$ over; 321 is a certain number of nines and $3 + 2 + 1$ over.

326 is a certain number of nines and $3 + 2 + 6$ over; but $3 + 2 + 6 = 11$, or another nine and 2 over.

2. Read the "Explanatory Note" carefully, and tell which of the following are exactly divisible by 9: 3256, 4266, 2314, 2574.

166. Problems.

1. 4625 is a certain number of 9's and ——— over.
2. 3526 is a certain number of 9's and ——— over.
3. 2154 is a certain number of 9's and ——— over.

* 25 is contained in 2450 $(4 \times 24) + 2$ times. Why?

† $33\frac{1}{3}$ is contained in $4666\frac{2}{3}$ $(3 \times 46) + 2$ times. Why?

Divisibility of Numbers.

167. Prime Factors and Exact Divisors.

1. Any integral number is exactly divisible by each of its prime factors and by the product of any two or more of its prime factors. Thus, 30, $(2 \times 3 \times 5)$, is exactly divisible by 2, by 3, by 5, and by (2×3), 6, and by (2×5), 10, and by (3×5), 15.

2. The exact integral divisors of 36, $(2 \times 2 \times 3 \times 3)$, are 2, 3, —, —, —, and ——.

168. Prime Factors, Common Divisors, and Greatest Common Divisors.

1. Any prime factor or any product of two or more prime factors common to two or more numbers is a common divisor of the numbers. Thus, the numbers 30, $(2 \times 3 \times 5)$, and 40, $(2 \times 2 \times 2 \times 5)$, have the factors 2 and 5 in common. So the common divisors of 30 and 40 are 2, 5, and 10, and the greatest common divisor is 10.

Rule.—*To find the greatest common divisor of two or more numbers, find the product of the prime factors common to the numbers.*

2. Find the g. c. d. of 50, 75, and 125.

Operation No. 1.

$$50 = 2 \times 5 \times 5.$$
$$75 = 3 \times 5 \times 5.$$
$$125 = 5 \times 5 \times 5.$$
$$5 \times 5 = 25, \text{ g. c. d.}$$

Operation No. 2.

$$\begin{array}{r|rrr} 5 & 50 & 75 & 125. \\ \hline 5 & 10 & 15 & 25. \\ \hline & 2 & 3 & 5 \end{array}$$

$$5 \times 5 = 25, \text{ g. c. d.}$$

3. Find the g. c. d. of 80, 100, 140.
4. Find the g. c. d. of 48, 60, 72.
5. Find the g. c. d. of 64, 96, 256.

Divisibility of Numbers.

6. Find the g. c. d. of 640 and 760.

Operation.

```
640)760(1
    640
    ———
    120)640(5
        600
        ———
         40)120(3
            120
            ———
```

Explanation.

The number 760 is an integral number of times the g. c. d., *whatever that may be;* so is the number 640. We make an incomplete division of 760 by 640 and have as a remainder the number 120. Since 640 and 760 are each an integral number of times the g. c. d., their difference, 120, must be an integral number of times the g. c. d.; for, taking an integral number of times a thing from an integral number of times a thing must leave an integral number of times the thing. Therefore, no number greater than 120 can be the g. c. d. But if 120 is an exact divisor of 640, it is also an exact divisor of 760, for it will be contained one more time in 760 than in 640. We make the trial, and find that 120 is not an exact divisor of 640; there is a remainder of 40. Since 600 (120×5) and 640 are each an integral number of times the g. c. d., 40 must be an integral number of times the g. c. d. But if 40 is an exact divisor of 120 it is an exact divisor of 600 (120×5) and 640 (40 more than 600) and 760 (120 more than 640). We make the trial, and find that it is an exact divisor of 120, and is therefore the g. c. d. of 640 and 760.

Observe that any number that is an exact divisor of two numbers is an exact divisor of their difference.

169. From the foregoing make a rule for finding the g. c. d. of two numbers and apply it to the following:

Find the g. c. d.:

1. Of 380 and 240.
2. Of 275 and 155.
3. Of 144 and 96.
4. Of 1728 and 288.
5. Of 650 and 175.
6. Of 540 and 450.
7. Of 320 and 860.
8. Of 475 and 350.
9. Of 390 and 520.
10. Of 450 and 600.

(a) Find the sum of the ten results.

Algebra—Equations.

170. An equation is the expression of the equality of two numbers or combinations of numbers.

EQUATIONS.

(1) $2+4+6=3+5+4$
(2) $a+b+c=40-12$

1. Every equation is made up of two members. The part of the equation which is on the left of the sign of equality is called the first member; the part on the right of the sign of equality, the second member.

2. If the same number be added to each member of an equation, the equality will not be destroyed.

If $x=8$, then $x+4=8+4$.
If $a+b=16$, then $a+b+c=16+c$.

3. If the same number be subtracted from each member of an equation, the equality will not be destroyed.

If $x=8$, then $x-3=8-3$.
If $a+b=16$, then $a+b-c=16-c$.

4. If each member of an equation be multiplied by the same number, the equality will not be destroyed.

If $x=8$, then $4x=4$ times 8, or 32.
If $a+b=16$, then $4a+4b=4$ times 16, or 64.

5. If each member of an equation be divided by the same number, the equality will not be destroyed.

If $x=8$, then $\frac{x}{4}=\frac{8}{4}$, or 2.

If $a+b=16$, then $\frac{a}{4}+\frac{b}{4}=\frac{16}{4}$, or 4.

Algebra—Equations.

7. Any term in an equation may be transposed from one member of the equation to the other; but its sign must be changed when the transposition is made.

If $x + 5 = 15$, then $x = 15 - 5$, or 10.
If $y - 6 = 27$, then $y = 27 + 6$, or 33.
If $a + b + c = 18$, then $a + b = 18 - c$.
If $x + y - z = 25$, then $x + y = 25 + z$.

171. TO FIND THE NUMBER FOR WHICH x STANDS IN AN EQUATION IN WHICH THERE IS NO OTHER UNKNOWN NUMBER.

EXAMPLE NO. 1.

Equation,	$x + 2x + 3x - 5 = 13$
Transposing,	$x + 2x + 3x = 13 + 5$
Uniting,	$6x = 18$
Dividing,	$x = 3$

EXAMPLE NO. 2.

Equation,	$2x + 3x + 6 = 5x - 2x + 18$
Transposing,	$2x + 3x - 5x + 2x = 18 - 6$
Uniting,	$2x = 12$
Dividing,	$x = 6$

PROBLEMS.

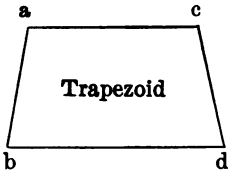
Find the value of x.

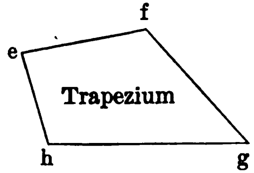
1. $x + 4 = 12$
2. $x + 3x = 8$
3. $5x - 2 = 23$
4. $3x - x = 44$
5. $7x + x = 144$.
6. $3x + 2x - 4 = x + 16$
7. $5x - 7 = 3x + 5$
8. $7x + 2x - x = 3x + 35$
9. $5x - 4x - 3x + 6x = 44$
10. $6x - 8 - 2x = 3x + 5$

(*a*) Find the sum of the ten results.

Geometry.

172. QUADRILATERALS THAT ARE NOT PARALLELOGRAMS.





1. Two of the sides of a trapezoid are parallel and two are not parallel. In the trapezoid represented above the side *ac* is parallel to the side ———.

2. No two of the bounding lines of a trapezium are parallel.

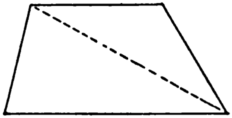
3. In the trapezoid represented above no one of the angles is a right angle. Name the angles that are greater than right angles; the angles that are less than right angles.

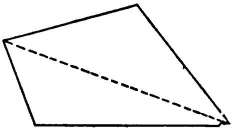
4. Draw a trapezoid two of whose angles are right angles.

5. Can you draw a trapezoid having one and only one right angle?

6. Draw a trapezium one of whose angles is a right angle.

7. Can you draw a trapezium having more than one right angle?





8. Every quadrilateral may be divided into two triangles. Remember that the sum of the angles of two triangles is equal to four right angles. Observe that the sum of the angles of the two triangles is equal to the sum of the angles of the quadrilateral. *So the sum of the angles of a quadrilateral is equal to four right angles.*

173. Miscellaneous Review.

1. If two of the angles of a trapezoid are right angles and the third is an angle of 60°, how many degrees in the fourth angle? Draw such a trapezoid.*

2. If the sum of three of the angles of a trapezium is 298°, how many degrees in the fourth angle? Draw such a trapezium.*

3. If one of the angles of a triangle is an angle of 80°, and the other two angles are equal, how many degrees in each of the other angles? Draw the figure.*

4. If one of the angles of a quadrilateral is a right angle, and the other three angles are equal, what kind of a quadrilateral is the figure?

5. One of the angles of a quadrilateral is a degrees; another is b degrees; the third is c degrees. How many degrees in the fourth angle?

6. The smallest angle of a triangle is x degrees; another angle is $2x$ degrees, and the third is $3x$ degrees:

$$\text{Then } x + 2x + 3x = 180.$$

Find the value of x; of $2x$; of $3x$.

7. 643,265,245,350. Without performing the division tell whether this number is exactly divisible by 9; by 5; by 10; by 25; by 50; by $12\frac{1}{2}$; by 18; by 6; by 15; by 30; by 90; by $16\frac{2}{3}$.†

8. A number is made up of the following prime factors: 2, 2, 3, 3, 5, 7, 11. Is the number exactly divisible by 18? by 26? by 35? by 77? by 21? by 30? by 45? by 8?

* It is not expected that this drawing will be accurate in its angular measurement—simply an approximation to accuracy, to aid the pupil in recognizing the comparative size of angles.

† A careful study of pages 211-215 inclusive will enable the pupil to make the statements called for with little hesitation.

FRACTIONS.

174. A fraction may be expressed by two numbers, one of them being written above and the other below a short horizontal line; thus, $\frac{3}{5}$, $\frac{11}{17}$, $\frac{240}{379}$.

175. The number above the line is the **numerator** of the fraction; the number below the line, the **denominator** of the fraction.

176. Kinds of Fractions.

1. A fraction whose numerator is less than its denominator is a **proper fraction.**

$\frac{3}{4}$, $\frac{2}{5}$, $\frac{12}{17}$, are proper fractions.

2. A fraction whose numerator is equal to or greater than its denominator is an **improper fraction.**

$\frac{3}{3}$, $\frac{6}{6}$, $\frac{27}{4}$, are improper fractions.

Note.—The fraction .7 is a proper fraction. 2.7 may be regarded as an improper fraction or as a mixed number. If it is to be considered an improper fraction it should be read, 27 tenths; if a mixed number, 2 and 7 tenths.

3. Such expressions as the following are **compound fractions:**

$\frac{3}{4}$ of $\frac{6}{7}$, $\frac{2}{5}$ of $\frac{1}{2}$, $\frac{5}{8}$ of $\frac{7}{11}$.

4. A fraction whose numerator or denominator is itself a fraction or a mixed number, is a **complex fraction.**

$\frac{\frac{2}{3}}{4}$, $\frac{2}{3\frac{1}{2}}$, $\frac{\frac{1}{2}}{2\frac{1}{2}}$, are complex fractions.

Fractions.

5. Any fraction that is neither compound nor complex is a **simple fraction.**

$\frac{3}{5}$, $\frac{12}{11}$, $\frac{24}{37}$, are simple fractions.

6. A fraction whose denominator is 1 with one or more zeros annexed to it is a **decimal fraction.**

$\frac{3}{10}$, .7, .25, $\frac{36}{100}$, are decimal fractions.

NOTE 1.—The denominator of a decimal fraction may be expressed by figures, or it may be indicated by the position of the right-hand figure of its numerator with reference to the decimal point. When the denominator is thus indicated, the fraction is called **a decimal**, and is said to be *written decimally*.

NOTE 2.—All fractions that are not decimal are called **common fractions.** A decimal fraction when not "written decimally" (or thought of as written decimally) is usually classed as a common fraction.

7. A **complex decimal** is a decimal and a common fraction combined in one number.

$.7\frac{1}{3}$, $.25\frac{1}{7}$, $.056\frac{2}{3}$, are complex decimals.

177. There are **three aspects** in which fractions should be considered.

I. THE FRACTIONAL UNIT ASPECT.

The numerator tells the *number of things* and the denominator *indicates their name.* In the fraction $\frac{5}{7}$ there are 5 things (magnitudes) called sevenths. In the fraction $\frac{5}{8}$ there are *five fractional units,* each of which is one eighth of some other unit called the *unit of the fraction.*

NOTE.—The function of the denominator is to show the number of parts into which the unit of the fraction is divided; the function of the numerator, to show the number of parts taken.

Fractions.

II. THE DIVISION ASPECT.

The numerator of a fraction is *a dividend*, the denominator *a divisor*, and the fraction itself *a quotient;* thus, in the fraction $\frac{5}{8}$, the dividend is 5, the divisor 8, and the quotient $\frac{5}{8}$.

NOTE.—In the case of an improper fraction, as $\frac{8}{4}$, it may be more readily seen by the pupil that the numerator is the dividend, the denominator the divisor, and the fraction ($\frac{8}{4} = 2$) the quotient; but the division relation is in every fraction, whether proper or improper, common or decimal, simple or complex.

III. THE RATIO ASPECT.*

The numerator of a fraction is *an antecedent*, the denominator *a consequent*, and the fraction itself *a ratio;* thus, in the fraction $\frac{7}{10}$, 7 is the antecedent, 10 the consequent, and $\frac{7}{10}$ the ratio.

NOTE 1.—This relation may be more readily seen by the pupil in the case of an improper fraction. In the fraction $\frac{12}{4}$, 12 is the antecedent, 4 the consequent, $\frac{12}{4}$, or 3, the ratio.

NOTE 2.—*Every integral number as well as every fraction is a ratio.* The number 8 is the ratio of a magnitude that is 8 times some unit of measurement to a magnitude that is 1 time the same unit of measurement.

178. REDUCTION OF FRACTIONS.

1. The numerator and the denominator of a fraction are its **terms**.

2. A fraction is said to be in its **lowest terms** when its numerator and denominator are integral numbers that are prime to each other.

* This may be omitted until the book is reviewed.

Fractions.

3. Reduce $\frac{160}{200}$ to its lowest terms.

Operation.

$$10)\frac{160}{200}=\frac{16}{20}.$$

$$4)\frac{16}{20}=\frac{4}{5}.$$

Explanation.

Dividing each term of $\frac{160}{200}$ by 10, we have 1 tenth as many parts, which are 10 times as large. Dividing each term of $\frac{16}{20}$ by 4, we have 1 fourth as many parts, which are 4 times as large. Hence, $\frac{160}{200}=\frac{4}{5}$. But 4 and 5 are prime to each other, and the fraction is in its lowest terms.

RULE.—*Divide each term of the fraction by any common divisor except 1, and divide each term of the fraction thus obtained by any common divisor except 1, and so continue until the terms are prime to each other.*

Reduce to lowest terms:

(1) $\frac{275}{375}$ (2) $\frac{520}{650}$ (3) $\frac{156}{270}$ (4) $\frac{235}{340}$

(5) $\frac{56}{210}$ (6) $\frac{36}{180}$ (7) $\frac{171}{405}$ (8) $\frac{63}{204}$

(9) $\frac{37\frac{1}{2}}{100}$* (10) $\frac{2\frac{1}{2}}{4}$†

(a) Find the sum of the ten results.‡

4. Reduce $\frac{5}{8}$ to higher terms — to 120ths.

Operation.

$$120 \div 8 = 15.$$

$$\frac{5 \times 15}{8 \times 15} = \frac{75}{120}$$

Explanation.

In $\frac{75}{120}$ there are 15 times as many parts as there are in $\frac{5}{8}$, and the parts are 1 fifteenth as large. Hence, $\frac{75}{120}=\frac{5}{8}$.

* Divide each term by $12\frac{1}{2}$.

† Divide each term by $\frac{1}{2}$.

‡ If the pupil has not had sufficient practice in addition of fractions to do this, the finding of the sum may be omitted until the book is reviewed.

Fractions.

Reduce to higher terms — to 160ths.

(1) $\frac{5}{8}$ (2) $\frac{11}{16}$ (3) $\frac{9}{20}$ (4) $\frac{37}{40}$ (5) $\frac{3}{40}$
(6) $\frac{3}{8}$ (7) $\frac{5}{16}$ (8) $\frac{11}{20}$ (9) $\frac{37}{80}$ (10) $\frac{43}{80}$

(a) Find the sum of the ten results.

5. Two or more fractions whose denominators are the same, are said to have **a common denominator.**

6. Two or more fractions that do not have a common denominator may be changed to equivalent fractions *having a common denominator.*

EXAMPLE.

$\frac{2}{3}$ and $\frac{3}{4}$ may be changed to 12ths, 24ths, or 36ths.

$$\frac{2}{3} = \frac{\quad}{24} \qquad \frac{3}{4} = \frac{\quad}{24} \qquad \frac{2}{3} = \frac{\quad}{36} \qquad \frac{3}{4} = \frac{\quad}{36}$$

7. Two or more fractions that do not have a common denominator may be changed to equivalent fractions having their *least common denominator.* The l. c. d. of two or more fractions is the l. c. m. of the given denominators.

EXAMPLE.

Change $\frac{11}{30}$, $\frac{9}{40}$, and $\frac{37}{60}$ to equivalent fractions having their least common denominator.

OPERATION.

(1) The l. c. m. of 30, 40, and 60 is 120.

(2) $120 \div 30 = 4 \quad \frac{11}{30} = \frac{44}{120}$

(3) $120 \div 40 = 3 \quad \frac{9}{40} = \frac{27}{120}$

(4) $120 \div 60 = 2 \quad \frac{37}{60} = \frac{74}{120}$

Fractions.

Reduce to equivalent fractions having their l. c. d.

1. $\frac{11}{15}$ and $\frac{9}{20}$.
2. $\frac{19}{25}$ and $\frac{13}{20}$.
3. $\frac{23}{30}$ and $\frac{17}{40}$.
4. $\frac{7}{35}$ and $\frac{5}{63}$.
5. $\frac{8}{45}$ and $\frac{37}{60}$.
6. $\frac{4}{15}$, $\frac{2}{5}$, and $\frac{11}{20}$.
7. $\frac{6}{25}$, $\frac{1}{5}$, and $\frac{7}{20}$.
8. $\frac{7}{30}$, $\frac{2}{5}$, and $\frac{23}{40}$.
9. $\frac{28}{35}$, $\frac{7}{10}$, and $\frac{58}{63}$.
10. $\frac{37}{45}$, $\frac{3}{10}$, and $\frac{23}{60}$.

(a) Find the sum of the twenty-five fractions.*

179. To Add Common Fractions.

Rule.—*Reduce the fractions if necessary to equivalent fractions having a common denominator, add their numerators, and write their sum over the common denominator.*

Example.

Add $\frac{11}{45}$, $\frac{17}{30}$, and $\frac{53}{60}$.

(1) The l. c. m. of 45, 30, and 60 is 180.

(2) $\frac{11}{45} = \frac{44}{180}$. $\frac{17}{30} = \frac{102}{180}$. $\frac{53}{60} = \frac{159}{180}$.

(3) $\frac{44}{180} + \frac{102}{180} + \frac{159}{180} = \frac{305}{180}$.

Find the sum of—

1. $\frac{5}{16}$ and $\frac{7}{20}$.
2. $\frac{4}{15}$ and $\frac{9}{18}$.
3. $\frac{11}{21}$ and $\frac{5}{28}$.
4. $\frac{7}{15}$ and $\frac{13}{45}$.
5. $\frac{5}{14}$ and $\frac{7}{10}$.
6. $\frac{11}{16}$, $\frac{1}{2}$, and $\frac{13}{20}$.
7. $\frac{11}{15}$, $\frac{1}{3}$, and $\frac{9}{18}$.
8. $\frac{10}{21}$, $\frac{1}{4}$, and $\frac{23}{28}$.
9. $\frac{8}{15}$, $\frac{2}{3}$, and $\frac{32}{45}$.
10. $\frac{9}{14}$, $\frac{1}{4}$, and $\frac{3}{10}$.

(a) Find the sum of the ten sums.*

(For a continuation of this work, see page 231.)

* This may be omitted until the subject of fractions is reviewed.

Algebraic Fractions.

180. The expressions $\frac{a}{b}$, $\frac{x}{4}$, $\frac{6}{cd}$, are algebraic fractions.

The above expressions are read, a divided by b, x divided by 4, 6 divided by cd.

181. Reduce to lowest terms:

1. $\frac{ab}{a^3} = \left(\frac{a \times b}{a \times a \times a}\right)$. $\frac{ab \div a}{a^3 \div a} = \frac{b}{a^2}$

2. $\frac{4a}{6b} = \left(\frac{2 \times 2 \times a}{2 \times 3 \times b}\right)$. $\frac{4a \div 2}{6b \div 2} = \frac{2a}{3b}$

3. $\frac{abc}{bcd} = \left(\frac{a \times b \times c}{b \times c \times d}\right)$. $\frac{abc \div bc}{bcd \div bc} = \frac{a}{d}$

Let $a = 2$, $b = 3$, $c = 5$, and $d = 7$, and verify.

Observe that to reduce a fraction to its lowest terms we have only to strike out the factors that are common to its numerator and denominator.

4. $\frac{a^2b}{a^3c}$. What factors are common to both numerator and denominator? Reduce and verify.

5. $\frac{x^2y^3}{y^2z^2}$. What factors are common to both numerator and denominator? Reduce and verify.

6. $\frac{a^2b^3}{a^5b^5} =$ 7. $\frac{xy^2}{x^3y^5} =$ 8. $\frac{4a + 4b}{6c + 8d} =$

9. $\frac{abc}{ax} =$ 10. $\frac{2ax}{4a^2x} =$ 11. $\frac{3x + 6y}{12} =$

Algebraic Fractions.

182. Reduce to higher terms:

1. Change $\frac{2a}{bc}$ to a fraction whose denominator is abc.

$\frac{2a \times a}{bc \times a} = \frac{2a^2}{abc}$ Let $a = 2$, $b = 3$, and $c = 5$, and verify the reduction.

2. Change $\frac{3x}{2ay}$ to a fraction whose denominator is $2ay^2$.

$\frac{3x \times y}{2ay \times y} = \frac{3xy}{2ay^2}$ Give any values you please to a, x, and y, and verify the reduction.

183. Reduce to equivalent fractions having a common denominator:

1. $\frac{x}{ab}$ and $\frac{y}{a^2d}$ Since the common denominator must be exactly divisible by each of the given denominators, it must contain all the prime factors* found in either of the given denominators. The new denominator must therefore be $a \times a \times b \times d = a^2bd$; $a^2bd \div ab = ad$; $a^2bd \div a^2d = b$.

$$\frac{x}{ab} \times \frac{ad}{ad} = \frac{adx}{a^2bd} \qquad \frac{y}{a^2d} \times \frac{b}{b} = \frac{by}{a^2bd}$$

Give any values you please to a, b, d, x, and y, and verify.

2. $\frac{4}{ab^2}$ and $\frac{3}{bc^2}$ The common denominator must contain the factors a, b, b, c, c. Reduce and verify.

3. $\frac{xy}{5}$ and $\frac{yz}{a}$ The common denominator is $5a$. Reduce and verify.

* Since the numerical values of the letters are unknown, each must be regarded as prime to all the others. The prime factors, then, in the first denominator are a and b; in the second, a, a, and d.

Geometry.

184. QUADRILATERALS.

1. All the geometrical figures on this page are quadrilaterals; that is, *each has four sides.*

2. The first four figures are parallelograms; that is, *the opposite sides of each figure are parallel.*

3. The first two figures are rectangular; that is, *their angles are right angles.*

4. The first and third are equilateral; that is, *the sides are equal.*

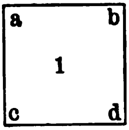
5. There is one equilateral rectangular parallelogram. Which is it?

6. There is one equilateral parallelogram that is not rectangular. Which is it?

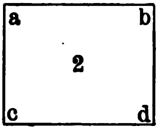
7. There is one rectangular parallelogram that is not equilateral. Which is it?

8. The sum of the angles of each figure on the page is equal to —— right angles.

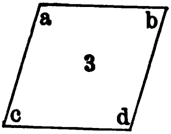
9. Tell as nearly as you can the size of each angle of each figure.



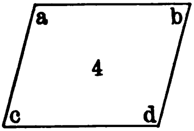
Square.



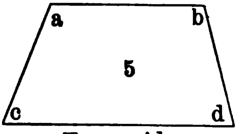
Oblong.



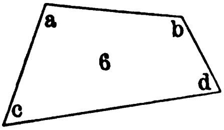
Rhombus.



Rhomboid.



Trapezoid.



Trapezium.

185. Miscellaneous Review.

1. The difference of two numbers is $374\frac{11}{40}$; the smaller number is $243\frac{17}{30}$. What is the larger number?

2. The difference of two numbers is a; the smaller number is b. What is the larger number?

3. James had a certain number of dollars and John had three times as many; together they had 196 dollars. How many had each? ($x + 3x = 196$.)

4. William had a certain number of marbles; Henry had twice as many as William, and George had twice as many as Henry; together they had 161. How many had each? ($x + 2x + 4x = 161$.)

5. Divide 140 dollars between two men, giving to one man 30 dollars more than to the other. ($x + x + 30 = 140$.)

6. By what integral numbers is 30 ($2 \times 3 \times 5$) exactly divisible besides itself and 1?

7. By what is abc ($a \times b \times c$) exactly divisible besides itself and 1?

(1) How many times is a contained in abc?
(2) How many times is b contained in abc?
(3) How many times is c contained in abc?
(4) How many times is ab contained in abc?
(5) How many times is ac contained in abc?
(6) How many times is bc contained in abc?

Observe that a number composed of three different prime factors has —— exact integral divisors.

8. Change $\frac{5}{8}$ to 60ths. Is $\frac{5}{8}$ more or less than $\frac{37}{60}$?

9. Change $\frac{3}{8}$ to 100ths. Change $\frac{2}{3}$ to 100ths.

10. Change $\frac{1}{6}$ to 100ths. Change $\frac{5}{6}$ to 100ths.

FRACTIONS.

(Continued from page 226.)

186. To Subtract Common Fractions.

Rule.—*Reduce the fractions if necessary to equivalent fractions having a common denominator, find the difference of their numerators, and write it over the common denominator.*

Example.

From $\frac{11}{25}$ subtract $\frac{7}{35}$.

(1) The l. c. m. of 25 and 35 is 175.

(2) $\frac{11}{25} = \frac{77}{175}$. $\quad \frac{7}{35} = \frac{35}{175}$.

(3) $\frac{77}{175} - \frac{35}{175} = \frac{42}{175}$

Compare the following:

77 175ths − 35 175ths = 42 175ths.
77 apples − 35 apples = 42 apples.

Find the difference of—

1. $\frac{3}{5}$ and $\frac{5}{18}$.
2. $\frac{2}{5}$ and $\frac{4}{18}$.
3. $\frac{9}{20}$ and $\frac{1}{3}$.
4. $\frac{11}{20}$ and $\frac{1}{6}$.
5. $\frac{5}{9}$ and $\frac{1}{2}$.
6. $\frac{2}{3}$ and $\frac{1}{4}$.
7. $\frac{1}{3}$ and $\frac{1}{4}$.
8. $\frac{4}{5}$ and $\frac{1}{3}$.
9. $\frac{1}{5}$ and $\frac{1}{6}$.
10. $\frac{4}{9}$ and $\frac{1}{2}$.
11. $\frac{3}{4}$ and $\frac{7}{20}$.
12. $\frac{3}{5}$ and $\frac{3}{20}$.
13. $\frac{2}{5}$ and $\frac{9}{20}$.
14. $\frac{1}{4}$ and $\frac{1}{20}$.
15. $\frac{7}{9}$ and $\frac{3}{10}$.
16. $\frac{2}{9}$ and $\frac{1}{5}$.
17. $\frac{5}{18}$ and $\frac{1}{3}$.
18. $\frac{13}{18}$ and $\frac{1}{6}$.

(a) Find the sum of the eighteen differences.

Fractions.

187. TO SUBTRACT ONE MIXED NUMBER FROM ANOTHER WHEN THE FRACTION IN THE SUBTRAHEND IS GREATER THAN THE FRACTION IN THE MINUEND.

EXAMPLE.

From $58\frac{3}{8}$ take $32\frac{2}{3}$.

Operation.

$$58\tfrac{3}{8} = 58\tfrac{9}{24}$$
$$32\tfrac{2}{3} = 32\tfrac{16}{24}$$
$$\text{Difference } 25\tfrac{17}{24}$$

Explanation.

$\frac{16}{24}$ is greater than $\frac{9}{24}$, therefore we take 1 unit from the 8 units, change it to 24ths, and add it to the 9 24ths.

$\frac{24}{24} + \frac{9}{24} = \frac{33}{24}$. $\frac{33}{24} - \frac{16}{24} = \frac{17}{24}$.

2 units from 7 (8 − 1) units = 5 units. 3 tens from 5 tens = 2 tens.

I. Find the difference of—

1. $24\frac{2}{7}$ and $16\frac{2}{5}$.
2. $29\frac{3}{8}$ and $15\frac{1}{4}$.
3. $46\frac{19}{56}$ and $18\frac{7}{20}$.
4. $52\frac{5}{8}$ and $31\frac{3}{5}$.
5. $47\frac{3}{8}$ and $18\frac{2}{5}$.
6. $35\frac{1}{4}$ and $26\frac{1}{2}$.
7. $28\frac{3}{7}$ and $14\frac{3}{5}$.
8. $36\frac{9}{28}$ and $8\frac{9}{10}$.
9. $65\frac{1}{2}$ and $22\frac{3}{4}$.
10. $34\frac{1}{2}$ and $27\frac{1}{4}$.

(a) Find the sum of the ten differences.

II. Reduce to simplest form—

1. $5\frac{1}{2} + 3\frac{1}{2} - 5\frac{1}{4}$.
2. $6\frac{2}{5} - 3\frac{1}{8} + 4\frac{1}{5}$.
3. $2\frac{1}{5} - 1\frac{1}{4} + 3\frac{2}{5}$.
4. $7\frac{3}{4} + 3\frac{3}{8} - 1\frac{3}{5}$.
5. $6\frac{1}{2} - 3\frac{1}{4} + 5\frac{1}{5}$.
6. $3\frac{1}{4} + 4\frac{1}{2} - 3\frac{1}{8} - 2\frac{3}{8} + 1\frac{3}{4}$.
7. $6\frac{3}{5} - 2\frac{1}{4} - 1\frac{3}{4} - 1\frac{21}{40} + 2\frac{1}{4}$.
8. $5\frac{1}{2} + 4\frac{3}{8} + 2\frac{1}{8} + 3\frac{7}{8} + 3\frac{1}{4}$.

(b) Find the sum of the eight results.

Fractions.

188. To Multiply a Fraction by an Integer.

Multiply $\frac{7}{24}$ by 6.

Operation No. 1.

6 times $\frac{7}{24}$ are $\frac{42}{24} = 1\frac{3}{4}$.

Operation No. 2.

6 times $\frac{7}{24} = \frac{7}{4} = 1\frac{3}{4}$.

1. *Observe* that by the first operation we obtain $\frac{42}{24}$; that in $\frac{42}{24}$ there are 6 times as many parts as there are in $\frac{7}{24}$, and that the parts are of the same size as those in $\frac{7}{24}$.

2. *Observe* that by the second operation we obtain $\frac{7}{4}$; that in $\frac{7}{4}$ there are the same number of parts as there are in $\frac{7}{24}$, and that the parts are 6 times as great as those in $\frac{7}{24}$.

Note.—The 7 of $\frac{7}{24}$ may be regarded as a dividend; the 24, as a divisor, and $\frac{7}{24}$ itself as a quotient. In $\frac{42}{24}$, we have a dividend 6 times as great as that in $\frac{7}{24}$, the divisor remaining unchanged. In $\frac{7}{4}$ we have a divisor 1 sixth as great as that in $\frac{7}{24}$, the dividend remaining unchanged. Multiplying the dividend or dividing the divisor by any number multiplies the quotient by the same number.

Rule.—*To multiply a fraction by an integer, multiply its numerator or divide its denominator by the integer.*

I. Find the product.

1. $\frac{5}{12} \times 4$.	5. $\frac{5}{4} \times 8$.	9. $\frac{7}{12} \times 4$.
2. $\frac{8}{15} \times 6$.	6. $\frac{3}{5} \times 9$.	10. $\frac{7}{15} \times 6$.
3. $\frac{11}{20} \times 5$.	7. $\frac{3}{4} \times 8$.	11. $\frac{9}{20} \times 5$.
4. $\frac{17}{30} \times 7$.	8. $\frac{2}{5} \times 9$.	12. $\frac{13}{30} \times 7$.

(a) Find the sum of the twelve products.

II. Find the product.

1. $3\frac{1}{2} \times 7$.	5. $4\frac{2}{5} \times 5$.	9. 6.3×5.
2. $5\frac{3}{8} \times 6$.	6. $1\frac{7}{8} \times 4$.	10. $2\frac{13}{20} \times 4$.
3. $7\frac{7}{20} \times 4$.	7. $5\frac{3}{5} \times 5$.	11. $4\frac{5}{8} \times 6$.
4. 3.7×5.	8. $8\frac{1}{8} \times 4$.	12. $6\frac{1}{2} \times 7$.

(b) Find the sum of the twelve products.

Fractions.

189. TO DIVIDE A FRACTION BY AN INTEGER.

Divide $\frac{6}{7}$ by 3.

Operation No. 1.
One third of $\frac{6}{7} = \frac{2}{7}$.

Operation No. 2.
One third of $\frac{1}{7} = \frac{1}{21}$.
One third of $\frac{6}{7} = \frac{6}{21} = \frac{2}{7}$.

1. *Observe* that by the first operation we obtain $\frac{2}{7}$; that in $\frac{2}{7}$ there are 1 third as many parts as there are in $\frac{6}{7}$, and that the parts are of the same size as those in $\frac{6}{7}$.

2. *Observe* that by the second operation we obtain $\frac{6}{21}$; that in $\frac{6}{21}$ there are the same number of parts as there are in $\frac{6}{7}$, and that the parts are 1 third as great as those in $\frac{6}{7}$.

NOTE 1.—The 6 of $\frac{6}{7}$ may be regarded as a dividend; the 7 as a divisor, and the $\frac{6}{7}$ itself as a quotient. In $\frac{2}{7}$ we have a dividend 1 third as great as that in $\frac{6}{7}$, the divisor remaining unchanged. In $\frac{6}{21}$ we have a divisor 3 times as great as that in $\frac{6}{7}$, the dividend remaining unchanged. Dividing the dividend or multiplying the divisor by any number divides the quotient by the same number.

RULE.—*To divide a fraction by an integer, divide its numerator or multiply its denominator by the integer.*

I. Find the quotient. (See p. 245, problems 15 and 16)

1. $\frac{7}{12} \div 4$.
2. $\frac{7}{12} \div 5$.
3. $\frac{7}{12} \div 20$.
4. $\frac{5}{6} \div 4$.
5. $\frac{5}{6} \div 5$.
6. $\frac{5}{6} \div 20$.
7. $\frac{17}{24} \div 4$.
8. $\frac{17}{24} \div 5$.
9. $\frac{17}{24} \div 20$.

(a) Find the sum of the nine quotients.

II. Find the quotient. (See p. 245, problems 17 and 18.)

1. $17\frac{1}{5} \div 3$.
2. $17\frac{1}{5} \div 4$.
3. $17\frac{1}{5} \div 6$.
4. $18\frac{3}{10} \div 3$.
5. $18\frac{3}{10} \div 4$.
6. $18\frac{3}{10} \div 6$.
7. $16\frac{1}{2} \div 3$.
8. $16\frac{1}{2} \div 4$.
9. $16\frac{1}{2} \div 6$.

(b) Find the sum of the nine quotients.

Fractions.

190. TO MULTIPLY BY A FRACTION.

\$6 multiplied by 3, means, take 3 times \$6. $\$6 \times 3 = \$18$.

\$6 multiplied by 2, means, take 2 times \$6. $\$6 \times 2 = \$12$.

\$6 multiplied by $2\frac{1}{2}$, means, take $2\frac{1}{2}$ times \$6; or 2 times \$6 $+ \frac{1}{2}$ of \$6. $\$6 \times 2\frac{1}{2} = \$15$.

\$6 multiplied by $\frac{1}{2}$, means, take $\frac{1}{2}$ of \$6. $\$6 \times \frac{1}{2} = \$3$.

\$6 multiplied by $\frac{2}{3}$, means, take $\frac{2}{3}$ of \$6. $\$6 \times \frac{2}{3} = \$4$.

TO THE TEACHER.—Require the pupil to examine the preceding statements until he clearly understands that to multiply by a fraction is to take such part of the multiplicand as is indicated by the fraction. Thus: to multiply 48 by $\frac{3}{4}$ is to take three fourths of 48; that is, *three times 1 fourth of 48.* It will thus be clear that multiplication by a fraction involves both multiplication and division.

EXAMPLE I.

Multiply 24 by $\frac{3}{4}$.
1 fourth of 24 is 6.
3 fourths of 24 are 18.

EXAMPLE II.

Multiply $\frac{3}{5}$ by $\frac{3}{4}$.
1 fourth of $\frac{3}{5}$ is $\frac{3}{20}$.
3 fourths of $\frac{3}{5}$ are $\frac{9}{20}$.

EXAMPLE III.

Multiply $275\frac{3}{5}$ by $\frac{3}{4}$.
1 fourth of $275\frac{3}{5}$ is $68\frac{9}{10}$.
3 fourths of $275\frac{3}{5}$ are $206\frac{7}{10}$.

EXAMPLE IV.

Multiply $346\frac{2}{5}$ by $2\frac{1}{2}$.
Two times $346\frac{2}{5} = 692\frac{4}{5}$.
1 half of $346\frac{2}{5} = 173\frac{1}{5}$.
$692\frac{4}{5} + 173\frac{1}{5} = 866$ Ans.

RULE.—*To multiply by a fraction, divide the multiplicand by the denominator of the fraction and multiply the quotient thus obtained by the numerator of the fraction.*

Observe that in practice we may, if more convenient, multiply the multiplicand by the numerator of the fraction, and divide the product thus obtained by the denominator. To multiply 12 by $\frac{3}{4}$ we may take 3 times 1 fourth of 12 or 1 fourth of 3 times 12, as we choose.

Fractions.

I. Find the product. (See p. 245, problems 19 and 20.)

1. $345 \times \frac{1}{2}$.
2. $345 \times \frac{3}{10}$.*
3. $345 \times \frac{1}{5}$.
4. $263 \times \frac{1}{3}$.
5. $263 \times \frac{3}{4}$.
6. $263 \times \frac{2}{3}$.
7. $263 \times \frac{1}{4}$.
8. $576 \times \frac{3}{8}$.
9. $576 \times \frac{3}{4}$.

(a) Find the sum of the nine products.

II. Find the product. (See p. 245, problems 21 and 22.)

1. $\frac{7}{16} \times \frac{1}{2}$.
2. $\frac{7}{16} \times \frac{3}{10}$.†
3. $\frac{7}{16} \times \frac{1}{5}$.
4. $\frac{5}{8} \times \frac{1}{3}$.
5. $\frac{5}{8} \times \frac{3}{4}$.
6. $\frac{5}{8} \times \frac{2}{3}$.
7. $\frac{5}{8} \times \frac{1}{4}$.
8. $\frac{1}{2} \times \frac{3}{8}$.
9. $\frac{1}{2} \times \frac{3}{4}$.

(b) Find the sum of the nine products.

III. Find the product. (See p. 246, problems 23 and 24.)

1. $372\frac{1}{4} \times \frac{1}{2}$.
2. $372\frac{1}{4} \times \frac{3}{10}$.
3. $372\frac{1}{4} \times \frac{1}{5}$.
4. $523\frac{2}{3} \times \frac{1}{3}$.
5. $523\frac{2}{3} \times \frac{3}{4}$.
6. $523\frac{2}{3} \times \frac{2}{3}$.
7. $523\frac{2}{3} \times \frac{1}{4}$.
8. $153\frac{1}{2} \times \frac{3}{8}$.
9. $153\frac{1}{2} \times \frac{3}{4}$.

(c) Find the sum of the nine products.

IV. Find the product. (See p. 246, problems 25 and 26.)

1. $462\frac{2}{3} \times 2\frac{1}{10}$.
2. $462\frac{2}{3} \times 3\frac{7}{10}$.
3. $462\frac{2}{3} \times 2\frac{1}{5}$.
4. $346\frac{1}{6} \times 2\frac{1}{3}$.
5. $346\frac{1}{6} \times 3\frac{3}{4}$.
6. $346\frac{1}{6} \times 3\frac{2}{3}$.
7. $346\frac{1}{6} \times 2\frac{1}{4}$.
8. $275\frac{1}{2} \times 4\frac{1}{2}$.
9. $275\frac{1}{2} \times 3\frac{1}{4}$.
10. $275\frac{1}{2} \times 2\frac{1}{4}$.

(d) Find the sum of the ten products.

* Take 3 times 1 tenth of 345, or 1 tenth of 3 times 345.

† Lead the pupil to see that in problems of this kind the correct result may be obtained by "*multiplying the numerators together for a new numerator and the denominators together for a new denominator*"; that in so doing he divides the multiplicand by the denominator of the multiplier and multiplies the quotient so obtained by the numerator of the multiplier.

Algebraic Fractions.

$\frac{a}{b}+\frac{c}{d}$. l. c. d. $= bd$	$\frac{2}{3}+\frac{5}{7}$. l. c. d. $= 21$
$bd \div b = d$ $\quad \frac{a\times d}{b\times d}=\frac{ad}{bd}$	$21 \div 3 = 7$. $\quad \frac{2\times 7=14}{3\times 7=21}$
$bd \div d = b$ $\quad \frac{c\times b}{d\times b}=\frac{bc}{bd}$	$21 \div 7 = 3$. $\quad \frac{5\times 3}{7\times 3}=\frac{15}{21}$
$\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$	$\frac{14}{21}+\frac{15}{21}=\frac{29}{21}$

Observe that in cases like the above, in which the denominators are prime to each other, the l. c. d. is the product of the given denominators, and each new numerator may be found by multiplying the given numerator by the denominator of the other fraction.

191. PROBLEMS IN ADDITION AND SUBTRACTION.

1. $\frac{a}{x}+\frac{b}{y}=\frac{ay}{xy}+\frac{bx}{xy}=\frac{ay+bx}{xy}$

Let $a = 2$, $b = 3$, $x = 5$, $y = 7$, and verify.

2. $\frac{a}{x}-\frac{b}{y}=\frac{ay}{xy}-\frac{bx}{xy}=\frac{ay-bx}{xy}$

Assign a numerical value to each letter and verify.

3. $\frac{2}{x}-\frac{1}{y}$

4. $\frac{3}{x}-\frac{2}{y}$

SOLVE.—Then let $x = 5$ and $y = 7$ and verify.

Algebraic Fractions.

192. PROBLEMS IN MULTIPLICATION AND DIVISION.

EXAMPLE I.

$$\frac{a}{b} \times c = \frac{ac}{b} \qquad \Big| \qquad \frac{2}{5} \times 3 = \frac{6}{5} = 1\tfrac{1}{5}$$

Let $a = 2$, $b = 5$, and $c = 3$; then $\dfrac{ac}{b} = \dfrac{2 \times 3}{5} = \dfrac{6}{5} = 1\tfrac{1}{5}$.

EXAMPLE II.

$$\frac{a}{b} \div c = \frac{a}{bc} \qquad \Big| \qquad \frac{2}{5} \div 3 = \frac{2}{15}$$

Let $a = 2$, $b = 5$, and $c = 3$: then $\dfrac{a}{bc} = \dfrac{2}{5 \times 3} = \dfrac{2}{15}$

EXAMPLE III.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad \Big| \qquad \frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$$

Let $a = 2$, $b = 5$, $c = 3$, and $d = 7$; then $\dfrac{ac}{bd} = \dfrac{2 \times 3}{5 \times 7} = \dfrac{6}{35}$

I. Find the product and verify as above.

1. $\dfrac{a^2}{b^3} \times c$ 3. $\dfrac{x}{y^2} \times y$ 5. $\dfrac{ax}{bx} \times 3a$

2. $\dfrac{x}{y^3} \times x$ 4. $\dfrac{ab}{cd} \times a$ 6. $\dfrac{x}{7} \times 5$

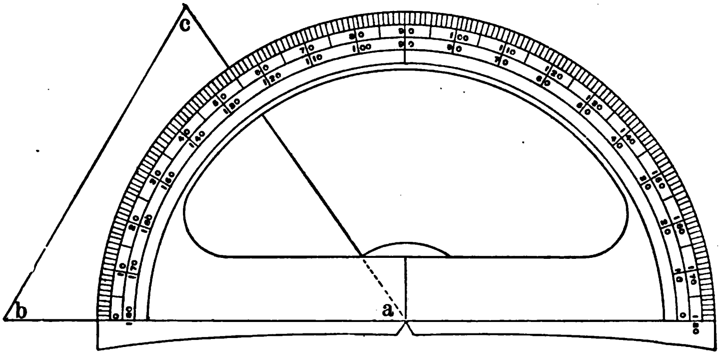
II. Find the quotient and verify as above.

1. $\dfrac{a^2}{b^3} \div c$ 3. $\dfrac{x}{y^2} \div y$ 5. $\dfrac{ax}{bx} \div 3a$

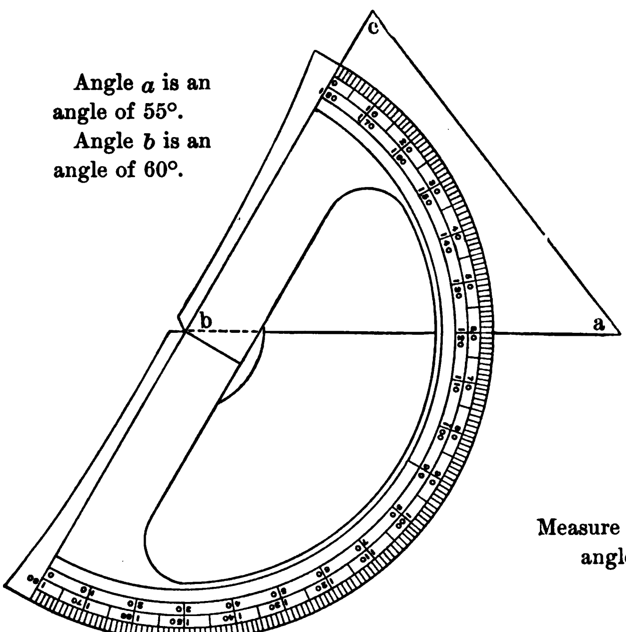
2. $\dfrac{x}{y^3} \div x$ 4. $\dfrac{ab}{cd} \div a$ 6. $\dfrac{x}{7} \div 5$

Geometry.

193. The Protractor.



Angle *a* is an angle of 55°.
Angle *b* is an angle of 60°.



Measure angle *c*.

194. Miscellaneous Review.

1. A piece of land in the form of an equilateral triangle measures on one side $46\frac{3}{11}$ rods. What is the distance around it?

2. The perimeter of a piece of land that is an exact square is $246\frac{3}{4}$ feet. How far across on one side?

3. The length of a certain rectangular field is three times its breadth; its perimeter is 360 rd. What is its breadth? Its length?

4. If $\frac{3}{7}$ of the value of a farm is $2154, what is $\frac{7}{8}$ of the value of the farm?

NOTE.—If $\frac{3}{7}$ of a certain number is 24, what is the number? What is $\frac{7}{8}$ of the number?

5. I spent $\frac{3}{5}$ of my money and had $3.60 remaining. (a) How much did I spend? (b) What I had remaining, equals what part of what I spent?

6. Change $\frac{5}{12}$ to an equivalent fraction whose denominator is 30.

7. Change $\frac{6}{7}$ to an equivalent fraction whose numerator is 30.

8. Change $\dfrac{a^2}{b}$ to an equivalent fraction whose denominator is bc.

9. Multiply $\frac{5}{9}$ by $\frac{3}{8}$ and multiply the product by 25.

10. Multiply $\frac{37}{40}$ by $\frac{13}{20}$ and multiply the product by 25.

11. If $\frac{3}{4}$ of an acre of land is worth $36, how much are $37\frac{1}{2}$ acres worth at the same rate?

12. The rent of a house for 2 yr. 4 mo. was $840. What was the rate per year?

195. To Divide by a Fraction.

Example I.

Divide 6 by $\frac{2}{3}$.

Operation No. 1.

$6 = \frac{18}{3}$.

$\frac{18}{3} \div \frac{2}{3} = 9$.*

Operation No. 2.

$1 \div \frac{2}{3} = \frac{3}{2}$.†

$6 \div \frac{2}{3} = 6$ times $\frac{3}{2} = \frac{18}{2} = 9$.

Example II.

Divide $\frac{7}{8}$ by $\frac{3}{5}$.

Operation No. 1.

$\frac{7}{8} = \frac{35}{40}$. $\quad \frac{3}{5} = \frac{24}{40}$.

$\frac{35}{40} \div \frac{24}{40} = 1\frac{11}{24}$.‡

Operation No. 2.

$1 \div \frac{3}{5} = \frac{5}{3}$.§

$\frac{7}{8} \div \frac{3}{5} = \frac{7}{8}$ of $\frac{5}{3} = \frac{35}{24} = 1\frac{11}{24}$.

From the foregoing operations the following rules for dividing by a fraction are obtained:

Rule I.—*Reduce the dividend and the divisor to like fractional units, then divide the numerator of the dividend by the numerator of the divisor.*

Rule II.—"*Invert the divisor and proceed as in multiplication.*"

Observe that the inverted divisor shows the number of times the divisor is contained in 1: then in 6 it is contained 6 times as many times; in 4, 4 times as many; in $\frac{7}{8}$, $\frac{7}{8}$ as many; in $\frac{3}{4}$, $\frac{3}{4}$ as many, etc.

* 18 thirds ÷ 2 thirds = 9.

† $1 = \frac{3}{3}$. 3 thirds ÷ 2 thirds = $1\frac{1}{2} = \frac{3}{2}$.

‡ 35 fortieths ÷ 24 fortieths = $1\frac{11}{24}$.

§ $1 = \frac{5}{5}$. 5 fifths ÷ 3 fifths = $1\frac{2}{3} = \frac{5}{3}$.

Fractions.

I. Find the quotient. (See page 246, problems 27 and 28.)

1. $46 \div \frac{1}{2}$.	4. $375 \div \frac{1}{2}$.	7. $196 \div \frac{1}{2}$.
2. $46 \div \frac{3}{4}$.	5. $375 \div \frac{3}{4}$.	8. $196 \div \frac{3}{4}$.
3. $46 \div \frac{3}{5}$.	6. $375 \div \frac{3}{5}$.	9. $196 \div \frac{3}{5}$.

(a) Find the sum of the nine quotients.

II. Find the quotient. (See page 246, problems 29 and 30.)

1. $\frac{7}{8} \div \frac{2}{3}$.	4. $\frac{5}{12} \div \frac{2}{3}$.	7. $\frac{11}{24} \div \frac{2}{3}$.
2. $\frac{7}{8} \div \frac{5}{6}$.	5. $\frac{5}{12} \div \frac{5}{6}$.	8. $\frac{11}{24} \div \frac{5}{6}$.
3. $\frac{7}{8} \div \frac{10}{13}$.	6. $\frac{5}{12} \div \frac{10}{13}$.	9. $\frac{11}{24} \div \frac{10}{13}$.

(b) Find the sum of the nine quotients.

III. Find the quotient. (See page 246, problems 31 and 32.)

1. $5\frac{1}{2} \div \frac{3}{4}$.	4. $24\frac{2}{3} \div \frac{3}{4}$.	7. $19\frac{5}{6} \div \frac{3}{4}$.
2. $5\frac{1}{2} \div \frac{6}{7}$.	5. $24\frac{2}{3} \div \frac{6}{7}$.	8. $19\frac{5}{6} \div \frac{6}{7}$.
3. $5\frac{1}{2} \div \frac{2}{5}$.	6. $24\frac{2}{3} \div \frac{2}{5}$.	9. $19\frac{5}{6} \div \frac{2}{5}$.

(c) Find the sum of the nine quotients.

IV. Find the quotient. (See page 246, problem 33.)

1. $325\frac{1}{2} \div 2\frac{1}{2}$.	6. $174\frac{1}{2} \div 2\frac{1}{2}$.
2. $325\frac{1}{2} \div 2\frac{2}{5}$.	7. $174\frac{1}{2} \div 2\frac{2}{5}$.
3. $325\frac{1}{2} \div 3\frac{1}{3}$.	8. $174\frac{1}{2} \div 3\frac{1}{3}$.
4. $325\frac{1}{2} \div 7\frac{1}{2}$.	9. $174\frac{1}{2} \div 7\frac{1}{2}$.
5. $325\frac{1}{2} \div 1\frac{1}{3}$.	10. $174\frac{1}{2} \div 1\frac{1}{3}$.

(d) Find the sum of the ten quotients.

Fractions.

196. To Reduce Complex Fractions to Simple Fractions.

$\frac{\frac{3}{4}}{\frac{7}{8}}$ is read, $\frac{3}{4}$ divided by $\frac{7}{8}$. $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4}$ of $\frac{8}{7} = \frac{24}{28} = \frac{6}{7}$.

I. Reduce to their simplest forms.

1. $\frac{\frac{5}{6}}{\frac{1}{2}}$ 2. $\frac{\frac{5}{6}}{\frac{3}{4}}$ 3. $\frac{\frac{5}{6}}{\frac{3}{5}}$ 4. $\frac{\frac{5}{6}}{\frac{1}{5}}$

(a) Find the sum of the four fractions.

Observe that a complex fraction may be reduced to a simple fraction by multiplying its numerator and denominator by some number that will in each case give an integral product. When this number can be easily discovered by inspection this is a convenient method of reduction: thus $\frac{\frac{1}{2}}{7} = \frac{\frac{1}{2} \times 2}{7 \times 2} = \frac{1}{14}$.

II. Reduce to their simplest forms. (See page 246, problem 34.)

1. $\frac{\frac{5}{6}}{5}$ 2. $\frac{\frac{1}{3}}{4}$ 3. $\frac{5}{\frac{1}{2}}$ 4. $\frac{1\frac{1}{2}}{2}$

(b) Find the sum of the four fractions.

III. Reduce to their simplest forms.

1. $\frac{8}{\frac{1}{2}}$ 2. $\frac{8}{\frac{3}{4}}$ 3. $\frac{8}{\frac{3}{5}}$ 4. $\frac{8}{\frac{1}{5}}$

(c) Find the sum of the four fractions.

IV. Reduce to their simplest form.

1. $\frac{12\frac{1}{2}}{100}$* 2. $\frac{16\frac{2}{3}}{100}$ 3. $\frac{66\frac{2}{3}}{100}$ 4. $\frac{87\frac{1}{2}}{100}$

* Divide the numerator and denominator by $12\frac{1}{2}$.

Fractions.

197. PRACTICAL APPLICATION OF THE PRECEDING RULES.

Page 224, problems 1 and 2.

1. B owned a farm of 375 acres; he gave to his son 275 acres. What part of the farm did the son receive?

2. Mr. L. earned $650 in one year; of this sum he expended $520. What part of his earnings did he expend?

Page 225, problems 1 and 2.

3. A lady owned $\frac{5}{8}$ of an acre of land. How many 160ths of an acre did she own?

4. Benton walked $\frac{11}{16}$ of a mile. Express the distance he walked in 160ths of a mile.

Page 226, Art. 179, problems 1 and 6.

5. In a certain furnace $\frac{5}{16}$ of a ton of coal was consumed in one day, and $\frac{7}{20}$ of a ton the next day. What part of a ton was consumed in the two days?

6. Mr. Luker has three lots of land; in the first lot there are $\frac{11}{16}$ of an acre; in the second, $\frac{1}{2}$ of an acre, and in the third, $\frac{13}{20}$ of an acre. How many acres in all?

Page 231, Art. 186, problems 3 and 4.

7. Of $\frac{9}{20}$ of a mile of board fence $\frac{1}{3}$ of a mile was burned. What part of a mile remained?

8. Mr. Reynolds had put into the bank $\frac{11}{20}$ of his annual salary; he drew from this money a sum equal to $\frac{1}{6}$ of his salary. What part of his salary remained in the bank?

Page 232, Art. 187, I., problems 4 and 5.

9. From $52\frac{5}{8}$ tons of hay, were sold and delivered $31\frac{3}{8}$ tons. How many tons remained of the unsold hay?

10. On Monday James rode $47\frac{3}{8}$ mi.; on Tuesday, $18\frac{2}{8}$ mi. How much further did he ride Monday than Tuesday?

Page 233, Art. 188, I., problems 1 and 5.

11. If a street car makes a round trip in $\frac{5}{12}$ of an hour, in how long a time can it make 4 such trips?

12. If $\frac{5}{4}$ yd. of ribbon are required to trim a hat, how much ribbon will be required to trim 8 such hats?

Page 233, Art. 188, II., problems 1 and 5.

13. At $3\frac{1}{2}$ dollars a cord, what is the cost of 7 cords of wood?

14. If Henry rides his wheel at the rate of $4\frac{2}{5}$ miles an hour, how far does he ride in 5 hours?

Page 234, Art. 189, I., problems 1 and 4.

15. If $\frac{7}{12}$ of a yd. of ribbon is cut into 4 equal pieces, what part of a yard is each piece?

16. John hoes 4 rows of corn in $\frac{5}{6}$ of an hour. In what part of an hour does he hoe 1 row?

Page 234, Art. 189, II., problems 1 and 5.

17. A horse traveled $17\frac{1}{5}$ miles in 3 hours. What was his rate per hour?

18. A farmer divided a field containing $18\frac{3}{10}$ acres into 4 equal lots. How many acres in each lot?

Page 236, I., problems 1 and 5.

19. At $345 an acre, what is the cost of $\frac{1}{2}$ acre?

20. At $263 an acre, what is the cost of $\frac{3}{4}$ acre?

Page 236, II., problems 1 and 9.

21. A piece of land in the form of a rectangle is $\frac{7}{16}$ of a mile long and $\frac{1}{2}$ of a mile wide. The piece is what part of a square mile?

22. At $\frac{1}{2}$ a dollar per yard, what is the cost of $\frac{3}{4}$ of a yard of silk?

Page 236, III., problems 1 and 9.

23. A strip of land $372\frac{1}{4}$ rods long and $\frac{1}{2}$ a rod wide contains how many square rods?

24. At $\$153\frac{1}{2}$ an acre, find the cost of $\frac{3}{4}$ of an acre.

Page 236, IV., problems 1 and 8.

25. At $\$462\frac{2}{3}$ a mile, what is the cost of grading $2\frac{1}{10}$ miles of road?

26. How many square feet in a piece of land $275\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft.?

Page 242, I., problems 1 and 5.

27. At $\frac{1}{2}$ a dollar a bushel, how many bushels of potatoes can be bought for 46 dollars?

28. At $\frac{3}{4}$ of a dollar a bushel, how many bushels of apples can be bought for 375 dollars?

Page 242, II., problems 1 and 5.

29. At $\frac{2}{3}$ of a dollar a yard, how many yards of cloth can be bought for $\frac{7}{8}$ of a dollar?

30. At $\frac{5}{6}$ of a dollar a yard, what part of a yard of cloth can be bought for $\frac{5}{12}$ of a dollar?

Page 242, III., problems 1 and 6.

31. If a rectangular diagram on the blackboard contains $5\frac{1}{2}$ square feet and is $\frac{3}{4}$ of a foot wide, how long is it? Make the diagram.

32. At $\$\frac{4}{5}$ a bushel, how many bushels of meal can be bought for $\$24\frac{2}{3}$?

Page 242, IV., problem 1.

33. A strip of land contains $325\frac{1}{2}$ square rods, and is $2\frac{1}{2}$ rods wide. How long is it?

Page 243, II., problem 1.

34. Five-sixths of an hour is what part of 5 hours?

Algebraic Fractions.

198. Problems in Division with a Fraction for a Divisor.

Example I.

See page 241, Rule II, and Observation.

$$a \div \frac{b}{c} = \frac{ac}{b} \qquad \Big| \qquad 7 \div \frac{2}{3} = \frac{21}{2} = 10\tfrac{1}{2}$$

Let $a = 7$, $b = 2$, and $c = 3$; then $\frac{ac}{b} = \frac{7 \times 3}{2} = \frac{21}{2} = 10\frac{1}{2}$

Example II.

See page 241, Rule II, and Observation.

$$\frac{a}{x} \div \frac{b}{c} = \frac{ac}{bx} \qquad \Big| \qquad \frac{7}{10} \div \frac{2}{3} = \frac{21}{20} = 1\tfrac{1}{20}$$

Let $a = 7$, $x = 10$, $b = 2$, and $c = 3$; then $\frac{ac}{bx} = \frac{7 \times 3}{2 \times 10} = \frac{21}{20} = 1\frac{1}{20}$

I. Find the quotient and verify as above.

1. $x \div \frac{a}{b}$
2. $y \div \frac{c}{d}$
3. $a \div \frac{b}{x}$
4. $b \div \frac{c}{y}$
5. $xy \div \frac{1}{a}$
6. $yz \div \frac{2}{a}$

II. Find the quotient and verify as above.

1. $\frac{a}{x} \div \frac{b}{y}$
2. $\frac{b}{x} \div \frac{c}{y}$
3. $\frac{c}{y} \div \frac{d}{z}$
4. $\frac{a}{z} \div \frac{b}{x}$
5. $\frac{b}{c} \div \frac{x}{y}$
6. $\frac{c}{d} \div \frac{x}{y}$

Algebraic Fractions.

199. Miscellaneous Exercises.

Example I.

$$a + \frac{b}{c} = \frac{ac + b}{c} \qquad \Big| \qquad 3\frac{2}{5} = \frac{17}{5} = 3\frac{2}{5}$$

Let $a = 3$, $b = 2$, and $c = 5$.

$$\text{Then } \frac{ac + b}{c} = \frac{(3 \times 5) + 2}{5} = \frac{17}{5} = 3\frac{2}{5}$$

I. Reduce to improper fractions and verify as above.

1. $x + \frac{a}{y}$ 2. $y + \frac{2}{x}$ 3. $b + \frac{x}{2}$

Example II.

$$\frac{ab + c}{b} = a + \frac{c}{b} \qquad \Big| \qquad \frac{11}{3} = 3\frac{2}{3}$$

Let $a = 2$, $b = 3$, and $c = 5$; then $a + \frac{c}{b} = 2 + \frac{5}{3} = 3\frac{2}{3}$

II. Reduce to mixed numbers and verify.

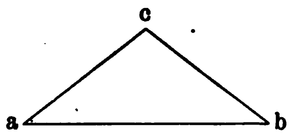
1. $\frac{ax + b}{x}$ 2. $\frac{by + c}{y}$ 3. $\frac{by + 2}{y}$

III. Reduce and verify.

1. $\dfrac{\frac{a}{x}}{\frac{b}{y}}$ 2. $\dfrac{\frac{2}{a}}{\frac{3}{b}}$ 3. $\dfrac{\frac{x}{b}}{\frac{y}{c}}$ 4. $\dfrac{\frac{x}{3}}{\frac{y}{2}}$

Geometry.

200. Construction Problems—Triangles.



1. Draw a triangle. Make the side *ab* 3 inches long. Make the angle *a*, 45°. Make the angle *b*, 45°. Prove your work by measuring the angle *c* which should be an angle of —— degrees.

Observe that if two angles of any triangle and the length of the included side are given, the triangle may be drawn.

2. Draw a triangle making one of the angles 40°, another 60°, and the included side 5 inches long. The third angle should measure —— degrees. Prove your work by measuring the third angle.

Measure the sides carefully and observe that the longest side is opposite the largest angle, and the shortest side opposite the smallest angle.

3. Draw several triangles of different shapes and sizes. Convince yourself by measurement with the protractor that *the sum of the three angles of any triangle is —— degrees.*

4. Draw several triangles of different shapes and sizes. Convince yourself by measurement with a ruler that *the sum of two sides of any triangle is greater than the third side of the same triangle.*

5. Attempt to draw a triangle whose sides are 6 inches, $3\frac{1}{2}$ inches, and $2\frac{1}{2}$ inches.

201. Miscellaneous Review.

1. If $\frac{3}{4}$ of a cord of wood cost \$4.50 how much will 27 cords cost at the same rate?

2. If $2\frac{1}{4}$ tons of coal cost \$12.60, how much will $17\frac{3}{5}$ tons cost at the same rate?

3. A man owned $7\frac{5}{10}$ acres of land; he sold $2\frac{3}{8}$ acres. What fractional part of his land did he sell?

4. The sum of two fractions is $1\frac{5}{13}$; one of the fractions is $\frac{5}{7}$. What is the other fraction?

5. The product of two fractions is $\frac{15}{63}$; one of the fractions is $\frac{3}{7}$. What is the other fraction?

6. If a furnace consumes $\frac{3}{16}$ of a ton of coal a day, in how many days will $5\frac{1}{4}$ tons be consumed?

7. How many pounds of sugar at 4¢ a pound must be given for $27\frac{1}{2}$ pounds of butter at 23¢ a pound?

8. How many pounds of coffee at $33\frac{1}{3}$¢ a pound must be given for $15\frac{1}{2}$ dozen eggs at 20¢ a dozen?

9. Which is the greater fraction, $\frac{7}{8}$ or $\frac{7}{9}$?

10. Multiplying both terms of a fraction by the same number does not change the value of the fraction. Does adding the same number to both terms of a fraction change the value?

11. Dividing both terms of a fraction by the same number does not change the value of the fraction. Does subtracting the same number from both terms of a fraction change the value?

12. Change $\dfrac{5}{11\frac{1}{2}}$ to a fraction whose denominator is 46.

13. Change $\dfrac{3}{7\frac{1}{2}}$ to a fraction whose denominator is 15.

FRACTIONS.

202. To Change Decimals to Common Fractions and Common Fractions to Decimals.

Example I.

Change .36 to a common fraction in its lowest terms.

$$.36 = \frac{36}{100} = \frac{9}{25}$$

Reduce to common fractions.

1. .45	3. .375	5. .55	7. .625
2. .045	4. .0375	6. .055	8. .0625

(a) Find the sum of the eight decimals.

(b) Find the sum of the eight common fractions.

Example II.

Change $\frac{3}{8}$ to a decimal.

Operation.

$$8\overline{)3.000}$$
$$.375$$

Explanation.

3 over 8 means 3 divided by 8. We therefore annex zeros to the numerator 3, and perform the division.

One eighth of 30 tenths is 3 tenths with a remainder of 6 tenths, etc.

Reduce to decimals.

1. $\frac{3}{4}$	3. $\frac{3}{5}$	5. $\frac{1}{4}$	7. $\frac{2}{5}$
2. $\frac{3}{40}$	4. $\frac{3}{50}$	6. $\frac{37}{40}$	8. $\frac{47}{50}$

(c) Find the sum of the eight decimals.

Fractions.

EXAMPLE III.

Change $\frac{2}{7}$ to a decimal.

Operation.

$$7)\underline{2.00}$$
$$.28+$$

$$7)\underline{2.000}$$
$$.285+$$

$$7)\underline{2.0000}$$
$$.2857+$$

Explanation.

It will be observed that, however far this division may be carried, there is always a remainder. The fact that there is a remainder is indicated by writing the plus sign after the last figure of the decimal. The first quotient may be read *28 hundredths, plus.*

Observe, too, that the error in the first answer is less than 1 hundredth, since the true quotient is more than 28 hundredths and less than 29 hundredths. We may therefore say that the first result is *true to hundredths;* the second, *true to thousandths.*

Reduce to decimals, true to thousandths.

1. $\frac{3}{7}$	4. $\frac{2}{11}$	7. $\frac{1}{6}$	10. $\frac{5}{9}$
2. $\frac{5}{7}$	5. $\frac{3}{11}$	8. $\frac{5}{6}$	11. $\frac{2}{3}$
3. $\frac{6}{7}$	6. $\frac{6}{11}$	9. $\frac{4}{9}$	12. $\frac{1}{3}$

(a) Find the sum of the twelve common fractions.

(b) Find the sum of the twelve decimals.*

Determine which of the following fractions can be reduced to terminating decimals† and which cannot.

1. $\frac{2}{3}$	3. $\frac{2}{5}$	5. $\frac{2}{7}$	7. $\frac{2}{9}$	9. $\frac{2}{11}$
2. $\frac{3}{4}$	4. $\frac{5}{6}$	6. $\frac{3}{8}$	8. $\frac{3}{10}$	10. $\frac{5}{12}$

Observe that if a fraction is in its lowest terms and the denominator contains any other prime factor besides 2's and 5's, the fraction cannot be reduced to a "terminating" decimal. Can you tell why?

* Observe that the difference between *a* and *b* must be less than 12 thousandths. Why?

† See foot-note, page 258.

Fractions.

203. To Reduce a Complex Decimal* to a Common Fraction.

Example.

Change $.27\frac{3}{11}$ to a common fraction.

Operation.

$$.27\frac{3}{11} = \frac{27\frac{3}{11}}{100}$$

$$27\frac{3}{11} = \frac{300}{11}$$

$$\frac{300}{11} \div 100 = \frac{300}{1100} = \frac{3}{11}$$

Explanation.

Writing the denominator, we have $27\frac{3}{11}$ divided by 100.

$27\frac{3}{11}$ reduced to 11ths $= \frac{300}{11}$.

$\frac{300}{11} \div 100 = \frac{300}{1100} = \frac{3}{11}$.

Reduce to common fractions.

1. $.38\frac{1}{3}$	4. $.24\frac{2}{11}$	7. $.61\frac{2}{3}$
2. $.83\frac{1}{6}$	5. $.35\frac{3}{11}$	8. $.16\frac{5}{6}$
3. $.45\frac{2}{7}$	6. $.40\frac{6}{11}$	9. $.54\frac{5}{7}$

(a) Find the sum of the nine common fractions.

(b) Find the sum of the nine complex decimals.

Note.—If the division be carried sufficiently far in any non-terminating decimal there will be found a certain figure or set of figures that is constantly repeated: thus, we may have, .3666666, or .27272727, or .5236236236. The part repeated is called *a repetend*, and may be written thus: $.3\dot{6}$, $.\dot{2}\dot{7}$, $.5\dot{2}3\dot{6}$. It is a curious fact that the real denominator of any repetend is as many 9's as there are figures in the repetend; $.3\dot{6} = .3\frac{6}{9}$, $.\dot{2}\dot{7} = \frac{27}{99}$, $.5\dot{2}3\dot{6} = .5\frac{236}{999}$.

* Decimals that are complete without the annexation of a common fraction are said to be *terminating decimals*. .24 is a terminating decimal. .666666 + is a non-terminating decimal. A decimal with a common fraction annexed, as $.33\frac{1}{3}$, is sometimes called a *complex decimal*.

Fractions.

204. To Reduce a Fraction to Hundredths.

EXAMPLES.

I. $\frac{3}{4}$ $\quad \dfrac{4)3.00}{.75}$; or $\frac{3}{4}$ of $\dfrac{100}{100}=\dfrac{75}{100}$ or .75.

II. $\frac{2}{3}$ $\quad \dfrac{3)2.00}{.66\frac{2}{3}}$; or $\frac{2}{3}$ of $\dfrac{100}{100}=\dfrac{66\frac{2}{3}}{100}$ or $.66\frac{2}{3}$.

III. $\frac{7}{8}$ $\quad \dfrac{8)7.00}{.87\frac{1}{2}}$; or $\frac{7}{8}$ of $\dfrac{100}{100}=\dfrac{87\frac{1}{2}}{100}$ or $.87\frac{1}{2}$.

IV. $\frac{1}{40}$ $\quad \dfrac{40)1.00}{.02\frac{1}{2}}$; or $\frac{1}{40}$ of $\dfrac{100}{100}=\dfrac{2\frac{1}{2}}{100}$ or $.02\frac{1}{2}$.

I. Reduce to hundredths.

1. $\frac{1}{2}$	6. $\frac{1}{4}$	11. $\frac{1}{10}$	16. $\frac{3}{20}$
2. $\frac{1}{3}$	7. $\frac{1}{6}$	12. $\frac{2}{3}$	17. $\frac{5}{6}$
3. $\frac{1}{8}$	8. $\frac{3}{8}$	13. $\frac{11}{40}$	18. $\frac{9}{40}$
4. $\frac{3}{10}$	9. $\frac{2}{5}$	14. $\frac{7}{25}$	19. $\frac{1}{50}$
5. $\frac{1}{7}$	10. $\frac{2}{7}$	15. $\frac{3}{14}$	20. $\frac{5}{14}$

(a) Find the sum of the twenty decimals.

(b) Find the sum of the twenty common fractions.

II. Reduce to hundredths.

1. $\frac{11}{175}$	4. $\frac{18}{175}$	7. $\frac{24}{175}$
2. $\frac{15}{175}$	5. $\frac{21}{175}$	8. $\frac{16}{175}$
3. $\frac{17}{175}$	6. $\frac{35}{175}$	9. $\frac{18}{175}$

(a) Find the sum of the nine decimals.

(b) Find the sum of the nine common fractions.

Fractions.

III. Reduce to hundredths.

Note.—Such fractions as the following may be easily reduced to hundredths by dividing the numerator and the denominator of each by that number which will change the denominator to 100.*

1. $\frac{37}{500}$	5. $\frac{45}{400}$	9. $\frac{36}{300}$
2. $\frac{46}{500}$	6. $\frac{92}{400}$	10. $\frac{40}{300}$
3. $\frac{94}{500}$	7. $\frac{120}{400}$	11. $\frac{85}{300}$
4. $\frac{323}{500}$	8. $\frac{143}{400}$	12. $\frac{139}{300}$

(a) Find the sum of the twelve results.

(b) Find the sum of the twelve common fractions.

IV. Reduce to hundredths.

Note.—Multiply the numerator and the denominator of each fraction by that number which will change the denominator to 100.

1. $\frac{7}{20}$	4. $\frac{8}{12\frac{1}{2}}$	7. $\frac{13}{20}$
2. $\frac{14}{25}$	5. $\frac{7}{10}$	8. $\frac{11}{25}$
3. $\frac{27}{50}$	6. $\frac{4\frac{1}{2}}{12\frac{1}{2}}$	9. $\frac{23}{50}$

(a) Find the sum of the nine decimals.

V. Reduce to hundredths.

1. $\frac{115}{250}$	4. $\frac{6}{40}$	7. $\frac{135}{250}$
2. $\frac{69}{150}$	5. $\frac{18}{40}$	8. $\frac{81}{150}$
3. $\frac{154}{350}$	6. $\frac{16}{40}$	9. $\frac{196}{350}$

(a) Find the sum of the nine decimals.

* Every common fraction can be changed to hundredths by annexing two zeros to the numerator and dividing by the denominator; but this method of reduction is not always the most simple.

Fractions.

205. DENOMINATE FRACTIONS.

1. One half inch is what part of a foot?
2. Two and $\frac{1}{2}$ inches are what part of a foot?
3. Five and $\frac{1}{2}$ inches are what part of a foot?
4. One half foot is what part of a rod?
5. Three and one half feet are what part of a rod?
6. Ten and one half feet are what part of a rod?
7. Sixty-four rods are what part of a mile?
8. Ninety-six rods are what part of a mile?
9. One hundred eighty rods are what part of a mile?
10. One and one half quarts are what part of a peck?
11. Two and one half quarts are what part of a gallon?
12. Twenty-four quarts are what part of a bushel?
13. Fourteen ounces are what part of a pound?
14. Seven and one half ounces are what part of a pound?
15. One and one fourth ounces are what part of a pound?
16. Six hundred pounds are what part of a ton?
17. Four hundred fifty pounds are what part of a ton?
18. Six hundred twenty-five pounds are what part of a ton?
19. Seventy-five square rods are what part of an acre?
20. Forty-five square rods are what part of an acre?
21. One hundred square rods are what part of an acre?
22. Thirty-two cubic feet are what part of a cord?
23. Fifty-six cubic feet are what part of a cord?
24. One hundred cubic feet are what part of a cord?
25. Seven and one half minutes are what part of an hour?
26. Forty minutes are what part of an 8-hour day?
27. Ninety minutes are what part of an 8-hour day?

Algebraic Fractions.

206. Fractions in Equations.

Example I.

$$\frac{x}{2} + 2x = 30$$

Multiplying both members of the equation (see page 217, Art. 170, Statement 4) by 2, the denominator of the fraction in the equation, we have—

$$x + 4x = 60$$

Uniting terms, $5x = 60$

Dividing by 5, $x = 12$

Example II.

$$\frac{x}{2} + \frac{x}{3} + 5x = 70$$

Multiplying by 2, $x + \frac{2x}{3} + 10x = 140$

Multiplying by 3, $3x + 2x + 30x = 420$*

Uniting terms, $35x = 420$

Dividing by 35, $x = 12$

Problems.

Find the value of x.

1. $\frac{x}{5} + \frac{x}{3} = 8.$

2. $\frac{x}{2} - \frac{x}{3} = 6.$

3. $\frac{x}{3} + 5x = 64.$

4. $2x - \frac{x}{3} = 50.$

* Observe that the equation might have been cleared of fractions by multiplying both its members by 6, the l. c. m. of 2 and 3.

Algebra.

207. PROBLEMS LEADING TO EQUATIONS CONTAINING ONE UNKNOWN QUANTITY, WITHOUT FRACTIONS.

EXAMPLE.

John and Henry together have 60 oranges, and Henry has three times as many as John. How many has each?

Let $x =$ the number John has,

then $3x =$ the number Henry has,

and $x + 3x =$ the number they together have.

Therefore $x + 3x = 60$.

Uniting $4x = 60$.

Dividing $x = 15$.

Multiplying $3x = 45$.

John has 15 oranges and Henry has 45 oranges.

PROBLEMS.

1. The sum of two numbers is 275, and the greater is four times the less. What are the numbers?

2. Robert has a certain sum of money and Harry has five times as much; together they have $216. How many dollars has each?

3. One number is four times another, and their difference is 270. What are the numbers?

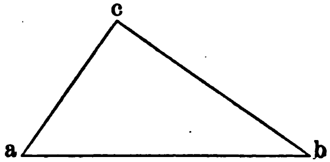
4. Peter has a certain number of marbles and William has 8 more than Peter; together they have 96 marbles. How many has each?

5. Sarah has a certain number of pennies and her sister has nine more than twice as many; together they have 93. How many has each?

6. Two times Reuben's money plus three times his money equals 175 dollars. How many dollars has he?

Geometry.

208. Construction Problems—Triangles.



1. Draw a triangle whose base, *ab*, is 3 inches long. Make the angle *a*, 55° and the angle *b*, 35°. The angle *c* should be —— degrees. Measure the three sides.

2. Draw a triangle two of whose sides are equal. Measure and compare the angles opposite the equal sides.

Observe that a triangle, two of whose sides are equal, has two angles equal; and conversely if two angles of a triangle are equal, two of the sides are equal.

3. If two triangles have the three sides of one equal to the three sides of the other, each to each, do you think the two triangles are alike in every respect?

4. If two triangles have the three angles of one equal to the three angles of the other, each to each, do you think the two triangles are necessarily alike in every respect?

5. Draw two triangles, the angles of one being equal to the angles of the other, and the sides of one *not* being equal to the sides of the other.

6. Is it possible to draw a triangle whose sides are equal, but whose angles are unequal?

7. Is it possible to draw a quadrilateral whose sides are equal but whose angles are unequal?

209. Miscellaneous Review.

1. Without a pencil, change each of the following fractions to hundredths: $\frac{3}{5}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{3}{25}$, $\frac{7}{50}$, $\frac{5}{12\frac{1}{2}}$, $\frac{7}{33\frac{1}{3}}$, $\frac{9}{16\frac{2}{3}}$, $\frac{60}{500}$, $\frac{30}{200}$, $\frac{45}{900}$, $\frac{60}{150}$, $\frac{60}{250}$, $\frac{60}{125}$, $\frac{60}{66\frac{2}{3}}$.

2. Butter that cost 25¢ a pound was sold for 29¢ a pound. The gain was equal to what part of the cost? The gain was equal to how many hundredths of the cost?

3. The taxes on an acre of land which was valued at $600 were $12. The taxes were equal to what part of the valuation? The taxes were equal to how many hundredths of the valuation?

4. Mr. Jones purchased 500 barrels of apples. He lost by decay a quantity equal to 75 barrels. What part of his apples did he lose? How many hundredths of his apples did he lose?

5. Regarding a month as 30 days and a year as 360 days, what part of a year is 7 months and 10 days? How many hundredths of a year in 7 months and 10 days? How many thousandths of a year? How many ten-thousandths of a year?

6. One cord 48 cubic feet is what part of 4 cords 16 cubic feet? Change the fraction to hundredths; to thousandths; to ten-thousandths.

7. One mile 240 rods is what part of 3 miles 160 rods? Change the fraction to hundredths; to thousandths; to ten-thousandths.

8. From a bill of $175 there was a discount of $14. The discount is equal to how many hundredths of the amount of the bill?

PERCENTAGE.

210. Per cent means *hundredth* or *hundredths.* Per cent may be expressed as a common fraction whose denominator is 100, or it may be expressed decimally; thus, 6 per cent = $\frac{6}{100}$ or .06; $28\frac{1}{2}$ per cent = $\frac{28\frac{1}{2}}{100}$ or $.28\frac{1}{2}$; $\frac{1}{2}$ per cent = $\frac{\frac{1}{2}}{100}$ or $.00\frac{1}{2}$.

NOTE.—Instead of the words *per cent* sometimes the sign (%) is used; thus 6 per cent may be written 6%.

211. The **base** in percentage is the number of which hundredths are taken; thus, in the problem, *find* 11% *of* 600, the base is 600; in the problem, 16 *is what per cent of* 800? the base is 800; in the problem, 18 *is* 3% *of what?* the base is not given, but is to be found by the student.

Observe that whenever the base is given in problems like the above, it follows the word OF.

212. There are three cases in percentage and only three.

Case I. To find some per cent (hundredths) of a number, as: find 15% of 600.

Case II. To find a number when some per cent of it is given, as: 24 is 8% of what number?

Case III. To find what per cent one number is of another, as: 12 is what % of 400?

Observe that a thorough knowledge of fractions is the necessary preparation for percentage. The work in percentage is work in fractions, *the denominator employed being 100.*

Percentage.

213. Case I.

Find 17 per cent (.17) of 8460.

Note.—We may find $\frac{3}{4}$ of a number by finding 3 times 1 fourth of it; that is, by multiplying it by $\frac{3}{4}$. So we may find .17 of a number by finding 17 times 1 hundredth of the number; that is, by multiplying by .17.

Operation.

$$\begin{array}{r} 84^{\vee}60 \\ .17 \\ \hline 592.20 \\ 846.0 \\ \hline 1438.20 \end{array}$$

Explanation.

One per cent (1 hundredth) of 8460 is 84.60; 17 per cent (hundredths) of 8460 is 17 times 84.60, or 1438.20.

Problems.

1. Find 17% of 6420; of 5252; of 31.40.
2. Find 35% of 6420; of 5252; of 31.40.
3. Find 43% of 6420; of 5252; of 31.40.
4. Find 25% of 6420; of 5252; of 31.40.
5. Find 50% of 6420; of 5252; of 31.40.
6. Find 30% of 6420; of 5252; of 31.40.
7. Find 35% of 6420; of 5252; of 31.40.
8. Find 65% of 6420; of 5252; of 31.40.

(a) Find the sum of the twenty-four results.

9. A sold goods for B. As remuneration for his services he received a sum equal to 12% of the sales. He sold $2146 worth of goods. How much did he receive?

10. C is a collector of money. For this service he charges a commission of 6%; that is, his pay is 6% of the amount collected. He collected for D $375. How much should he pay over to D, and how much should he retain as pay for collecting?

Percentage.

214. CASE II.

673.20 is 17 per cent (.17) of what number?

Operation No. 1.

```
17)673.20(39.60
   51          100
   ---        -----
   163      3960.
   153
   ---
    102
    102
    ---
```

Explanation.

Since 673.20 is 17 hundredths of the number, 1 hundredth of the number is 1 seventeenth of 673.20, or 39.60; and 100 hundredths = 100 times 39.60, or 3960.

Operation No. 2.

```
.17)673.20ˇ(3960
    51
    ---
    163
    153
    ---
     102
     102
     ---
```

Explanation.

We may find 100 seventeenths of a number by finding 1 seventeenth of a hundred times the number. 100 times 673.20 is 67320. 1 seventeenth of 67320 is 3960.

Observe that dividing by .17 is finding $\frac{100}{17}$ of the dividend, just as dividing by $\frac{3}{4}$ is finding $\frac{4}{3}$ of the dividend, and dividing by $\frac{1}{3}$ is finding $\frac{3}{1}$ of the dividend.

NOTE.—Sometimes the process may be shortened by writing the per cent as a common fraction and reducing it to its lowest terms; then using the reduced fraction instead of the one whose denominator is 100.

PROBLEMS.

1. 360 is 15% of what number?
2. 360 is 25% of what number?
3. 360 is 50% of what number?
4. 360 is 75% of what number?
5. 360 is 40% of what number?

(a) Find the sum of the five results

Percentage.

Case II.—*Continued.*

Problems.

1. $34.32 is 13 per cent of what?
2. $34.32 is 15 per cent of what?
3. $34.32 is 25 per cent of what?
4. $34.32 is $33\frac{1}{3}$ per cent of what?
5. $34.32 is 50 per cent of what?
6. $64.98 is 19 per cent of what?
7. $64.98 is 12 per cent of what?
8. $64.98 is 20 per cent of what?
9. $64.98 is 24 per cent of what?
10. $64.98 is 25 per cent of what?

(a) Find the sum of the ten results.

11. A sold goods for B. As remuneration for his services A received a sum equal to 12% of the sales. He received $33.06. What was the amount of his sales? How much money does B have left of what he received for the goods after paying out of it A's commission?

12. C is a collector of money. For this service he charges a commission of 6%; that is, his pay is 6% of the amount collected. His commission on a certain collection was $74.40. What was the amount collected? How much should the man for whom he collected the money, receive?

13. C collected a sum of money for D, deducted his commission of 6%, and paid the remainder of the sum collected to D. He paid D $350.15.* What was the sum collected? How much money did C retain as his commission for collecting?

* $350.15 is what % of the amount collected?

Percentage.

215. CASE III.

625 is what per cent of 900 ?

Operation No. 1.

$$
\begin{array}{l}
900)625^{\vee}00(.69\tfrac{4}{9} \\
\underline{5400} \\
\quad 8500 \\
\quad \underline{8100} \\
\quad\ \ \dfrac{400}{900}=\dfrac{4}{9}
\end{array}
$$

Explanation.

625 is $\frac{625}{900}$ of 900.

This fraction may be changed to hundredths in the usual manner, viz., by performing the division indicated, *"carrying out" to hundredths only.*

Operation No. 2.

$$\frac{625}{900}=\frac{25}{36}$$

$$
\begin{array}{l}
36)25^{\vee}00(.69\tfrac{4}{9} = 69\tfrac{4}{9}\% \\
\underline{216} \\
\ \ 340 \\
\ \ \underline{324} \\
\quad \dfrac{16}{36}=\dfrac{4}{9}
\end{array}
$$

Explanation.

625 is $\frac{625}{900}$ or $\frac{25}{36}$ of 900.

The fraction, $\frac{25}{36}$, may be changed to hundredths in the usual manner.

Operation and Explanation No. 3.

$$625 \text{ is } \frac{625}{900} \text{ of } 900. \quad \frac{625 \div 9}{900 \div 9}=\frac{69\frac{4}{9}}{100}=.69\tfrac{4}{9}=69\tfrac{4}{9}\%$$

Operation and Explanation No. 4.

One hundredth of 900 is 9; then 625 is as many hundredths of 900 as 9 is contained times in 625. 9 is contained in 625, $69\frac{4}{9}$ times; so 625 is $69\frac{4}{9}\%$ (hundredths) of 900.

PROBLEMS.

What per cent (how many hundredths) of 800 is—

(1) 250 ? (2) 375 ? (3) 475 ? (4) 350 ? (5) 150 ?

(a) Find the sum of the five results.

Percentage.

CASE III.—*Continued.*

PROBLEMS.

What % is—

1. 32 of 600 ?	6. 50 of 750 ?	11. 40 of 325 ?
2. 54 of 600 ?	7. 90 of 750 ?	12. 50 of 325 ?
3. 75 of 600 ?	8. 85 of 750 ?	13. 62 of 325 ?
4. 95 of 600 ?	9. 80 of 750 ?	14. 78 of 325 ?
5. 344 of 600 ?	10. 445 of 750 ?	15. 95 of 325 ?

(a) Find the sum of the first five results.
(b) Find the sum of the second five results.
(c) Find the sum of the third five results.

What % is—

16. $.76 of $38 ?	(What is one % of $38 ?)
17. $9.02 of $225.50 ?	(What is one % of $225.50 ?)
18. $17.13 of $342.60 ?	(What is one % of $343.60 ?)
19. $58.45 of $835 ?	(What is one % of $835 ?)
20. $1.29 of $6.45 ?	(What is one % of $6.45 ?)
21. $113.10 of $754 ?	(What is one % of $754 ?)
22. $21 of $175 ?	(What is one % of $175 ?)

23. A sold goods for B to the amount of $346.25; he received as his commission for selling the goods, $41.55. His commission was what per cent of the sales ?

24. C collected for D $643.50; he retained as his commission for collecting the money, $38.61. His commission was what per cent of the sales ?

25. F purchased 256 barrels of apples; he lost by decay a quantity equal to 16 barrels. What % of his apples did he lose ?

Algebra.

216. PROBLEMS LEADING TO EQUATIONS CONTAINING FRACTIONS.

If to $\frac{1}{3}$ of Bernie's money you add $\frac{1}{4}$ of his money the sum is \$63. How much money has he?

Let $x =$ the number of dollars he has.

$$\text{Then } \frac{x}{3} + \frac{x}{4} = 63.$$

$$\begin{array}{lr}
\text{Multiplying by 3,} & x + \dfrac{3x}{4} = 189 \\
\text{Multiplying by 4,} & 4x + 3x = 756^{*} \\
\text{Uniting terms,} & 7x = 756 \\
\text{Dividing by 7,} & x = 108
\end{array}$$

Thus we find that Bernie had 108 dollars.

1. If to $\frac{1}{4}$ of a certain number you add $\frac{1}{5}$ of the same number, the sum is 45. What is the number?

2. A lady upon being asked her age replied: If from $\frac{1}{2}$ of my age you subtract $\frac{1}{5}$ of my age the remainder will be 9 years. How old was she?

3. The sum of two numbers is 72, and the less number equals $\frac{3}{5}$ of the greater number. What are the numbers?

Let $x =$ the greater number.

4. The sum of two numbers is 75, and the less number equals $\frac{2}{3}$ of the greater number. What are the numbers?

5. The difference of two numbers is 30, and the less number equals $\frac{3}{5}$ of the greater number. What are the numbers?

* Observe that the equation might have been cleared of fractions by multiplying both its members by 12, the l. c. m. of 3 and 4.

Algebra.

217. Miscellaneous Problems.

1. Harry has some marbles; Joseph has 8 more than $\frac{1}{2}$ as many as Harry; together they have 68. How many has each?

2. William has 12 more than $\frac{1}{2}$ as many cents as Lucius; together they have 87 cents. How many cents has each?

3. If to the half of a certain number you add a third of the number, the sum will be 5 more than 3 fourths of the number. What is the number?

Let $x =$ the number,

$$\text{Then } \frac{x}{2} + \frac{x}{3} - 5 = \frac{3x}{4}$$

4. If to $\frac{1}{2}$ of a number you add $\frac{1}{4}$ of the same number, the sum will be 12 more than $\frac{3}{8}$ of the number. What is the number?

5. If to a certain number you add 5 times the number, the sum will be 24 more than 4 times the number. What is the number?

6. If to $\frac{1}{2}$ of a number you add $\frac{1}{3}$ of the number the sum will be a. What is the number?

Let $x =$ the number.

$$\text{Then } \frac{x}{2} + \frac{x}{3} = a$$

Multiplying by 6, $\quad 3x + 2x = 6a$

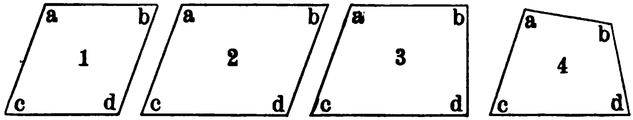
Uniting $\quad 5x = 6a$

Dividing by 5 $\quad x = \dfrac{6a}{5}$

Observe that you may put in the place of a any number you please; hence any number is $\frac{6}{5}$ of the sum of its half and third.

Geometry.

218. Construction Problems—Quadrilaterals.



1. Draw a rhombus. Make the angle *a*, 110°. Use the protractor for measuring the angle *a* only. Make *ab* and *ac* each 2 inches. Then draw *bd* parallel to *ac* and *cd* parallel to *ab*. Prove your work by measuring the other angles.

Observe that when one side and one angle of a rhombus are given the rhombus may be drawn.

2. If the angle *a* of a rhomboid is 110°, how many degrees in angle *b*? In angle *c*? In angle *d*?

3. Draw a trapezoid. Make the side *ab* 3 inches long. Make the angle *a*, 110 degrees, and the line *ac* 2 inches long. Make the angle *b*, 90 degrees; and the line *bd* of indefinite length, but long enough to form the side *bd*. Draw *cd* parallel to *ab*. Prove the work by finding the sum of the four angles.

4. Draw a trapezium. Make the angle *a*, 100°; the line *ab* 3 inches long, and the line *ac* 2 inches long. Make the angle *b*, 120°, and the side *bd* of indefinite length. Make the angle *c*, 85°. Angle *d* should be an angle of how many degrees? Prove the work by measuring angle *d*.

5. Draw four lines that together make a very irregular trapezium; then using the protractor, find the sum of the four angles.

219. Miscellaneous Review.

1. If I lose $\frac{3}{25}$ of my money, what % of my money do I lose?

2. Mr. Button spent $5 of the $12 which he had earned. What per cent of what he earned did he spend?

3. Mr. Thomas earns $150 per month. The monthly rent of the house in which he lives is equal to 15% of what he earns. How much is his rent per year?

4. Mr. Jones purchased 500 barrels of apples. He lost by decay a quantity equal to 75 barrels. What per cent of the apples purchased remained sound?

5. Mr. Brown borrowed $625. He used 87% of this money to pay debts. How much of the money borrowed did he have left?

6. Mr. Green paid $24.60 for a suit of clothes. If this was exactly 15% of his monthly earnings, how much does he earn per month?

7. Mr. White earns $1500 per year. He spends $312 for board, $200 for clothes, $75 for books and papers, $80 for traveling and amusements; gives to his church $40, for charitable purposes, $35, and all his other expenses amount to $23. The remainder of his salary he puts into a savings bank. What per cent of his salary does he save?

8. If to $\frac{1}{2}$ of my money you add $\frac{1}{5}$ of my money the sum will be $63. How much money have I?

9. The sum of two numbers is 120, and the less number is $\frac{3}{5}$ of the greater. What are the numbers?

10. If one angle of a rhomboid is 80°, what is the size of each of the other angles?

11. Three twenty-fifths is what % of $\frac{2}{5}$?

220. Problems in Case III, somewhat disguised.

Example No. 1.

75 is how many % more than 60?

75 is 15 more than 60; so the percentage part of this problem is, *15 is what % of 60?* Ans. 25%. Therefore 75 is 25% more than 60; that is 75 is 25 hundredths of 60 more than 60.

Example No. 2.

60 is how many % less than 75?

60 is 15 less than 75; so the percentage part of this problem is, *15 is what % of 75?* Ans. 20%. Therefore 60 is 20% less than 75; that is, 60 is 20 hundredths of 75 less than 75.

Observe that in example No. 1, 60 is the base, and that in example No. 2, 75 is the base; and that in problems of this kind the base always follows the word *than*.

Problems.

1. 60 is how many per cent more than 45?
2. 45 is how many per cent less than 60?

3. 150 is how many per cent more than 125?
4. 125 is how many per cent less than 150?

5. 225 is how many per cent more than 200?
6. 200 is how many per cent less than 225?

7. James has $345; Peter has $414. Peter has how many per cent more than James? James has how many per cent less than Peter?

Percentage.

221. APPLICATION OF ART. 220 TO "LOSS AND GAIN."

Find the per cent of loss or gain in each of the following:*

1. Bought for 25¢ and sold for 30¢.
2. Bought for 25¢ and sold for 23¢.
3. Bought for 25¢ and sold for 27¢.
4. Bought for 25¢ and sold for 21¢.
5. Bought for $40 and sold for $48.
6. Bought for $40 and sold for $35.
7. Bought for $40 and sold for $52.
8. Bought for $40 and sold for $36.
9. Sold for 65¢ that which cost 50¢.
10. Sold for 18¢ that which cost 20¢.
11. Sold for 30¢ that which cost 36¢.
12. Sold for 35¢ that which cost 30¢.
13. Sold for $50 that which cost $40.
14. Sold for $40 that which cost $50.
15. Sold for $30 that which cost $40.

16. Mr. Watson bought 60 lbs. of tea at 32¢ a pound and sold it at 50¢. What was his per cent of gain if he sold as many pounds as he bought? If he lost 4 lb. by "down-weights" and wastage, how much money did he gain? What was his real per cent of gain?

17. Mr. Jenkins bought gloves at $4.50 per dozen and sold them at 50¢ a pair. What was his per cent of gain?

18. Mr. Warner bought apples at 40¢ a bushel. He lost 25% of them by decay and sold the remainder at 50¢ a bushel. Did he gain or lose by the transaction? What was his per cent of gain or loss?

* In speaking of the per cent of loss or of gain the *cost* is regarded as the *base* unless otherwise specified.

Percentage.

222. PERCENTAGE PROBLEMS UNDER CASE I, IN WHICH THE PER CENT IS MORE THAN 100.

Find 175% ($\frac{175}{100}$ or 1.75) of \$632.60.

Operation No. 1.	*Explanation.*
\$6'32.60	One % of \$632.60 is \$6.326.
1.75	175% of \$632.60 is 175 times \$6.326. or \$1107.05.
\$31.6300	*Observe* that 5% of \$632.60 is \$31.63; that 70% of \$632.60 is \$442.82; that 100% of \$632.60 is \$632.60.
\$442.820	
\$632.60	
\$1107.0500	

Operation No. 2.

$175\% = \frac{175}{100} = \frac{7}{4}$. $\frac{1}{4}$ of \$632.60 = \$158.15.
$\frac{7}{4}$ of \$632.60 = 7 times \$158.15, or \$1107.05.

PROBLEMS.

1. Find 175% of 356; of 276; of 540.20.
2. Find 155% of 356; of 276; of 540.20.
3. Find 145% of 356; of 276; of 540.20.
4. Find 125% of 356; of 276; of 540.20.
5. Find 200% of 356; of 276; of 540.20.
6. Find 150% of 356; of 276; of 540.20.
7. Find 250% of 356; of 276; of 540.20.

(a) Find the sum of the twenty-one results.

8. David's money is equal to 150% of Henry's money. Henry has \$240. How much has David?

9. If goods cost \$260, and the profit on them is 125%, what is the selling price?

10. If a Chicago lot at the beginning of a certain year was worth \$8000, and during the year increased in value 250% what was it worth at the end of the year?

Percentage.

223. PERCENTAGE PROBLEMS UNDER CASE II, IN WHICH THE PER CENT IS MORE THAN 100.

\$1107.05 is 175% of how much money?

Operation No. 1.

```
175)$1107ᵛ.05($6.326
     1050          100
     ----      -------
      570      $632.60
      525
      ---
       455
       350
      ----
       1050
       1050
       ----
```

Explanation.

Since \$1107.05 is 175 hundredths of the money, 1 hundredth of the money is one 175th of \$1107.05, or \$6.326; and 100 hundredths, 100 times \$6.326, or \$632.60.

Operation No. 2.

```
1.75)$1107.05ᵛ($632.60
      1050
      ----
       570
       525
       ---
        455
        350
       ----
        1050
        1050
        ----
```

Explanation.

We may find one hundred 175ths of a number by finding one 175th of 100 times the number. One hundred times \$1107.05 is \$110705. One 175th of \$110705 is \$632.60.

Operation and Explanation No. 3.

175% = $\frac{175}{100}$ = $\frac{7}{4}$. If \$1107.05 is 7 fourths of the money, 1 seventh of \$1107.05, or \$158.15, is 1 fourth of the money, and 4 fourths, or the whole of it, is 4 times \$158.15, or \$632.60

PROBLEMS.

1. 43.50 is 125% of what number?
2. 65.10 is 150% of what number?
3. 48.50 is 200% of what number?
4. 59.20 is 250% of what number?
5. 29.44 is 115% of what number?

(a) Find the sum of the five answers.

Percentage.

224. Percentage Problems under Case III, in which the per cent is more than 100.

$1107.05 is what per cent of $632.60 ?

Operation.

$$
\begin{array}{l}
\$6.326)\$1107.050^{\vee}(175 \\
\quad 6326 \\
\quad \overline{47445} \\
\quad 44282 \\
\quad \overline{\ \ 31630} \\
\quad \ \ 31630 \\
\end{array}
$$

Explanation.

One per cent of $632.60 is $6.326; then $1107.05 is as many per cent of $632.60 as $6.326 is contained times in $1107.05. It is contained 175 times, so $1107.05 is 175% of 632.60.

Note.—The above problem may be solved as a similar problem is solved on page 265, operation No. 1. $1107.05 is $\frac{1107.05}{632.60}$ of $632.60. Perform the division indicated, carrying out to hundredths only. The quotient will be 1.75 or $\frac{175}{100}$ = 175%.

Problems.

1. What per cent of 845 is 2112.5 ?
2. What per cent of 845 is 1056.25 ?
3. What per cent of 845 is 1352 ?
4. What per cent of 845 is 1183 ?
5. What per cent of 845 is 2746.25 ?

(a) Find the sum of the 5 answers.

6. Reuben has $2420; Bernie has $4961. Bernie's money equals how many per cent of Reuben's money ?

7. A certain house cost $6425. The lot upon which it stands cost $2325. (a) The cost of the house equals how many per cent of the cost of the lot ? (b) The cost of the lot equals how many per cent of the cost of the house ?

Percentage.

225. MISCELLANEOUS PROBLEMS.

1. Find $\frac{1}{2}\%$* of 632; of 356; of 272.
2. Find $\frac{1}{4}\%$ of 632; of 356; of 272.
3. Find .25%† of 632; of 356; of 272.

(a) Find the sum of the nine results.

4. Find $3\frac{1}{2}\%$ of 496; of 532; of 720.
5. Find $4\frac{3}{4}\%$ of 496; of 532; of 720.
6. Find 1.75% of 496; of 532; of 720.

(b) Find the sum of the nine results.

7. What part of 94 is 11?‡ What per cent?
8. What part of 94 is 36? What per cent?
9. What part of 94 is 47? What per cent?

(c) Find the sum of the three "per cents."

10. 15 is 3% of what number? 4%? 5%?
11. 24 is 3% of what number? 4%? 5%?
12. 42 is 3% of what number? 4%? 5%?

(d) Find the sum of the nine answers.

13. 375 is 125% ($\frac{125}{100}$) of what number?
14. 436 is 109% ($\frac{109}{100}$) of what number?
15. 598 is 115% ($\frac{115}{100}$) of what number?

(e) Find the sum of the three answers.

16. 544 is 15% less than what number?
17. 545 is 25% more than what number?
18. 510 is 170% of what number?

(f) Find the sum of the three answers.

* This means $\frac{1}{2}$ of 1 per cent.

† This means find 25 hundredths of 1 per cent.

‡ Answer with a common fraction in its lowest terms.

Algebra.

226. To Find Two Numbers when their Sum and Difference are Given.

1. The difference of two numbers is 4 and their sum is 20. What are the numbers?

$$\begin{array}{lrl} \text{Let} & x &= \text{the smaller number,} \\ \text{then} & x+4 &= \text{the larger number,} \\ \text{and} & x+x+4 &= 20. \\ \text{Transposing,} & x+x &= 20-4. \\ \text{Uniting,} & 2x &= 16. \\ \text{Dividing,} & x &= 8\text{, the smaller number.} \\ & x+4 &= 12\text{, the larger number.} \end{array}$$

2. The difference of two numbers is 9 and their sum is 119. What are the numbers?

3. The difference of two fractions is $\frac{9}{20}$ and their sum is $\frac{3}{4}$. What are the fractions?

4. The difference of two numbers is d and their sum is s. What are the numbers?

$$\begin{array}{lrl} \text{Let} & x &= \text{the smaller number,} \\ \text{then} & x+d &= \text{the larger number,} \\ \text{and} & x+x+d &= s \\ \text{Transposing,} & x+x &= s-d \\ \text{Uniting,} & 2x &= s-d \\ \text{Dividing,} & x &= \dfrac{s-d}{2} \end{array}$$

Observe that any number you please may be put in the place of s, and any number less than s in the place of d; so when the sum and the difference of two numbers are given, the smaller number may be found by subtracting the difference from the sum and dividing the remainder by 2.

Algebra.

227. ANOTHER METHOD OF FINDING TWO NUMBERS WHEN THEIR SUM AND DIFFERENCE ARE GIVEN.

1. The difference of two numbers is 17 and their sum is 69. What are the numbers?

$$\begin{array}{lrl} \text{Let} & x &= \text{the larger number,} \\ \text{then} & x-17 &= \text{the smaller number,} \\ \text{and} & x+x-17 &= 69. \\ \text{Transposing,} & x+x &= 69+17. \\ \text{Uniting,} & 2x &= 86. \\ \text{Dividing,} & x &= 43, \text{ the larger number.} \\ & x-17 &= 26, \text{ the smaller number.} \end{array}$$

2. The difference of two numbers is 8.4 and their sum 75.6. What are the numbers?

3. The difference of two numbers is d and their sum is s. What are the numbers?

$$\begin{array}{lrl} \text{Let} & x &= \text{the larger number,} \\ \text{then} & x-d &= \text{the smaller number,} \\ \text{and} & x+x-d &= s \\ \text{Transposing,} & x+x &= s+d \\ \text{Uniting,} & 2x &= s+d \\ \text{Dividing,} & x &= \dfrac{s+d}{2} \end{array}$$

Observe that any number you please may be put in the place of s, and any number less than s in the place of d; so when the sum and difference of two numbers are given, the larger may be found by adding the difference to the sum and dividing the amount by 2.

4. A horse and a harness together are worth \$146, and the horse is worth \$74 more than the harness. Find the value of each.

Geometry.

228. How Many Degrees in Each Angle of a Regular Pentagon?

Fig. 1.

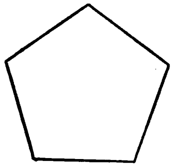
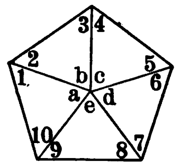


Fig. 2.



1. Every regular pentagon may be divided into —— equal isosceles triangles.

2. The sum of the angles of one triangle is equal to —— right angles; then the sum of the angles of 5 triangles is equal to —— right angles.

3. But the sum of the central angles in figure 2, $(a + b + c + d + e)$ is equal to —— right angles; then the sum of all the other angles of the five triangles is equal to 10 right angles, less 4 right angles, or 6 right angles $= 540°$. But the angular space that measures 540°, as shown in figure 2, is made up of 10 equal angles, so each one of the angles is 1 tenth of 540° or 54°. Two of these angles, as 1 and 2, make one of the angles of the pentagon; therefore each angle of the pentagon measures 2 times 54° or 108°.

4. Using the protractor construct a regular pentagon as follows:

(a) Draw two lines that meet in a point, each line being 2 inches long and the angular space between them being 108°.

(b) Regarding the two lines as two sides of a regular pentagon, draw two more sides each 2 inches long and joining those already drawn at an angle of 108°.

(c) Complete the figure by drawing the fifth side, and prove your work by measuring the last line drawn and the other two angles.

229. Miscellaneous Review.

Remembering that in speaking of the per cent of loss or gain, the *cost is the base* unless otherwise specified, tell the per cent of loss or gain in each of the following:

1. Bought for 2 and sold for 3.
2. Bought for 3 and sold for 2.
3. Bought for 4 and sold for 5.
4. Bought for 5 and sold for 4.
5. Bought for 5 and sold for 6.
6. Bought for 6 and sold for 5.
7. Bought for 8 and sold for 10; for 12.
8. Bought for 8 and sold for 14; for 16.
9. Bought for 8 and sold for 18; for 20.
10. Bought for 8 and sold for 4; for 2.

11. Mr. Parker sold goods at a profit of 25%; the amount of his sales on a certain day was $24.60. How much was his profit?

12. Mr. Jewell sold goods at a loss of 25%; the amount of his sales on a certain day was $24.60. How much was his loss.

13. By selling a horse for $156 there was a loss to the seller of 20%. What would have been his gain per cent if he had sold the horse for $234?

14. A bill was made for wood that was supposed to be 4 feet long. It was afterwards found to be only 46 inches long. What % should be deducted from the bill?

15. The marked price on a pair of boots was 25% above cost. If the dealer sells them for 25% less than the marked price will he receive more or less than the cost of the boots?

16. If by selling goods at a profit of 12% a man gains $6.60, what was the cost of the goods?

230. Discounting Bills.

Many kinds of goods are usually sold "on time"; that is, the buyer may have 30, 60, or 90 days in which to pay for them. If he pays for such goods at the time of purchase, or within ten days from the time of purchase, his bill is "discounted" from 1 % to 6 %, according to agreement; that is, a certain part of the amount of the bill is deducted from the amount.

Example.

Mr. Smith bought of Marshall Field & Co. a bill of goods amounting to $350.20. The discount for immediate payment ("spot cash") was 1%. How much must he pay for the goods?

1% of $350.20 is $3.50. $350.20 – $3.50 = $346.70.

Problems.

"Figure the discounts" on the following bills.

1. Bill of $324.37, discounted at 2%.
2. Bill of $276.45, discounted at 1%.
3. Bill of $356.50, discounted at 3%.
4. Bill of $536.50, discounted at 6%.
5. Bill of $561.80, discounted at 4%.

(a) Find the sum of the five bills before they are discounted.

(b) Find the sum of the discounts.

(c) Find the sum of the five bills after discounting.

Applications of Percentage.

231. Discounts from List Price.

Dealers in hardware, rubber boots and shoes, belting, rubber hose, and many other kinds of goods, sell from a list price agreed upon by the manufacturers. The actual price is usually less than the list price. "20% off" means that the list price is to be discounted 20%. "20 and 10 off" means that the list price is to be discounted 20%, and what remains is to be discounted 10%. Sometimes as many as nine successive discounts are allowed. Observe that in computing these the *base changes* with each discount.

Problems.

Find the actual cost of—

1. 500 ft. $\frac{3}{4}$-inch gas pipe (list, 7¢ per ft.) at 50 and 10 off.

2. 350 ft. $\frac{1}{2}$-inch gas pipe (list, 8¢ per ft.) at 50 and 10 off.

3. 200 ft. $1\frac{1}{2}$-inch gas pipe (list, 26¢ per ft.) at 55 and 10 off.

4. 260 ft. 2-inch gas pipe (list, 35¢ per ft.) at 55 and 10 off.

5. 48 $\frac{1}{2}$-in. elbows (list, 7¢ each) at 65 and 20 off.

6. 36 $\frac{3}{4}$-in. elbows (list, 9¢ each) at 65 and 20 off.

(a) Find the entire cost of the six items.

7. Find the cost of 12 pairs men's rubber boots (list, $3.00 per pair) at 25 and 10 off. Find the cost of the same at 35 off. Why are the results unlike?

8. Which is the lower price, 50 and 10 off or 60 off?

9. Bought for 40 off from list price and sold for 10 off from list price. What was my gain per cent?

10. Bought for 70 off from list and sold for 50 and 20 off from list. Did I lose or gain and how many per cent?

Applications of Percentage.

232. SELLING "on Commission."

When goods are sold "on commission" *the selling price is the base*; that is, the seller receives a certain per cent of the selling price as remuneration for services.

Commission is the sum paid an agent, or commission merchant, for transacting business.

PROBLEMS.

1. At 40%, what is the commission for selling $275 worth of books? If the salesman sells and collects for 40% of the selling price, how much of the $275 will he retain and how much "pay over" to the man for whom he sells the books?

2. While selling books on a commission of 40% my commission amounted to $56. What was the selling price of the books? If I not only sold but collected for 40% of the selling price, how much money should I "pay over" to my employer?

3. A real estate agent sold a house and lot for $4250. If his commission is 5%, how much should he receive for his services?

4. A real estate agent sold a piece of property upon which his commission at 5% amounted to $275. What was the selling price of the property. How much should the owner receive for the property after deducting the commission?

5. A commission merchant sold 2140 lbs. of butter at 23¢ a pound. After deducting his commission of 5% and paying freight charges of $36.50, and storage charges of $21.40, how much should he send to the man for whom he made the sale?

Applications of Percentage.

233. TAXES.

A **tax** is a sum of money paid for public purposes. A tax on property is reckoned at a certain per cent of the **assessed value** of the property. The assessed value may or may not be the real value. It is often much below the real value.

PROBLEMS.

1. Mr. Hardy has a farm of 240 acres which he values at \$24000. Its assessed value is \$22 per acre. If his state tax is $\frac{1}{2}\%$, his county tax $1\frac{1}{4}\%$, his town tax $\frac{1}{4}\%$, his school tax 2%, and his special road and bridge tax 1%, how much money must he pay as taxes on his farm?

2. The assessed value of the taxable property of a certain school district is \$176,242.25. If the school tax is $2\frac{1}{2}\%$ and the collector receives 2% of the amount collected as his commission, and collects the entire amount of the tax, how much should the district officers receive from this source for school purposes?

3. The assessed value of Mr. Randall's property is \$3400. At the rate of 15 mills on a dollar,* how much tax must he pay?

4. The assessed value of the property of a district of a certain city is \$250,000. (a) What must be the per cent of taxation to raise \$10,000? (b) What will be the net sum realized for public purposes if the collector is able to collect only 95% of this tax and he receives for his services 2% of the amount collected?

5. Mr. Evans's tax is \$35.60; the rate of taxation is $2\frac{1}{2}\%$. What is the assessed value of his property?

* "15 mills on a dollar" is the same as $1\frac{1}{2}\%$.

Applications of Percentage.

234. Insurance is a guaranty by one party to pay a certain sum to another party in the event of loss or damage.

The **policy** is the written contract given by the underwriter to the insured. The **premium** is the sum paid for insurance.

PROBLEMS.

1. A store valued at $7500 was insured for $5000 for 1 year. The rate of insurance was 2%. What was the amount of the premium?

2. A stock of goods valued at $10000 was insured for $5000. A fire occurred, but part of the goods were saved. It was found that the entire loss to the owner of the goods was $4750. (a) How much should he receive? (b) How much should he receive if the loss were $5750?*

3. An insurance agent offers to insure my farm buildings for $3500 for 1 year at 1%, or for 5 years at 3%; the entire premium in either case to be paid in advance. (a) If I accept the first proposition, how much is the premium to be paid? (b) How much if I accept the second?

4. What is the rate of insurance on the nearest store and stock of goods? On farm property? On village or city property other than stores?†

5. A large building was insured in one company for $25000, in another company for $15000. It was damaged by fire to the extent of $12800. How much of the damage should each company pay?‡

*In case of total loss the owner would receive $5000. In case of partial loss the owner should receive the full amount of the loss, provided it does not exceed $5000.

†Any insurance agent will be willing to answer these questions for you.

‡The companies must share the loss in proportion to the amount of insurance carried by them.

235. Miscellaneous Problems in Applications of Percentage.

1. A dealer who had marked goods 50% above the cost, sold them after deducting 10% from the marked price. His profit on that sale was what per cent of the cost of the goods?

2. By selling a suit of clothes for $7.20 I would lose an amount equal to 10% of the cost. For what must I sell the suit to gain a sum equal to 10% of the cost?

3. I sold goods at a loss of 7%. My actual loss was $3.50. What was the cost of the goods?

4. The real value of a stock of goods was $8250. They were insured for $5500. A fire occurred and the salvage amounted to only $575. If the insurance company promptly settles in accordance with the above facts what is the actual loss to the owner of the goods?

5. If I sell goods on a commission of $12\frac{1}{2}$%, what must be the amount of my sales in order that I may receive an annual salary of $2500?

6. A school numbers 140 pupils. The absence for one week was as follows: Monday, 3 days; Tues., 5 days; Wed., 4 days; Thurs., 5 days; Friday, 3 days. (a) What was the per cent of absence? (b) What was the per cent of attendance?

7. Sold a horse for $120 and gained 25%. What did the horse cost me?

8. When the cost is $\frac{3}{4}$ of the selling price what is the gain per cent?

9. When the selling price is $\frac{3}{4}$ of the cost what is the loss per cent?

Algebra.

236. Miscellaneous Problems.

1. In a school there are 896 pupils. There are three times as many boys as girls. How many girls? How many boys?

2. A man had 235 sheep. In the second flock there were 15 more sheep than in the first. In the third flock there were 20 fewer than in the first. How many sheep in each flock?

Note.—Let $x =$ the number in the first flock; then $x + 15 =$ the number in the second, and $x - 20$, the number in the third.

3. A man owns three farms. In the second there are half as many acres as in the first. In the third there are twice as many acres as in the first. In all there are 560 acres. How many acres in each farm?

4. In an apple and pear orchard containing 296 trees, there were 5 more than twice as many apple trees as pear trees. How many of each kind?

5. To a number I add one half of itself and 15, and have 150. What is the number?

6. From three times a number I subtract $\frac{2}{3}$ of the number and 5, and have 37 remaining. What is the number?

Note.—Let $x =$ the number; then $3x - \left(\frac{2x}{3} + 5\right) = 37$. On removing the parenthesis, what signs must be changed? See page 207, II.

7. If to three times a number I add $\frac{2}{3}$ of the number and 18, the sum will be 238. What is the number?

8. Two thirds of a number is equal to the number decreased by 56. What is the number?

Algebra.

237. Miscellaneous Problems.

1. A is 50 years old. B is 20 years old. In how many years will A be only twice as old as B?

Note.—Let $x =$ the number of years; then $(20 + x) \times 2 = 50 + x$.

2. Find four consecutive numbers whose sum is 150.

Note.—Let $x =$ the first; then $x + 1 =$ the second; $x + 2 =$ the third, etc.

3. Find three consecutive numbers whose sum is 87.

4. Two numbers have the same ratio as 2 and 3 and their sum is 360. What are the numbers?

5. Two numbers have the same ratio as 3 and 4, and their sum is 168. What are the numbers?

6. Two numbers have the same ratio as 2 and 5, and their difference is 87. What are the numbers?

7. A has $350. B has $220. How many dollars must A give to B so that each may have the same sum?

Note.—Let $x =$ the number of dollars that must be given by A to B; then $220 + x = 350 - x$.

8. C has $560. D has $340. How many dollars must C give to D so that each may have the same sum?

9. E has $630. F has $240. How many dollars must E give to F so that E will have exactly twice as many dollars as F?

10. The fourth and the fifth of a certain number are together equal to 279. What is the number?

11. The difference between 1 fourth and 1 fifth of a certain number is 28. What is the number?

Geometry.

238. How many Degrees in each Angle of a Regular Hexagon?

Fig. 1.

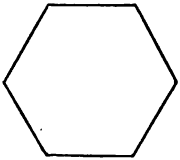
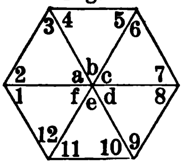


Fig. 2.



1. Every regular hexagon may be divided into —— equal isosceles triangles.

2. The sum of the angles of one triangle is equal to —— right angles, then the sum of the angles of 6 triangles is equal to —— right angles.

3. But the sum of the central angles in Fig. 2 ($a + b + c + d + e + f$) is equal to —— right angles; then the sum of all the other angles of the six triangles is equal to 12 right angles less 4 right angles, or 8 right angles = 720°. But the angular space that measures 720°, as shown in Fig. 2, is made up of 12 equal angles; so each one of the angles is one 12th of 720°, or 60°. Two of these angles, as 1 and 2, make one of the angles of the hexagon; therefore each angle of the hexagon measures 2 times 60°, or 120°.

4. Using the protractor, construct a regular hexagon, making each side 2 inches long.

Observe that since all the angular space about a point is equal to 4 right angles, or 360°, and since the space around the central point of the hexagon is divided into 6 equal angles, each of these angles is an angle of (360° ÷ 6) 60°. But each of the other angles of these triangles has been shown to be an angle of 60°; so each triangle is equiangular. Are the triangles equilateral?

239. Miscellaneous Review.

1. A man buys goods for $60 and sells them for $75. He gains —— dollars.

(a) The gain equals what part of the cost? What %?

(b) The gain equals what part of the selling price? What per cent?

(c) The cost equals what part of the selling price? What per cent?

2. When the cost is 2 thirds of the selling price what is the per cent of gain?

3. When the selling price is 2 thirds of the cost what is the per cent of loss?

4. Bought for $200 and sold for $300. What was the per cent of gain?

5. Bought for $300 and sold for $200. What was the per cent of loss?

6. A tax of 15 mills on a dollar was levied in a certain town, the assessed value of the taxable property being $475,250. If 5% of the tax is non-collectable and if the collector is allowed 2% of the amount collected, for his services, how much will be realized from the levy?

7. Which is the greater discount, "20 and 10 and 5 off" or "35 off"?

8. A sold goods for B on a commission of 15%. His sales for a certain period amounted to $780. If the goods cost B exactly $600 was B's net profit more or less than 10%?

9. A offers rubber boots at "50 and 20 off"; B offers them at "20 and 50 off." The quality and list price being the same, which offer shall I accept?

INTEREST.

240. **Interest** is compensation for the use of money.

241. The money for which interest is paid is called the **principal.**

242. The principal and interest together are called the **amount.**

NOTE.—Interest is usually reckoned in per cent, the principal being the base; that is, the borrower pays for the use of money a sum equal to a certain per cent of the principal. When a man loans money "at 6%" he expects to receive back the principal, and a sum equal to 6% of the principal for every year the money is loaned and at that rate for fractions of years.

EXAMPLE.

Find the interest of \$257 for two years at 6%.

Operation.	*Explanation.*
\$2˅57 .12 5.14 25.7 \$30.84	The interest of any sum for 2 years at 6% is 12 hundredths of the principal. One hundredth of \$257 is \$2.57, and 12 hundredths of \$257 is 12 times \$2.57 or \$30.84.

1. Find the interest of \$242 for 3 yr. at 7%.
2. Find the interest of \$375 for 2 yr. at 6%.
3. Find the interest of \$146 for 1 yr. at 5%.
4. Find the interest of \$274 for 3 yr. at 5%.
5. Find the interest of \$375 for 2 yr. at 8%.
6. Find the interest of \$864 for 3 yr. at 7%.

(a) Find the sum of the six results.

Interest.

243. To Compute Interest for any Number of Years and Months.

Note.—The interest for 1 month is 1 twelfth as much as it is for 1 year; for 2 months, 2 twelfths or 1 sixth, etc.

Example.

Find the interest of $324.50 for 2 yr. 5 mo. at 6%.

Operation and Explanation No. 1.

Interest of $324.50 for 1 year at 1% =	$3.245	
Interest of $324.50 for 1 year at 6% =	$19.470	
Interest of $324.50 for 2 years at 6% =		$38.940
Interest of $324.50 for 1 mo. at 6% =	$1.6225	
Interest of $324.50 for 5 mo. at 6% =		8.1125
Interest of $324.50 for 2 yr. 5 mo. at 6% =		$47.0525

Operation No. 2.

2 yr. 5 mo. = $2\frac{5}{12}$ years.

$2\frac{5}{12}$ times .06 = .14½.

$$\begin{array}{r} \$324.50 \\ .14\frac{1}{2} \\ \hline 1.6225 \\ 12.9800 \\ 32.450 \\ \hline \$47.0525 \end{array}$$

Explanation.

The interest of any sum for 2 yr. 5 mo. is 14½ hundredths of the principal.

1 hundredth of $324.50 is	$3.2450
½ hundredth of $324.50 is	$1.6225
4 hundredths of $324.50 is	12.9800
10 hundredths of $324.50 is	32.450
14½ hundredths of $324.50 is	$47.0525

Problems.

1. Find the interest of $325.40 for 1 yr. 6 mo. at 7%.
2. Find the interest of $420.38 for 2 yr. 10 mo. at 6%
3. Find the interest of $221.60 for 2 yr. 3 mo. at 6%.
4. Find the interest of $145.20 for 1 yr. 9 mo. at 5%.
5. Find the interest of $340.10 for 3 yr. 1 mo. at 4%.

(a) Find the sum of the five results.

Interest.

244. To Compute Interest for any Number of Years, Months, and Days.

NOTE.—In computing interest, 30 days is usually regarded as 1 month, and 360 days as 1 year; so each day is $\frac{1}{30}$ of a month or $\frac{1}{360}$ of a year.

EXAMPLE.

Find the interest of $256.20 for 2 yr. 7 mo. 13 days at 6%.

Operation and Explanation No. 1.

Interest of $256.20 for 1 yr. at 1% =	$2.5620	
Interest of $256.20 for 1 yr. at 6% =	$15.3720	
Interest of $256.20 for 2 yr. at 6% =		$30.7440
Interest of $256.20 for 1 mo. at 6% =	$1.2810	
Interest of $256.20 for 7 mo. at 6% =		$8.9670
Interest of $256.20 for 1 da. at 6% =	$.04270	
Interest of $256.20 for 13 da. at 6% =		.5551
Interest of $256.20 for 2 yr. 7 mo. 13 da. at 6% =		$40.2661

Operation No. 2.

2 yr. 7 mo. 13 da. $2\frac{223}{360}$ yr.

$2\frac{223}{360}$ times .06 = $.15\frac{43}{60}$

$2ˇ56.20
$.15\frac{43}{60}$

1.8361
12.8100
25.620

$40.2661

Explanation.

The interest of any sum for 2 yr. 7 mo. 13 da. at 6% is $.15\frac{43}{60}$ of the principal.

1 hundredth of $256.20 is	$2.5620
$\frac{43}{60}$ of 1 hundredth of $256.20 is	$1.8361
5 hundredths of $256.20 is	$12.81
10 hundredths of $256.20 is	$25.62
$15\frac{43}{60}$ hundredths of $256.20 is	$40.2661

PROBLEMS.

1. Find the interest of $350.40 for 2 yr. 5 mo. 7 da. at 6%.
2. Find the interest of $145.30 for 1 yr. 7 mo. 10 da. at 8%.
3. Find the interest of $174.20 for 2 yr. 3 mo. 15 da. at 7%.

(a) Find the sum of the three results.

245. The "six-per-cent method."

NOTE.—If the teacher so prefers, the problems on the three preceding pages, as well as those that follow, may be solved by the "*six-per-cent method.*"

EXPLANATORY.

The interest at 6% for 1 yr. = .06 of the principal.
The interest at 6% for 1 mo. = $\frac{1}{12}$ of .06, or .005 of the principal.
The interest at 6% for 1 day = $\frac{1}{30}$ of .005, or .000$\frac{1}{6}$ of the principal.

READING EXERCISE.

1. Interest for 1 yr. at 6% = ——100ths of the principal;

for 2 yr. = ——100ths;	for 3 yr. = ——100ths;
for 1 mo. = ——1000ths;	for 2 mo. = ——1000ths;
for 3 mo. = ——1000ths;	for 4 mo. = ——1000ths;
for 5 mo. = ——1000ths;	for 6 mo. = ——1000ths;
for 1 da. = ——1000th;	for 3 da. = ——1000th;
for 6 da. = ——1000th;	for 12 da. = ——1000ths;
for 18 da. = ——1000ths;	for 24 da. = ——1000ths;
for 25 da. = ——1000ths;	for 27 da. = ——1000ths.

Find the interest of $243.25 for 2 yr. 5 mo. 18 da. at 6%.

Operation and Explanation.

Interest for 2 yr. = .12 of the principal.
Interest for 5 mo. = .025 of the principal.
Interest for 18 da. = .003 of the principal.
Total interest = .148 of the principal.
$243.25 × .148 = $36.001.

(1) $36.001 plus $\frac{1}{6}$ of $36.001 = int. of same prin. for the same time at 7%.

(2) $36.001 less $\frac{1}{6}$ of $36.001 = int. of same prin. for the same time at 5%.

(3) How find the interest at 8%? At 9%? At 4%? At 3%?

Interest.

246. PROBLEMS TO BE SOLVED BY THE "SIX PER CENT METHOD."

1. Find the interest of $265 for 1 yr. 3 mo. 13 da. at 6%.*
2. Find the interest of $346 for 2 yr. 5 mo. 20 da. at 6%.†
3. Find the interest of $537 for 1 yr. 7 mo. 10 da. at 6%.‡
4. Find the interest of $428 for 3 yr. 3 mo. 14 da. at 6%.
5. Find the interest of $150 for 1 yr. 6 mo. 15 da. at 6%.

(a) Find the sum of the five results.

6. Find the interest of $245.30 for 6 mo. 18 da. at 7%.
7. Find the interest of $136.25 for 8 mo. 10 da. at 7%.
8. Find the interest of $321.42 for 5 mo. 15 da. at 7%.
9. Find the interest of $108.00 for 10 mo. 8 da. at 7%.
10. Find the interest of $210.80 for 7 mo. 21 da. at 7%.

(b) Find the sum of the five results.

11. Find the amount of $56.25 for 2 yr. 4 mo. 2 da. at 6%.
12. Find the amount of $31.48 for 1 yr. 5 mo. 11 da. at 6%.
13. Find the amount of $55.36 for 2 yr. 8 mo. 12 da. at 6%.
14. Find the amount of $82.75 for 2 yr. 10 mo. 8 da. at 6%.
15. Find the amount of $27.35 for 1 yr. 1 mo. 1 da. at 6%.

(c) Find the sum of the five results.

16. Find the amount of $875 for 3 yr. 8 mo. 15 da. at 5%.
17. Find the amount of $346 for 2 yr. 6 mo. 12 da. at 4%.
18. Find the amount of $500 for 1 yr. 7 mo. 18 da. at 3%.
19. Find the amount of $600 for 2 yr. 5 mo. 21 da. at 8%.
20. Find the amount of $825 for 1 yr. 9 mo. 24 da. at 9%.

(d) Find the sum of the five results.

*1 yr.	.06	†2 yr.	.12	‡1 yr.	.06
3 mo.	.015	5 mo.	.025	7 mo.	.035
13 da.	.002⅙	20 da.	.003⅓	10 da.	.001⅔
Total	.077⅙	Total	.148⅓	Total	.096⅔

247. To Find the Time Between Two Dates.

Example.

How many years, months, and days from Sept. 25, 1892, to June 10, 1896?

The Usual Method.

1896 - 6 - 10 1892 - 9 - 25 3 - 8 - 15	From the 1896th yr., the 6th mo., and the 10th day, subtract the 1892nd yr., the 9th mo., and the 25th day. Regard a month as 30 days.

A Better Method.

From Sept. 25, 1892, to Sept. 25, 1895, is 3 years.
From Sept. 25, 1895, to May 25, 1896, is 8 months.
From May 25, 1896, to June 10, 1896, is 16 days.

Note.—The two methods will not always produce the same results. The greatest possible variation is two days. Find the time from Jan. 22, 1895, to March 10, 1897, by each method, and compare results. The difference arises from the fact that the month as a measure of time is a variable unit—sometimes 28 days, sometimes 31. The "*usual method*" regards each month as 30 days; the "*better method*" counts first the whole years, then the whole months, then the days remaining. By the "usual method," the time from Feb. 28, 1897, to March 1, 1897, is 3 days; by the "better method," it is 1 day.

Problems.

Find the time by both methods and compare the results.

1. From March 15, 1894, to Sept. 10, 1897.
2. From March 15, 1894, to Sept. 20, 1897.
3. From May 25, 1895, to Sept. 4, 1898.
4. From May 25, 1895, to Oct. 4, 1898.
5. From June 28, 1894, to Mch. 1, 1898.
6. From June 28, 1894, to May 1, 1898.
7. From Jan. 10, 1892, to Jan. 25, 1898.
8. From Jan. 10, 1892, to Dec. 25, 1898.
9. From April 15, 1893, to Aug. 15, 1898.

248. Algebra Applied to Problems in Percentage.

Example.

75 is 15 per cent of what number?

Let $x =$ the number sought.

Then $\frac{15}{100}$ of x, or $\frac{15x}{100} = 75.$

Multiplying by 100, $15x = 7500.$

Dividing by 15, $x = 500.$ Ans.

Problems.

1. 56 is 2 per cent of what number?
2. 45 is 5 per cent of what number?
3. 60 is 12 per cent of what number?
4. 37 is 4 per cent of what number?
5. 53 is 8 per cent of what number?
6. n is r per cent of what number?

Let $x =$ the number sought.

Then $\frac{rx}{100} = n$

Multiplying by 100, $rx = 100n$

Dividing by r $x = \frac{100n}{r}$ *

7. James has $54.20, and James's money equals 40 per cent of Henry's money. How much money has Henry?

8. Mr. Williams's annual expenses are $791.20; this is 92 per cent of his annual income. How much is his annual income?

*Observe that every problem on this page can be solved by this *formula*.

249. ALGEBRA APPLIED TO PROBLEMS IN PERCENTAGE.

EXAMPLE.

60 is what per cent of 75?

Let $x =$ the per cent (number of hundredths).

Then $\frac{x}{100}$ of 75, or $\frac{75x}{100} = 60.$

Multiplying by 100, $75x = 6000$

Dividing by 75, $x = 80.$ Ans.

PROBLEMS.

1. 180 is what per cent of 200?
2. 17 is what per cent of 340?
3. $87.50 is what per cent of $250?
4. 81 is what per cent of 540?
5. $75.60 is what per cent of $630?
6. n is what per cent of b?*

Let $x =$ the per cent.

Then $\frac{x}{100}$ of b, or $\frac{bx}{100} = n.$

Multiplying by 100, $bx = 100n$

Dividing by b, $x = \frac{100n}{b}$*

Solve the first five problems in four ways:

(1) Let $x =$ the per cent and solve as the "example" is solved.

(2) Using one hundredth of each base as a divisor and the other number mentioned in the problem as a dividend, find the quotient.

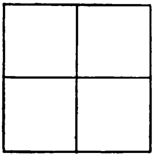
(3) Find what part the first number mentioned in each problem is of the base, and change the fraction thus obtained to hundredths.

(4) Apply the formula, $\frac{100n}{b} = x.$

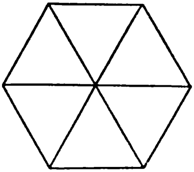
*The b in problem 6 and in the formula may be thought of as standing for the base.

Geometry.

250. Some Interesting Facts about Squares, Triangles, and Hexagons.

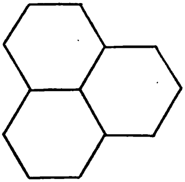


1. Four equal squares may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 90°. Four times 90° = —— degrees.



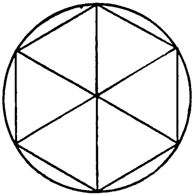
2. Six equal equilateral triangles may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 60°. Six times 60° = —— degrees.

Cut from paper 6 equal equilateral triangles and join them as shown in the figure.



3. Three equal hexagons may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 120°. Three times 120° = —— degrees.

Cut from paper 3 equal hexagons and join them as shown in the figure.



4. Since every regular hexagon may be divided into six equal equilateral triangles, as shown in the figure, it follows that the side of a regular hexagon is exactly equal to the radius of the circle that circumscribes the hexagon.

251. Miscellaneous Reviews.

1. Find the interest of \$250 from Sept. 5, 1896, to Jan. 17, 1898, at 6%.

2. Find the amount of \$340 from April 19, 1895, to Oct. 1, 1896, at 6%.

3. Find the amount of \$500 from May 20, 1897, to Feb 28, 1898, at 5%.

4. Find the amount of \$630 from July 1, 1896, to Nov. 1, 1896, at 7%.

5. Find the amount of \$800 from Jan. 1, 1897, to Jan. 25, 1897, at 6%.

6. If a 60-day bill of \$400 is discounted 2% for cash, how much ready money will be required to pay the bill?

7. Find the amount of \$392 for 60 days (two months) at 6%.

NOTE.—Observe that \$392 is the answer to problem 6. The result of problem 7, then, is the amount that the goods mentioned in problem 6 would cost at the end of 60 days if the purchaser borrowed the money at 6% to pay for them. Compare this result with \$400. How much does the purchaser of the bill save by borrowing the money at 6% to pay the bill instead of letting the bill run 60 days and then paying \$400?

8. Find the cost of goods, the list price being \$46, and the discounts "50 and 10 off and 2 off for cash."

9. My remuneration for selling goods on a commission of 40% amounted to \$56.20. How much should the man for whom the goods were sold receive?

NOTE.—\$56.20 is 40% of the selling price of the goods. The man for whom the goods were sold should receive 60% of the selling price. When goods are sold on a commission of 40%, what the agent receives equals what part of what the employer receives? What the employer receives is how many times what the agent receives?

PROMISSORY NOTES.

252. When a man borrows money at a bank he gives his note for a specified sum to be paid at a specified time, and he receives therefor, not the sum named in the note, but that sum less the interest upon it from the time the note is given to the bank officials to the time the note is due.

253. The acceptance of a note by the bank officials and the payment of a sum less than it will be worth at maturity is called "*discounting the note.*"

254. The **proceeds** of a note is the sum paid for it. As a rule, the notes discounted at a bank are those which bear no interest, and the date of discounting is usually the same as the date of the note, though not necessarily so.

EXAMPLE.

Find the discount and proceeds of the following:

$800. JACKSONVILLE, ILL., Jan. 10, 1898.

Sixty days after date I promise to pay James Rice, or order, eight hundred dollars, value received.

ARTHUR WILLIAMS.

Discounted at 6%, Jan. 10, 1898.

From the date of discount to the date of maturity it is 60 days. Interest of $800 for 60 days = $8.00.

Proceeds of note = $800 − $8 = $792.

Observe that the bank receives the interest on $800 for two months at 6% for the use of $792 for two months. The actual rate of interest is therefore a little more than the rate named.

255. PROBLEMS IN BANK DISCOUNT.

1. $375.00. CHICAGO, ILL., Apr. 5, 1898.

Thirty days after date I promise to pay to the order of John Smith, three hundred seventy-five dollars, at the Union National Bank. Value received. JAMES WHITE.

Discounted April 5, at 6 %. Find proceeds.

2. $450.00. AURORA, ILL., March 10, 1898.

Sixty days after date I promise to pay to Wm. George, or order, four hundred fifty dollars, at the Old Second National Bank. Value received. F. D. WINSLOW.

Discounted March 10, at 7 %. Find proceeds.

3. $2300.00. WAUKEGAN, ILL., Feb. 9, 1898.

Ninety days after date I promise to pay to the order of John Mulhall, two thousand three hundred dollars, at the Security Savings Bank. Value received.

CHAS. WHITNEY.

Discounted Feb. 9, at 7 %. Find proceeds.

4. $5000.00. SERENA, ILL., Jan. 20, 1898.

Sixty days after date I promise to pay to John Parr, or order, five thousand dollars. Value received.

HARRY BROWN.

Discounted Jan. 20, at 7 %. Find proceeds.

5. $3500. BOSTON, MASS., April 12, 1898.

Sixty days after date I promise to pay to W. J. Button, or order, three thousand five hundred dollars. Value received. HARRY WILSON.

Discounted Apr. 24, at 6 %. Find proceeds.

Promissory Notes.

256. THE DISCOUNTING OF INTEREST-BEARING NOTES.

Whenever a note is presented at a bank to be discounted, *its value at maturity* is regarded as the sum upon which the discount is to be reckoned. When the note is not interest-bearing, its face value is its value at maturity. In the discounting of short-time notes at banks it is customary to find the exact number of days from the date of discounting to the date of maturity and to regard each day as $\frac{1}{360}$ of a year.

EXAMPLE.

$750.00. AUSTIN, ILL., Jan. 1, 1898.

Six months after date, I promise to pay to the order of N. D. Gilbert, seven hundred fifty dollars, with interest at the rate of six per cent per annum. O. T. BRIGHT.

Discounted at a bank, May 10, 1898, at 7%.

Value of the note at maturity,	$772.50
Discount, 52 days at 7%,	7.81
Proceeds,	$764.69

1. $540.00. TOPEKA, KANSAS, Dec. 10, 1897.

Five months after date, I promise to pay to the order of J. C. Thomas, five hundred forty dollars, with interest at the rate of six per cent per annum. HIRAM BAKER.

Discounted at a bank, April 1, 1898, at 7%.

2. $325.00. EARLVILLE, ILL., Feb. 1, 1898.

Six months after date, I promise to pay to the order of Wm. R. Haight, three hundred twenty-five dollars, with interest at the rate of six per cent per annum.

CHAS. HOSS

Discounted at a bank, June 1, 1898, at 7%.

257. Partial Payments on Notes.

A **partial payment** is a part payment made upon a note before the time of final settlement.

There are two methods in common use of finding the value of a note at maturity, upon which one or more partial payments have been made. The first method is often applied to computation of "short-time notes," such as run one year or less. A formal statement of this method is called the—

MERCHANTS' RULE.—*Find the amount of the face of the note from the date to maturity. Then find the amount of each payment from the time it was made to the maturity of the note. From the amount of the face of the note subtract the sum of the amounts of the payments.*

$450. WAUKEGAN, ILL., Jan. 1, 1897.

One year after date, I promise to pay to the order of Wm. E. Toll, four hundred fifty dollars, with interest at six per cent. Value received. JEROME BIDDLECOM.

Part payments were made upon this note as follows; March 1, 1897, $250; May 1, 1897, $150.

Amount of the face of the note for one year,		$477.00
Amount of $250, March 1, '97, to Jan 1, '98,	$262.50	
Amount of $150, May 1, '97, to Jan. 1, '98,	156.00	$418.50
Value at maturity,		$ 58.50

1. $1000.00. SPRINGFIELD, ILL., Jan. 1, 1898.

Eight months after date, I promise to pay to the order of Fred H. Wines, one thousand dollars, with interest at six per cent. Value received. J. H. FREEMAN.

Part payments were made upon this note as follows: Apr. 1, 1898, $350; June 15, 1898, $240.

How much was due on this note at maturity, Sept. 1, 1898?

Promissory Notes.

258. A decision of the United States Supreme Court many years ago led to the very general adoption in the solution of "partial payment problems," of what is now known as the United States Rule.—*Find the amount of the principal at the time of the first payment. Subtract the first payment from this amount. The remainder is a new principal, upon which find the amount at the time of the second payment. Subtract the second payment from this amount. Continue this process to the time of settlement. The last amount is the sum due.*

An exception to the foregoing rule is required when there is a payment which is less than the interest due at the time the payment is made. In such case the payment is treated as though made at the time of the next payment; and if the two payments together do not equal the interest due, the sum of both is again carried forward.

$950. PETERSBURG, ILL., Jan. 1, 1895.

On or before Jan. 1, 1898, I promise to pay to N. W. Branson, or order, nine hundred fifty dollars, with interest at six per cent. Value received. B. LANING.

Part payments were made on this note as follows: July 1, 1895, $150; Jan. 1, 1896, $200; Jan. 1, 1897, $250. Find the amount due Jan. 1, 1898.

Amount of $950, July 1, 1895 (6 mo.),	$978.50
Subtract first payment,	150.00
New principal,	$828.50
Amount of $828.50, Jan. 1, 1896 (6 mo.),	$853.35
Subtract second payment,	200.00
New principal,	$653.35
Amount of $653.35, Jan. 1, 1897 (1 year),	$692.55
Subtract third payment,	250.00
New principal,	$442.55
Amount of $442.55, Jan. 1, 1898 (1 year),	$469.10

259. PROBLEMS IN "PARTIAL PAYMENTS."

1. Date of note, Jan. 1, 1896.
   Face of note, $800.
   Rate of interest, six per cent.
   Payment May 1, 1896, $125.
   Payment Dec. 1, 1896, $230.
   Find amount due Jan. 1, 1898.

2. Date of note, July 10, 1896.
   Face of note, $500.
   Rate of interest, six per cent.
   Payment Dec. 22, 1896, $200.
   Payment July 15, 1897, $200.
   Find amount due Jan. 1, 1898.

3. Date of note, Jan. 1, 1897.
   Face of note, $600.
   Rate of interest, six per cent.
   Payment Sept. 1, 1897, $100.
   Find amount due Jan. 1, 1898.

NOTE.—Solve problem 3 by the Merchants' Rule and by the United States Rule. The answer obtained by the latter will be 48 cents larger than the answer obtained by the former.

Observe that the Merchants' Rule is based upon the supposition that interest is not due until the time of settlement of the note, while the U. S. Rule is based upon the supposition that interest is due whenever a payment is made. By applying the latter rule to problem 3, $24 of interest must be paid Sept. 1; by applying the former rule to the same problem the entire $100 paid Sept. 1 applies in payment of principal. The answer, then, by the U. S. Rule must be greater than the answer by the Merchants' Rule, by the interest on $24 for four months, or 48 cents.

Algebra.

260. Algebra Applied to Some Problems in Interest

Example.

What principal at 6% will gain \$96 in 2 yrs.?

Let $x =$ the principal.

Since the interest at 6% for 2 years equals $\frac{12}{100}$ of the principal

$$\text{then } \frac{12}{100}x, \text{ or } \frac{12x}{100} = 96.$$

$$\text{Multiplying by 100} \qquad 12x = 9600$$

$$x = 800.*$$

Problems.

1. What principal at 6% will gain \$67.50 in 2 years 6 months?
2. What principal at 6% will gain \$27.20 in 1 year 4 months?
3. What principal at 7% will gain \$87.50 in 2 years 6 months?
4. What principal at 5% will gain \$187.50 in five years?
5. What principal at 8% will gain \$64 in 1 year 3 months?
6. What principal at 6% will gain \$61.20 in 2 years 6 months 18 days?

$$\text{Let } x = \text{the principal, then } \frac{153}{1000}x, \text{ or } \frac{153x}{1000} = \$61.20.$$

(a) Find the sum of the six answers.

* What principal at a% will gain b dollars in c years?

261. Algebra Applied to Some Problems in Interest.

Example.

What principal at 6% will amount to $828.80 in 2 years?

Let $x =$ the principal.

Then $x + \dfrac{12x}{100} = 828.80.$

Multiplying by 100, $100x + 12x = 82880.$

Uniting, $112x = 82880$

$x = 740.$*

Problems.

1. What principal at 6% will amount to $368 in 2 years 6 months? †

2. What principal at 6% will amount to $588.30 in 1 year 10 months?

3. What principal at 5% will amount to $393.75 in 2 years 6 months?

4. What principal at 5% will amount to $287.50 in three years?

5. What principal at 6% will amount to $458.80 in 2 years 5 months and 12 days?

Let $x =$ the principal; then $\dfrac{147}{1000}x$, or $\dfrac{147x}{1000} =$ the interest,

and $x + \dfrac{147x}{1000} =$ the amount, $458.80.

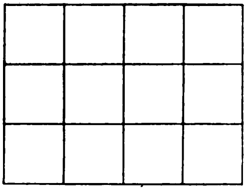
(a) Find the sum of the five answers.

* What principal at a% will amount to b dollars in c years?

† To the Pupil.—Prove each answer obtained by finding its amount for the given time at the given rate

Geometry.

262. The Area of a Rectangle.



1. One side of every rectangle may be regarded as its base. The side perpendicular to its base is its altitude.

2. The number of square units in the row of square units next to the base of a rectangle, taken as many times as there are linear units in its altitude, equals the number of square units in its area. In the figure given, we have 4 sq. units $\times$ 3 = 12 sq. units.

Note 1.—In the above, it is assumed that the base and altitude are measured by the same linear unit, and that the square unit takes its name from the linear unit.

Note 2.—In the actual finding of the area of rectangles for practical purposes, the work is done mainly with abstract numbers and the proper interpretation is given to the result. There can be no serious objection to the rule for finding the area of rectangles as given in the old books, provided the pupil is able to interpret it.

Rule.—To find the area of a rectangle, "*multiply its base by its altitude.*"

Problems.

1. Find the area of the surface of a cubical block whose edge is 9 inches in length.

2. Find the area in square yards of a rectangular piece of ground that is 36 feet by 45 feet.

3. Find the area in acres of a rectangular piece of land that is 92 rods by 16 rods.

4. Find the area in square rods of a piece of ground that is 99 feet by 66 feet.

263. Miscellaneous Review.

1. Clarence Marshall wished to borrow some money at a bank. He was told by the president of the bank that they (the bank officials) were "discounting good 30-day paper" at 7%. Mr. Marshall's name being regarded as "good," he drew his note upon one of the forms in use at the bank, for $1000 payable in thirty days without interest. On the presentation of this note to the cashier, how much money should he receive?

2. If Mr. Marshall's note described in problem 1 was dated April 10, 1898, (a) when must it be paid? (b) How much money will he pay when he "takes up" the note? (c) Does he pay for the use of the money borrowed, at the rate of exactly 7% per annum?

3. If a bank is discounting at 7%, how much should be given for a note of $200 due in two months from the time it is discounted and bearing interest for the two months at the rate of 6% per annum?

4. Find the value at the time of settlement of the following note:

Date of note, Apr. 1, 1896.
Face of note, $300. Rate, 6%.
Payment, Aug. 1, 1896, $75.
Payment, Apr. 1, 1897, $80.
Settled, Aug. 1, 1898.

5. What principal at 6% will gain $6 in 1 year 4 months?

6. What principal at 6% will amount to $81 in 1 year 4 months?

7. Find the area, in square feet, of a walk 4 feet wide around a rectangular flower-bed that is 40 feet long and 12 feet wide.

STOCKS AND BONDS.

264. Some kinds of business require so much capital that many persons combine to provide the necessary money. Such a combination of men organized under the laws of a State, the capital being divided into shares, is known as **a corporation,** or **stock-company.** Those who own the shares are called **stock-holders.** The stock-holders elect from their own number certain men to manage the business. These managers are called **directors.**

265. The **nominal value** of a share is its face value; that is, the sum named on its face. Large corporations, usually, though not always, divide their capital into $100 shares.

If the business is prosperous, shares may sell on the market for more than their nominal value. The stock is then said to be "*above par*," or "*at a premium.*"

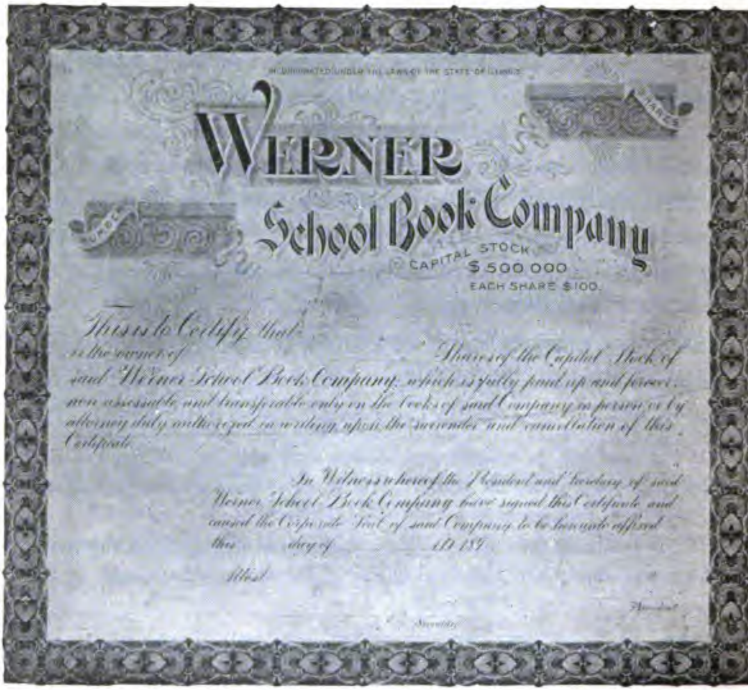
If the business does not prosper, the shares may sell on the market for less than their nominal value. The stock is then said to be "*below par*," or "*at a discount.*"

266. If the business is profitable, a part or all of the earnings is periodically divided among the stock-holders. The sum divided is called **a dividend.**

Dividends are always reckoned on the nominal or par value of the stock. If a corporation declares a 2% dividend, it pays to each stock-holder a sum equal to 2% of the nominal value of the stock which he owns.

267. The kinds of business which are usually conducted by corporations, are: The mining of coal, silver, gold, etc.; the operation of gas works, railroads, large manufacturing establishments of all kinds, creameries, etc.

Certificate of Stock.


INCORPORATED UNDER THE LAWS OF THE STATE OF ILLINOIS

SHARES

Werner School Book Company

CAPITAL STOCK $500 000

EACH SHARE $100.

This is to Certify that
is the owner of Shares of the Capital Stock of said Werner School Book Company, which is fully paid up and forever non-assessable, and transferable only on the books of said Company in person or by attorney duly authorized in writing, upon the surrender and cancellation of this Certificate.

In Witness whereof the President and Secretary of said Werner School Book Company have signed this Certificate and caused the Corporate Seal of said Company to be hereunto affixed this day of A.D. 189

Attest

President

Secretary

1. Examine the above certificate. What part of the entire stock of the Werner School Book Company would the owner of one hundred shares have?

2. A 3% dividend would require how much money from the treasury of the Company? How much money should the owner of one hundred shares receive?

3. If a 4% dividend is declared how much money should the owner of seven hundred and fifty shares of stock receive as his share of the dividend?

Stocks.

268. History of a Stock Company.

The farmers of a certain community agreed to combine in the building and management of a creamery. It was determined that a capital of $5000 was necessary. This was divided into 50 shares of $100 each. Men then came forward and contributed as follows:

A took 5 shares and paid in $500.
B took 3 shares and paid in $300.
C took 6 shares and paid in $600.
D took 4 shares and paid in $400.
E took 7 shares and paid in $700.
F took 2 shares and paid in $200.
G took 10 shares and paid in $1000.

H, I, J, K, L, M, N, O, P, Q, R, S, and T took 1 share each and paid in $100 each.

(a) At the end of the first year the directors declared an 8% dividend. How much did A receive? B? C? K?

(b) What was the entire amount of the money divided among the stock-holders at the end of the first year?

(c) Soon after this dividend was paid, A sold his stock to X at a premium of 10%. How much did A receive for his stock?

(d) At the end of the second year the profits were found to be comparatively small, and the directors could pay a dividend of only 3%. How much did X receive? B? C? K?

(e) What was the entire amount of the money divided among the stock-holders at the end of the second year?

(f) Soon after this dividend was paid, B, C, D, E, and F sold their stock to Y at a discount of 10%. How much did this stock cost Y?

HISTORY OF A STOCK COMPANY—*Continued.*

(g) At the end of the third year, the profits were so small that no dividend was declared. The stock-holders became disheartened and many of them offered to sell their stock at a large discount. Z appeared in the market and bought at "50¢ on the dollar" all the stock of the company except that owned by X and Y. How much did this stock cost Z?

(h) At the end of the fourth year, the directors, X, Y, and Z, declared a 10% dividend. How much money was divided and how much did each receive?

(i) Before the close of the fifth year, the property burned and the lot upon which it stood was sold. After the insurance money had been received, the book accounts collected, and all debts paid, there remained in the treasury of the company $4350. How much of this money should each stock-holder, X, Y, and Z, receive?

(j) Did this creamery enterprise prove a good investment for X? For Y? For Z? For A? For B? For M?

MISCELLANEOUS PROBLEMS.

1. The directors of a company whose capital is $50000 determined to distribute among the stock-holders $2500 of profits. (a) A dividend of what per cent shall be declared? (b) How much will a man receive who owns 15 100-dollar shares?

2. A company whose capital is $75000 pays a dividend of 3%. (a) How much money is divided among the stockholders?

3. Mr. Steele owns 20 shares ($100) in the C., B. & Q. R. R. He receives as his part of a certain dividend $110. What is the per cent of the dividend?

Bonds.

269. A **bond** is a very formal promissory note given by a government or other corporate body, as a railway or a gas company, for money borrowed. Bonds usually have attached to them small certificates called **coupons**. These are really little notes for the interest that will be due at different times Thus, a 10-year bond for $1000 with interest at 6% payable semi-annually may have 20 coupons attached, each calling for $30 of interest.

270. Money invested in bonds yields a specified income; but the income from money invested in stocks depends upon the profits of the company.

271. Bonds, like stocks, are sometimes sold for more than their face value. They are then said to be "*above par*," or "*at a premium*." Like stocks, too, they are sometimes "*below par*," or "*at a discount*."

1. What is the semi-annual interest on two 1000-dollar U. S. 5% bonds?

2. What sum should be named on each coupon of a 1000-dollar city bond if the interest is payable annually at the rate of 7%?

3. To raise the money to build a court-house, a certain county issued $50000 worth of 6% ten-year bonds. These sold upon the market at 2% premium. (a) How much money was received for the bonds? (b) How much did A pay, who bought three 1000-dollar bonds? (c) If the interest was payable semi-annually, how much should A receive each 6 months on this investment?

4. Has the county or city in which you live any "bonded indebtedness"? If so, how much, and what is the rate of interest?

City Bond with Coupons Attached.

NOTE.—Bonds are made in great variety both as to form and content; but in all, indebtedness is acknowledged, and the amount, rate of interest, and time of payment for both principal and interest, named. The above is a very short and concise form of Bond (much reduced in size) and is an exact copy of one prepared for actual use.

1. Examine the above Bond. If the time it is to run is five years, how many coupons should be attached?

2. If the Bond is dated Jan. 1, 1898, what date should be written in each coupon?

3. If the face of the Bond is $100 and the rate 5%, what sum should be written in each coupon?

272. ALGEBRA APPLIED TO SOME PROBLEMS IN INTEREST.

EXAMPLE.

At what rate per cent will \$500 gain \$55 in 2 yrs.?*

Let $x =$ the rate,

then $\frac{2x}{100}$ of 500, or $\frac{1000x}{100}$, or $10x =$ the interest,

and $10x = 55$

Dividing $x = 5\frac{1}{2}$

PROBLEMS.

1. At what rate per cent will \$450 gain \$72 in 2 years?
2. At what rate per cent will \$320 gain \$48 in 3 years?
3. At what rate per cent will \$560 gain \$84 in 2 years 6 months?
4. At what rate per cent will \$600 gain \$75 in 2 years 6 months?
5. At what rate per cent will \$600 gain \$114 in 2 years 4 months 15 days?

$$2 \text{ yr. } 4 \text{ mo. } 15 \text{ da.} = 2\tfrac{3}{8} \text{ years.}$$

$$\text{Let } x = \text{the rate.} \quad \text{Then } \frac{2\frac{3}{8}x}{100} \text{ of } 600 = 114.$$

NOTE.—Problem 5 may be solved arithmetically by finding the interest of \$600 for 2 yr. 4 mo. 15 da. at 6%. Divide this interest by 6 (to find the interest at 1%) and find how many times the quotient is contained in \$114.

* The arithmetical solution of this problem is as follows: The interest of \$500 for 2 years at 1% is $\frac{2}{100}$ of \$500. $\frac{2}{100}$ of \$500 = \$10. To gain \$55 in 2 years, \$500 must be loaned at as many per cent as \$10 is contained times in \$55. It is contained $5\frac{1}{2}$ times; so \$500 must be loaned at $5\frac{1}{2}$% to gain \$55 in 2 years. *Observe* that by this method we *divide the given interest by the interest of the principal for the given time at one per cent.*

Algebra.

273. ALGEBRA APPLIED TO SOME PROBLEMS IN INTEREST.

EXAMPLE.

In how long a time will \$650 gain \$97.50 at 6%?

Let $x =$ the number of years,

then $\frac{6x}{100}$ of $650 = 97.50$

Simplifying, $39x = 97.50$

Dividing, $x = 2.5$ *

PROBLEMS.

1. In how long a time will \$400 gain \$30 at 5%?
2. In how long a time will \$600 gain \$96 at 6%?
3. In how long a time will \$800 gain \$68 at 6%?
4. In how long a time will \$500 gain \$56 at 6%?
5. In how long a time will \$400 gain \$29 at 6%?

REVIEW PROBLEMS.

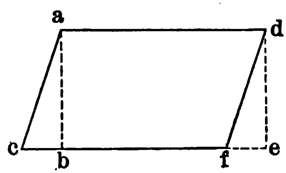
6. What principal at 8% will gain \$124.80 in 3 years?
7. What principal at 7% will amount to \$410.40 in 2 years?
8. At what rate per cent will \$900 gain \$72 in 2 years?
9. In how long a time will \$1000 gain \$160 at 6 per cent?
10. What principal at 5% will amount to \$736 in 3 years?

TO THE PUPIL.—Prove each answer by finding the interest on the given principal at the given rate for the time obtained.

* The arithmetical solution of this problem is as follows: The interest of \$650 for one year at 6% is \$39. As many years will be required to gain \$97.50 as \$39.00 is contained times in \$97.50. It is contained $2\frac{1}{2}$ times; so in $2\frac{1}{2}$ years \$650 will gain \$97.50. Observe that by either method we *divide the given interest by the interest of the principal for 1 year at the given rate.*

Geometry.

274. THE AREA OF A RHOMBOID.*

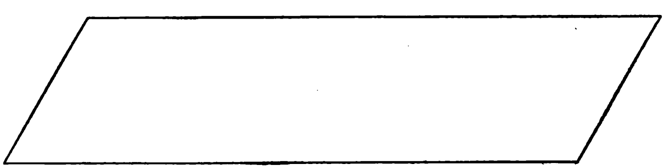


1. One side of a rhomboid may be regarded as its *base.* The perpendicular distance from the base to the opposite side is its *altitude.*

2. Convince yourself by measurements and by paper-cutting that from every rhomboid there may be cut a triangle, (*abc*), which when placed upon the opposite side, (*def*), converts the rhomboid into a rectangle (*adeb*).

Observe that the base of the rectangle is equal to the base of the rhomboid, and the altitude of the rectangle equal to the altitude of the rhomboid.

3. A rhomboid is equivalent to a rectangle having the same base and altitude. Hence, to find the area of a rhomboid, *find the area of a rectangle whose base and altitude are the same as the base and altitude of the rhomboid.* Or, as the rule is given in the older books,—"*Multiply the base by the altitude.*"



PROBLEM.—If the above figure represents a piece of land, and is drawn on a scale of $\frac{1}{8}$ inch to the rod, how many acres of land?

* The statements upon this page apply to the rhombus as well as to the rhomboid.

275. Miscellaneous Review.

1. Mr. Watson purchased 15 shares of C., B. & Q. R. R. stock at 12% discount. (a) How much did he pay for the stock? (b) When a 3% dividend is declared and paid, how much does he receive?*

2. James Cooper bought 12 shares of stock in the Sugar Grove Creamery at 8% below par, and a few days after sold the stock at 5% above par. How much more did he receive for the stock than he gave for it?

3. A certain city borrowed a large sum of money and issued therefor 10-year 5% bonds with the interest payable semi-annually. (a) How many coupons were attached to each bond? (b) On a $1000 bond, each coupon should call for how much money?

4. Sometimes such bonds as those described in problem 3 are offered for sale to the highest bidder, in "blocks" of $10000, $20000, or $50000. If a $20000 "block" is "bid off" at $2\frac{1}{4}$% premium, how much should the city receive for the block"?

5. What must be the nominal value of 5% bonds that will yield to their owner an annual income of $750?

$$\text{Let } x = \text{the nominal value; then } \frac{5x}{100} = \$750.$$

6. What must be the nominal value of 4% bonds that will yield to their owner an annual income of $720?

7. A owns $6000 of 5% bonds; B owns $8000 of $4\frac{1}{2}$% bonds. How much greater is the annual income from B's bonds than from A's?

8. Find the area of a piece of land in the form of a rhomboid, whose base is 32 rods and whose altitude is 15 rods.

* The par value of each share of stock mentioned on this page is $100.

RATIO.

276. **Ratio** is relation by quotient. The two numbers (magnitudes) of which the ratio is to be found are called the **terms of the ratio.** The first term is called the **antecedent** and the second term the **consequent.** The ratio is the quotient of the antecedent divided by the consequent.

The usual sign of ratio is the colon. It indicates that the ratio of the two numbers between which it stands is to be found, the number preceding the colon being the antecedent, and the number following it, the consequent. The expression, $12 : 4 = 3$, is read, the ratio of 12 to 4 is 3.

EXERCISE.

Read and complete the following:

1. $12:4 =$	$4:12 =$	$12:2 =$
2. $18:9 =$	$9:18 =$	$18:6 =$
3. $15:5 =$	$5:15 =$	$15:10 =$

NOTE.—It will be observed that the sign of ratio is the sign of division ($\div$) with the line omitted.

277. *Every integral number is a ratio.* The number 4 is the ratio of a magnitude 4 (inches, ounces, bushels) to the measuring unit 1 (inch, ounce, bushel). The number 7 is the ratio of 7 yards to 1 yard; of 7 dollars to 1 dollar, or of 7 seconds to 1 second, etc.

NOTE.—The ratio aspect of numbers is not the aspect most frequently uppermost in consciousness; neither ought it to be. But the pupil should now see that number *is* ratio; that while it implies aggregation and often stands in consciousness for magnitude, its essence is relation—ratio.

Ratio.

278. *Every fractional number is a ratio.* The fraction $\frac{3}{4}$ is the ratio of the magnitude 3 to the magnitude 4.

So $\frac{12}{4}$, (3), is the ratio of 12 to 4. *Observe* that in every case the terms of a ratio may be written as the terms of a fraction; the antecedent becoming the numerator and the consequent the denominator of the fraction. *The fraction itself is the ratio.*

Exercise I.

Make the terms of the ratio the terms of a fraction; then reduce the fraction to its simplest form.

1. The ratio of 20 to 6 is $\frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$.
2. The ratio of 6 to 20 is $\frac{6}{20} = \frac{3}{10}$.
3. The ratio of 7 to 5 is —; of 5 to 7, —.
4. The ratio of 12 to 1 is —; of 1 to 12, —.

Exercise II.

1. $\frac{5}{7}$ is the ratio of 5 to 7; of 10 to 14; of 15 to 21, etc.
2. $\frac{3}{4}$ is the ratio of — to —; of — to —; of — to —, etc.
3. $\frac{6}{8}$ is the ratio of — to —; of — to —; of — to —, etc.
4. 8 is the ratio of 8 to 1; of — to —; of — to —, etc.

Exercise III.

Make the necessary reduction and find the ratio: *

1. Of 2 feet to 8 inches.
2. Of 3 yards to 6 inches.
3. Of 6 rods to 3 yards.
4. Of 2 rods 5 yards to 1 yard 1 foot.

* The comparison of two magnitudes involves their measurement by the same standard. To compare feet with inches, the inches may be changed to feet or the feet to inches, or both may be changed to yards.

Ratio.

279. Not only is number itself ratio, but a large part of the work in arithmetic is merely the changing of the form of the expression of ratios.

EXERCISE IV.

(Reducing fractions to their lowest terms.)

1. Express the ratio of 30 to 40 in its simplest form.
2. Express the ratio of 560 to 720 in its simplest form.
3. Express the ratio of 425 to 875 in its simplest form.
4. Express the ratio of 5 min. to 2 hours in its simplest form.
5. Express the ratio of 1 lb. 4 oz. to 5 lb. 8 oz. in its simplest form.

EXERCISE V.

(Reducing improper fractions to integers.)

1. Express the ratio of 400 to 50 in its simplest form.
2. Express the ratio of 375 to 25 in its simplest form.
3. Express the ratio of 256 to 16 in its simplest form.
4. Express the ratio of 3 hours 20 minutes to 50 minutes in its simplest form.

EXERCISE VI.

(Reducing complex fractions to simple fractions.)

1. Express the ratio of $\frac{1}{2}$ to $\frac{3}{4}$ in its simplest form.
2. Express the ratio of $\frac{3}{5}$ to $\frac{3}{8}$ in its simplest form.
3. Express the ratio of $2\frac{1}{2}$ to $8\frac{1}{2}$ in its simplest form.
4. Express the ratio of $\frac{3}{4}$ of an inch to 1 foot in its simplest form.

NOTE.—Observe that the denominator in fractions corresponds to the consequent in ratio.

Ratio.

Exercise VII.

(Changing common fractions to decimals.)

1. Express the ratio of 3 to 4 ($\frac{3}{4}$), in hundredths.
2. Express the ratio of 20 to 50, in tenths.
3. Express the ratio of 30 to 80, in thousandths.
4. Express the ratio of 50 sq. rd. to 1 acre 40 rd., in hundredths.

Exercise VIII.

(Finding what per cent one number is of another.)

1. Express the ratio of 15 to 20, in hundredths.
2. Express the ratio of 14 to 200, in hundredths.
3. Express the ratio of 17 to 25, in hundredths.
4. Express the ratio of 16 to $33\frac{1}{3}$, in hundredths.
5. Express the ratio of 27 to 500, in hundredths.

Exercise IX.

(Changing "per cent" to a common fraction in its lowest terms, or to a whole or mixed number.)

1. A's money equals 40% of B's money. (a) Express the ratio of A's money to B's money in the form of a fraction in its lowest terms. (b) Express the ratio of B's money to A's money in its simplest form.

2. One number is 50% more than another number. (a) Express the ratio of the smaller to the larger number in the form of a fraction in its lowest terms. (b) Express the ratio of the larger to the smaller number in its simplest form.

Note.—Observe that the base in percentage corresponds to the consequent in ratio.

Ratio.

Exercise X.

The specific gravity of a liquid or solid is the ratio of its weight to the weight of the same bulk of water.

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. A cubic foot of cork weighs 15 lb. What is the ratio of the weight of the cork to the weight of the water? Express the ratio in hundredths. What is the specific gravity of cork?

2. A certain piece of limestone weighs 37 ounces. Water equal in bulk to the piece of limestone weighs 15 ounces. What is the ratio of the weight of the limestone to the weight of the water? What is the specific gravity of the limestone?

3. A certain bottle holds 10 ounces of water or $9\frac{1}{2}$ ounces of oil. What is the ratio of the weight of the oil to the weight of the water? Express the ratio in hundredths. What is the specific gravity of the oil?

Note.—*Observe* that in specific gravity problems, the weight of water corresponds to the consequent in ratio problems.

280. Miscellaneous Questions.

1. What is the ratio of a unit of the first integral order to a unit of the first decimal order?

2. What is the ratio of a unit of any order to a unit of the next lower order?

3. What ratio corresponds to 6 per cent?

4. What is the ratio of a dollar to a dime? Of a dime to a cent? Of a cent to a mill?

5. What is the ratio of 1 to $\frac{1}{10}$? Of 1 tenth to 1 hundredth?

6. What is the ratio of a rod to a yard? Of a yard to a foot? Of a foot to an inch?

Ratio.

281. Some Old Problems in New Forms.*

1. What is the ratio of the area of a 2-inch square to the area of a 6-in. square?* Of a 6-in. square to a 2-in. square?

2. What is the ratio of the perimeter of a 2-in. square to the perimeter of a 6-in. square? Of the perimeter of a 6-in. square to the perimeter of a 2-in. square?

3. What is the ratio of the area of a 3-ft. square to the area of a 6-ft. square? Of a 6-yd. square to a 3-yd. square?

4. What is the ratio of the perimeter of a 3-ft. square to the perimeter of a 6-ft. square? Of the perimeter of a 6 ft. square to the perimeter of a 3-ft. square?

5. What is the ratio of the solid content of a 2-inch cube to the solid content of a 6-in. cube? Of a 6-in. cube to a 2-in. cube?

6. What is the ratio of the surface of a 2-in. cube to the surface of a 6-in. cube? Of the surface of a 6-in. cube to the surface of a 3-in. cube?

7. What is the ratio of the solid content of a 3-ft. cube to the solid content of a 6-ft. cube? Of a 6-yd. cube to a 3-ft. cube?

8. What is the ratio of the surface of a 3-ft. cube to the surface of a 6-ft. cube? Of the surface of a 6-yd. cube to the surface of a 3-yd. cube?

9. What is the ratio of a square inch to a square foot? Of a cubic inch to a cubic foot?

* If pupils image the magnitudes compared, they will find no difficulty in the solution of these problems.

Algebra.

282. Algebra Applied to Some Problems in Ratio.

Example I.

The consequent is c; the ratio is r. What is the antecedent?

Let $x =$ the antecedent.

Then $\frac{x}{c} = r$.

and $x = cr$, the antecedent.

From the above, learn that the antecedent is always equal to the product of the consequent and the ratio.

1. Consequent 75; ratio 11. Antecedent?
2. Consequent 92; ratio $\frac{3}{4}$. Antecedent?
3. Consequent .56; ratio $\frac{1}{8}$. Antecedent?

Example II.

The antecedent is a; the ratio is r. What is the consequent?

Let $x =$ the consequent.

Then $\frac{a}{x} = r$.

and $a = rx$, or $rx = a$

dividing by r, $x = \frac{a}{r}$, the consequent.

From the above, learn that the consequent is always equal to the quotient of the antecedent divided by the ratio.

1. Antecedent 75; ratio 5. Consequent?
2. Antecedent 96; ratio $\frac{3}{4}$. Consequent?
3. Antecedent $\frac{2}{5}$; ratio $\frac{3}{8}$. Consequent?

283. To Find Two Numbers when their Sum and Ratio are Given.

Example.

The sum of two numbers is 36 and their ratio is 3. What are the numbers?

$$\begin{array}{lrl} \text{Let} & x &= \text{the smaller number.} \\ \text{Then} & 3x &= \text{the larger number,} \\ \text{and} & x + 3x &= 36 \\ & 4x &= 36 \\ & x &= 9, \text{ the smaller number.} \\ & 3x &= 27, \text{ the larger number.} \end{array}$$

Problems.

1. The sum of two numbers is 196, and their ratio is 3. What are the numbers?

2. The sum of two numbers is 294, and their ratio is $2\frac{1}{2}$. What are the numbers?

3. The sum of two decimals is .42, and their ratio is $2\frac{1}{2}$. What are the decimals?

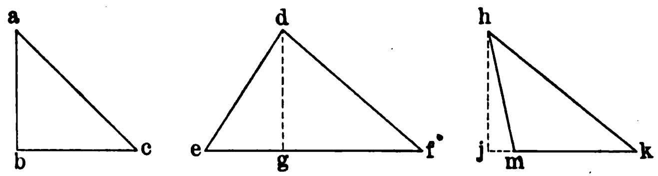
4. The sum of two numbers is s, and the ratio of the larger to the smaller is r. What are the numbers?

$$\begin{array}{lrl} \text{Let} & x &= \text{the smaller number.} \\ \text{Then} & rx &= \text{the larger number,} \\ \text{and} & rx + x &= s \\ \text{or} & (r+1)\,x &= s \\ \text{Dividing,} & x &= \dfrac{s}{r+1}\text{*} \end{array}$$

* Observe that any number you please may be put in the place of s, and any number greater than 1 in the place of r; therefore when the sum of two numbers and the ratio of the larger to the smaller are given, the smaller number may be found by dividing the sum by the ratio plus 1.

Geometry.

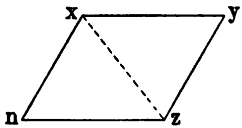
284. The Area of a Triangle.



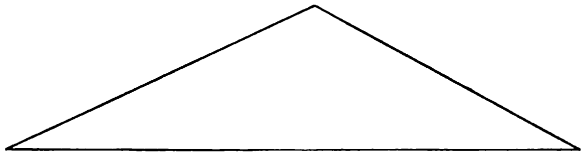
1. One side of a triangle may be regarded as its base. *The perpendicular* distance from its base (or from its base extended) to the opposite angle is its altitude.

2. What is the altitude of the first of the above triangles? Of the second? Of the third?

3. Convince yourself by measurement and by paper cutting that every triangle is one half of a parallelogram having the same base and the same altitude as the triangle.



4. To find the area of a triangle, *Find the area of the parallelogram having the same base and altitude, and take one half of the result.*



PROBLEM.—If the above figure represents a piece of land, and is drawn on a scale of $\frac{1}{8}$ inch to the rod, what part of an acre of land does it represent?

285. Miscellaneous Review.

1. The specific gravity of granite is 2.7.* How much does a cubic foot of granite weigh?

2. A certain vessel is exactly large enough to contain 1000 grains of water. It will contain only 700 grains of petroleum. What is the specific gravity of the petroleum?†

3. The specific gravity of gold is 19.3. How much does a cubic foot of gold weigh?

4. A cubic foot of sulphur weighs 125 lbs. What is the specific gravity of sulphur?

5. A cubic foot of steel weighs 487.5 lbs. What is the specific gravity of steel?

6. What is the ratio of 1 bu. to 1 pk.? Of 1 pk. to 1 qt.?

7. What is the ratio of \$37½ to \$15? Of \$15 to \$37½?

8. What is the area of a rhomboid whose base is 16 inches and whose altitude is 16 inches?

9. Is the rhomboid described in problem 8, equilateral?

10. The ratio of the perimeter of one square to the perimeter of another square is 4. What is the ratio of the areas of the two squares?

11. Draw three triangles, the base of each being 4 inches and the altitude of each being 2 inches. Make one of them a right-triangle; another, an isosceles triangle, and the third having angles unlike either of the other two. What can you say of the area of the right-triangle as compared with each of the others?

* See page 325, exercise 10.

† This means, what is the ratio of the weight of the petroleum to the weight of the same bulk of water?

PROPORTION.

286. The terms of a ratio are together called **a couplet.** Two couplets whose ratios are equal are called **a proportion.**

The two couplets of a proportion are often written thus: $6:18 = 10:30$, and should be read, the ratio of 6 to 18 equals the ratio of 10 to 30.

Couplets are sometimes written thus: $20:4::50:10$, and read, 20 is to 4 as 50 is to 10.*

287. To Find a Missing Term in a Proportion.

Example I. $36:12::x:25$.

The ratio of the first couplet is 3; that is, the antecedent is 3 times the consequent. Since the ratios of the couplets are equal, the ratio of the second couplet must be 3, and its antecedent must be 3 times its consequent. Three times $25 = 75$, the missing term.

Problems.

Find the missing term.

1. $90:45::x:180$.
2. $48:12::x:150$.
3. $75:30::x:140$.
4. $20:60 = x:225$.
5. $30:50 = x:175$.
6. $90:20 = x:140$.

* The ratio sign (:) may be regarded as the sign of division (÷) with the horizontal line omitted, and the proportion sign (::) the sign of equality (=) with an erasure through its center, thus: (= =).

Proportion.

EXAMPLE II. $36:12 = 48:x$.

Since the ratio of the first couplet is 3, the ratio of the second couplet must be 3, and x must equal 1 third of 48. 1 third of 48 is 16.

PROBLEMS.

Find the missing term.

1. $84:21 = 172:x$.
2. $96:16 :: 45:x$.
3. $75:30 = 125:x$.
4. $20:60 :: 120:x$.
5. $25:35 = 45:x$.
6. $50:25 :: 14:x$.

EXAMPLE III. $36:x = 45:15$.

The ratio of each couplet is 3; so each consequent must be 1 third its antecedent, and x, 1 third of 36, or 12.

PROBLEMS.

Find the missing term.

1. $54:x = 90:30$.
2. $75:x :: 125:25$.
3. $50:x = 40:16$.
4. $18:x :: 65:195$.
5. $50:x = 12:18$.
6. $35:x :: 21:3$.

EXAMPLE IV. $x:12 = 100:25$.

The ratio of each couplet is 4; so each antecedent must be 4 times its consequent, and x, 4 times 12, or 48.

PROBLEMS.

Find the missing term.

1. $x:16 :: 51:17$.
2. $x:22 = 76:19$.
3. $x:11 :: 24:3$.
4. $x:96 = 23:92$.
5. $x:40 :: 36:48$.
6. $x:27 = 42:14$.

288. Practical Problems.

1. If 75 yd. of cloth cost $115.25, how much will 15 yd. cost at the same rate?

$$75 \text{ yd.} : 15 \text{ yd.} = \$115.25 : x.$$

2. If $2\frac{1}{2}$ acres of land cost $76.20, how much will 15 acres cost at the same rate?

3. If 7 tons of coal can be bought for $26, how many tons can be bought for $39?

$$7 \text{ tons} : x \text{ tons} :: \$26 : \$39.$$

4. If 36 lb. coffee can be bought for $7, how many pounds can be bought for $$17\frac{1}{2}$?

5. If sugar sells at the rate of 18 lb. for $1, how much should 63 lb. of sugar cost?

6. If a post 6 ft. high casts a shadow 4 feet long, how high is that telegraph pole which at the same time and place casts a shadow 20 feet long?

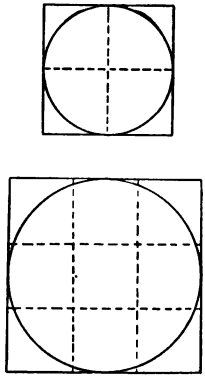
7. If a post 5 feet high casts a shadow 8 feet long, how high is that steeple which casts a shadow 152 feet long?

8. If a train moves 50 miles in 1 hr. 20 min., at the same rate how far would it move in 2 hours?

9. If a boy riding a bicycle at a uniform rate goes 12 miles in 1 hr. 15 min., how far does he travel in 25 minutes?

TO THE TEACHER.—After the pupil has solved the above problems by making use of the fact of the equality of the ratios, he should solve them by an analysis somewhat as follows: Prob. 1. Since 75 yd. cost $115.25, 1 yd. costs $\frac{1}{75}$ of $115.25; but 15 yd. cost 15 times as much as 1 yd., so 15 yd. cost 15 times $\frac{1}{75}$ of $115.25.

289. Magnitudes Which Are Proportional to the Squares of Other Magnitudes.



The areas of two squares are to each other as the squares of their lengths.

The areas of two circles are to each other as the squares of their diameters.

Observe that the ratio of the areas of the above squares is $\frac{4}{9}$ (or $\frac{2^2}{3^2}$). But the area of each circle is about $\frac{3}{4}$ (more accurately, .785+) of its circumscribed square; so the ratio of the areas of the circles is $\frac{4}{9}$ (or $\frac{2^2}{3^2}$).

1. The area of a 6-inch circle is how many times as great as the area of a 3-inch circle?

2. If a 4-inch circle of brass plate weighs 3 ounces, how much will a 6-inch circle weigh, the thickness being the same in each case?

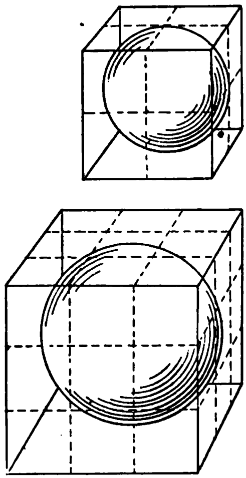
3. If a piece of rolled dough 1 foot in diameter is enough for 17 cookies, how many cookies can be made from a piece 2 feet in diameter, the thickness of the dough and the size of the cookies being the same in each case?

4. If a piece of wire $\frac{1}{8}$ of an inch in diameter will sustain a weight of 1000 lbs., how many pounds will a wire $\frac{1}{4}$ of an inch in diameter sustain?

Proportion.

290. Magnitudes Which Are Proportional to the Cubes of Other Magnitudes.

The solid contents of two cubes are to each other as the cubes of their lengths.



The solid contents of two spheres are to each other as the cubes of their diameters.

Observe that the ratio of the solid contents of the above cubes is $\frac{8}{27}$ (or $\frac{27}{8}$). But the solid content of each sphere is about $\frac{1}{2}$ (more accurately, .5236—) of its circumscribed cube; so the ratio of the solid contents of the spheres is $\frac{8}{27}$ (or $\frac{27}{8}$).

1. The solid content of a 6-inch sphere is how many times as great as the solid content of a 3-inch sphere?

2. If a 4-inch sphere of brass weighs 10 lbs., how many pounds will a 6-inch sphere of brass weigh?

3. If a sphere of dough 1 foot in diameter is enough for 20 loaves of bread, how many loaves can be made from a sphere of dough 2 feet in diameter?

4. If the half of a solid 8-inch globe weighs 4 lbs., how much will the half of a solid 5-inch globe weigh, the material being of the same quality?

291. Magnitudes Which Are Inversely Proportional to Other Magnitudes or to the Squares of Other Magnitudes.

Example.

If 5 men do a piece of work in 16 days, how long will it take 8 men to do a similar piece of work?

Operation and Explanation.

It is evident that the time required will be inversely proportional to the number of men employed; that is, if twice as many men are employed, not twice as much, but $\frac{1}{2}$ as much time will be required. Hence the proportion is not $5:8=16:x$, but, $5:8=x:16$; hence, $5:8=10:16$.

The interpretation of the above equation is, if 5 men can do a piece of work in 16 days, 8 men can do it in 10 days.

1. If 4 men can do a piece of work in 20 days, how long will it take 5 men to do a similar piece of work?

2. If 8 men can do a piece of work in 12 days, how long will it take 3 men to do a similar piece of work?

It can be shown that the intensity of light upon an object diminishes as the square of the distance between the luminous body and the illuminated object increases; that is, if the distance be twice as great in one case as in another, the intensity is not twice as great, not $\frac{1}{2}$ as great, but $\frac{1}{4}$ as great; if the distances are as 2 to 3 the intensities are, not as 2 to 3, not as 3 to 2, but as 9 to 4. The intensity at 2 feet is $\frac{9}{4}$ as great as at 3 feet.

3. Object A is 15 feet from an incandescent electric light. Object B is 20 feet from the same light. Object C is 30 feet from the same light. (a) How does the intensity of the light at B compare with the intensity at A? (b) How does the intensity at C compare with the intensity at A?

Algebra.

292. TO FIND THE MISSING TERM OF A PROPORTION WITHOUT FINDING THE RATIO.

The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means;** thus, in the proportion $12:6 = 8:4$, 12 and 4 are the extremes and 6 and 8 are the means.

Observe that in the following proportions the product of the means equals the product of the extremes:

$$6:3 = 8:4; \text{ then } 6 \times 4 = 3 \times 8$$
$$\tfrac{1}{2}:\tfrac{1}{4} = 4:2; \text{ then } \tfrac{1}{2} \times 2 = \tfrac{1}{4} \times 4$$

Let $a:b = c:d$, stand for any proportion.

Then $$\frac{a}{b} = \frac{c}{d}$$

Clearing of fractions, $ad = bc$

But a and d are the extremes and b and c the means; hence, in any proportion in which abstract numbers are em-employed, *the product of the means equals the product of the extremes.*

EXAMPLE I.

$$30:20 = 18:x.$$

$30x =$ the product of the extremes.

$20 \times 18 =$ the product of the means.

Then $30x = 20 \times 18$, or 360,

and $x = 12.$

EXAMPLE II.

$$10:25 = x:50.$$

Then $25x = 10 \times 50$, or 500,

and $x = 20.$

Algebra.

EXAMPLE III.

$$40 : x = 25 : 5.$$

Then $25x = 40 \times 5$, or 200,

and $x = 8$.

EXAMPLE IV.

$$x : 35 = 4 : 28.$$

Then $28x = 35 \times 4$, or 140,

and $x = 5$.

Find the missing terms:

1. $24 : 72 = x : 69.$
2. $45 : 12 = 75 : x.$
3. $35 : x = 14 : 40.$
4. $x : 70 = 3 : 21.$
5. $55 : 25 = x : 10.$
6. $\frac{2}{3} : 4 = x : 30.$
7. $x : \frac{3}{4} = 40 : 6.$
8. $\frac{1}{2} : \frac{2}{5} = x : 8.$
9. $.6 : .8 = 15 : x.$
10. $.25 : 5 = x : 40.$

11. If 8 acres of land cost $360, how much will 15 acres cost at the same rate?

$$8 : 15 = 360 : x.*$$

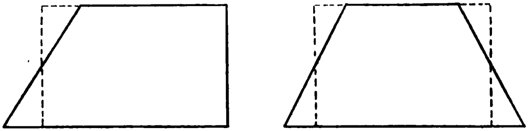
12. If 12 horses consume 3500 lb. of hay per month, how many pounds will 15 horses consume?

13. If 11 cows cost $280.50, how many cows can be bought for $433.50 at the same rate?

* Observe that in the solution of concrete problems by the method here given the numbers must be regarded as abstract. It would be absurd to talk or think of finding the product of 15 acres and 360 dollars and dividing this by 8 acres. It is true, however, that the ratio of 8 acres to 15 acres equals the ratio of 360 dollars to x dollars. It is also true that in the proportion $8 : 15 = 360 : x$, the product of the means is equal to the product of the extremes.

Geometry.

293. The Area of a Trapezoid.



1. Convince yourself by measurement and by paper cutting that from every trapezoid there may be cut a triangle (or triangles) which when properly adjusted to another part (or parts) of the trapezoid, will convert the trapezoid into a rectangle.

2. Convince yourself that the rectangle made from a trapezoid is not so long as the longer of the parallel sides of the trapezoid, and not so short as the shorter of the parallel sides of the trapezoid—that *its length is midway between the lengths of the two parallel sides of the trapezoid.*

Note.—*Observe* that the length of the rectangle thus formed may be found by adding half the difference of the parallel sides of the trapezoid to its shorter side, or by dividing the sum of its parallel sides by 2.

3. To find the area of a trapezoid, *find the area of the rectangle to which it is equivalent*, or, as the rule is usually given,—"*Multiply one half the sum of the parallel sides by the altitude.*"

4. Find the area of a trapezoid whose parallel sides are 10 inches and 15 inches respectively, and whose altitude is 8 inches.

5. How many acres in a trapezoidal piece of land, the parallel sides being 28 rods and 36 rods respectively, and the breadth (altitude) 25 rods?

294. Miscellaneous Review.

1. If 3 men can build 72 feet of sidewalk in a day, how many feet can 4 men build?

2. If 3 men can do a piece of work in 12 hours, in how many hours can 4 men do an equal amount of work?

3. If a piece of land 8 rods square is worth $500, how much is a piece of land 16 rods square worth at the same rate?

4. If a ball of yarn 3 inches in diameter is enough for a pair of stockings, how many pairs of stockings can be made from a ball 6 inches in diameter? *

5. If a grindstone 12 inches in diameter weighs 40 lb., how much will a grindstone 18 inches in diameter weigh, the thickness and quality of material being the same?

6. The opening in an 8-inch drain tile is how many times as large as the opening in a 2-inch drain tile? †

7. Find the area of a rhomboidal piece of land whose length (base) is 64 rods and whose width (altitude) is 15 rods.

8. Find the area of a trapezoidal piece of land, the length of the parallel sides being 44 rods and 52 rods respectively, and the width (altitude) being 18 rods. ‡

9. Find the area of a triangular piece of land whose base is 42 rods and whose altitude is 20 rods.

* Compare a 3-inch cube and a 6-inch cube. Remember that a 3-inch sphere is a little more than half of a 3-inch cube, and a 6-inch sphere a little more than one half of a 6-inch cube.

† Compare a 6-inch square with a 2-inch square. Remember that a 2-inch circle is about $\frac{3}{4}$ of a 2-inch square, and an 8-inch circle about $\frac{3}{4}$ of an 8-inch square.

‡ Draw a diagram of the land on a scale of $\frac{1}{8}$ inch to the rod.

POWERS AND ROOTS.

295. A product obtained by using a number twice as a factor is called **the second power** or **the square** of the number; thus, 25, (5×5), is the second power, or the square of 5.

NOTE.—Twenty-five is called the *second* power of 5, because it may be obtained by using 5, *twice* as a factor. It is called the *square* of 5, because it is the number of square units in a square whose side is 5 linear units.

1. What is the second power of 2? 8? 3? 5?
2. What is the square of 4? 7? 1? 6? 9? 10?

$11^2 = ?$ $12^2 = ?$ $13^2 = ?$ $14^2 = ?$

$15^2 = ?$ $16^2 = ?$ $17^2 = ?$ $18^2 = ?$

(a) Find the sum of the eighteen squares.

296. The **square root** of a number is one of the two equal factors of the number.

The radical sign, $\sqrt{}$, (without a figure above it) indicates that the square root of the number following it, is to be taken; thus $\sqrt{64}$, means the square root of 64.

1. What is the square root of 144? 81? 49?
2. What is the square root of 36? 25? 16?

$\sqrt{9} = ?$ $\sqrt{64} = ?$ $\sqrt{121} = ?$ $\sqrt{100} = ?$

$\sqrt{4} = ?$ $\sqrt{1} = ?$ $\sqrt{400} = ?$ $\sqrt{169} = ?$

(b) Find the sum of the fourteen results.

Powers and Roots.

297. Any number that can be resolved into two equal factors is a **perfect square.**

1. Tell which of the following are perfect squares and which are not:

9, 10, 12, 16, 18, 25, 32, 36.

NOTE.—It is a curious fact that no number, either integral or mixed, can be found which, when multiplied by itself, will give as a product 10, or 12, or 14, or any number that is not a *perfect square.*

2. Any integral number that is a perfect square is composed of an even number of like prime factors; that is, its prime factors are an even number of 2's, 3's, 5's, 7's, etc.

3. Tell which of the following are perfect squares:

144, $(2 \times 2 \times 2 \times 2 \times 3 \times 3)$; 250, $(2 \times 5 \times 5 \times 5)$; 225, $(5 \times 5 \times 3 \times 3)$.

RULE.—*To find the square root of an integral number, that is a perfect square, resolve the number into its prime factors and take half of them as factors of the root; that is, one half as many 2's, 3's, or 5's, etc., as there are 2's, 3's, or 5's, etc., in the factors of the number.*

4. Find the square root of 1225.

$$1225 = 5 \times 5 \times 7 \times 7. \qquad \sqrt{1225} = 5 \times 7 = 35.$$

5. Find the square root of 441; of 400.
6. Find the square root of 576; of 324.
7. Find the square root of 784; of 2025.
8. Find the square root of 625; of 3025.

(a) Find the sum of the last eight results.

Powers and Roots.

298. The Square of Common Fractions.

1. The square of $\frac{1}{2}$, $(\frac{1}{2} \times \frac{1}{2})$, is — —.

NOTE.—A square whose side is $\frac{1}{2}$ (of a linear unit) has an area of $\frac{1}{4}$ (of a square unit). Show this by diagram.

2. Answer the following and illustrate by diagram if necessary:

$(\frac{1}{3})^2 = ?$ $(\frac{1}{4})^2 = ?$ $(\frac{1}{5})^2 = ?$ $(\frac{1}{6})^2 = ?$

$(\frac{2}{3})^2 = ?$ $(\frac{3}{4})^2 = ?$ $(\frac{2}{5})^2 = ?$ $(\frac{5}{6})^2 = ?$

(a) Find the sum of the eight results.

3. A square of sheet brass whose edge is $\frac{5}{12}$ of a foot is what part of a square foot?

299. The Square Root of Common Fractions.

1. The square root of $\frac{9}{16}$ is — —.

NOTE 1.—A square whose area is $\frac{9}{16}$ (of a square unit) is $\frac{3}{4}$ (of a linear unit) in length. Show this by diagram.

NOTE 2.—Only those fractions are perfect squares which, when in their lowest terms, have perfect squares for numerators and perfect squares for denominators.

2. What is the square root of $\frac{25}{36}$? Of $\frac{16}{25}$? Of $\frac{1}{4}$?

$\sqrt{\frac{144}{25}} = ?$ $\sqrt{\frac{169}{36}} = ?$ $\sqrt{\frac{81}{16}} = ?$ $\sqrt{\frac{121}{144}} = ?$

(b) Find the sum of the seven results.

3. The area of a square piece of sheet brass is $\frac{81}{144}$ of a square foot. What is the length of the side of the square?

4. How long is the side of a square of zinc the area of which is $\frac{25}{36}$ of a square yard?

Powers and Roots.

300. The Square of Decimals.

1. The square of .5 is — —.

Note.—A square whose side is .5 (of a linear unit) has an area of .25 (of a square unit). Show this by diagram.

2. Answer the following and illustrate by diagram if necessary:

$.1^2 = ?$ $.2^2 = ?$ $.3^2 = ?$ $.4^2 = ?$

$.5^2 = ?$ $.6^2 = ?$ $.7^2 = ?$ $.8^2 = ?$

$1.2^2 = ?$ $1.5^2 = ?$ $1.6^2 = ?$ $1.8^2 = ?$

(a) Find the sum of the twelve results.

3. A square of sheet brass whose edge is .9 of a foot is what part of a square foot?

301. The Square Root of Decimals.

1. The square root of .25 is — —.

Note 1.—A square whose area is .25 (of a square unit) is .5 (of a linear unit) in length. Show this by diagram.

Note 2.—Only those decimals are perfect squares which, when in their lowest decimal terms, have numerators that are perfect squares and denominators that are perfect squares. The decimal denominators that are perfect squares are 100, 10000, 1000000, etc.

2. What is the square root of $\frac{49}{100}$? Of .36? Of .64?

$\sqrt{\frac{144}{100}} = ?$ $\sqrt{1.44} = ?$ $\sqrt{2.25} = ?$ $\sqrt{6.25} = ?$

(b) Find the sum of the seven results.

3. How long is the edge of a square of zinc whose area is 4.84 square feet?*

*4.84 feet is $\frac{484}{100}$ feet.

Powers and Roots.

302. A product obtained by using a number three times as a factor is called **the third power,** or **the cube,** of the number; thus, 125 ($5 \times 5 \times 5$) is the third power, or the cube, of 5.

NOTE.—One hundred twenty-five is called the *third* power of 5, because it may be obtained by using 5 *three* times as a factor. It is called the *cube* of 5 because it is the number of *cubic units* in a cube whose edge is 5 linear units.

$1^3 = 1$ $3^3 = 27$ $5^3 = 125$ $7^3 = 343$ $9^3 = 729$

1. Find the cube of 12; of 13; of 14; of 15.

$16^3 = ?$ $17^3 = ?$ $18^3 = ?$ $19^3 = ?$ $20^3 = ?$

(a) Find the sum of the nine results.

303. The **cube root** of a number is one of its three equal factors.

The radical sign with a figure 3 over it indicates that the cube root of the number following it is to be taken; thus, $\sqrt[3]{512}$, means, the cube root of 512.

RULE.—*To find the cube root of an integral number that is a perfect cube, resolve the number into its prime factors and take one third of them as factors of the root; that is, one third as many 2's, 3's, or 5's, etc., as there are 2's, 3's, or 5's in the factors of the number.*

1. Find the cube root of 216.

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3. \quad \sqrt[3]{216} = 2 \times 3$$

2. Find the cube root of 1728; of 3375; of 2744; of 10648; of 5832.

(b) Find the sum of the five results.

Powers and Roots.

304. Miscellaneous Problems.

1. Square 42. Then resolve the square of 42 into its prime factors and compare them with the prime factors of 42.

2. Cube 42. Then resolve the cube of 42 into its prime factors and compare them with the prime factors of 42.

3. Square 45. Then resolve the square of 45 into its prime factors and compare them with the prime factors of 45.

4. Cube 45. Then resolve the cube of 45 into its prime factors and compare them with the prime factors of 45.

5. Divide the cube of 15 by the square of 15.

6. Divide the cube of $\frac{3}{4}$ by the square of $\frac{3}{4}$.

7. Divide the cube of .7 by the square of .7.

8. Divide the cube of 2.5 by the square of 2.5.

9. Find the square root of $5 \times 5 \times 7 \times 7$.

10. Find the cube root of $3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$.

11. Find the square root of each of the following perfect squares:

(1) 3025	(2) 4225	(3) 5625	(4) 7225
(5) $\frac{144}{625}$	(6) $\frac{25}{576}$	(7) $\frac{49}{324}$	(8) $\frac{16}{529}$
(9) .64	(10) .0064	(11) .0625	(12) 2.56 *

12. Find the cube root of each of the following perfect cubes:

(1) 1728	(2) 15625	(3) 3375	(4) 9261
(5) $\frac{8}{125}$	(6) $\frac{27}{512}$	(7) $\frac{64}{343}$	(8) $\frac{1}{729}$

* Think of this number as $\frac{256}{100}$.

Algebra.

305. To Find the Square Root of Numbers Represented by Letters and Figures.

Explanation.

Since the square root of a number is one of its two equal factors, the square root of a^4, $(a \times a \times a \times a)$, is a^2, $(a \times a)$. The square root of a^2 is a. The square root of a^6 is a^3. Let $a = 3$, and verify each of the foregoing statements.

1. $\sqrt{b^4} = ?$ $\sqrt{b^6} = ?$ $\sqrt{b^2} = ?$ Verify.

2. $\sqrt{a^2b^2} = ?$ $\sqrt{a^2b^4} = ?$ $\sqrt{a^4b^4} = ?$ Verify.

3. $\sqrt{4a^2b^2} = ?$ $\sqrt{9a^4b^2} = ?$ $\sqrt{16a^2b^4} = ?$

4. $\sqrt{25x^2y^2} = ?$ $\sqrt{36x^2y^4} = ?$ $\sqrt{49x^4y^4} = ?$

5. Let $a = 2$, $b = 3$, and $c = 5$, and find the numerical value of each of the following:

(1) $\sqrt{a^2b^4}$ (2) $\sqrt{a^4c^2}$ (3) $\sqrt{b^2c^4}$

(4) $\sqrt{a^4b^4}$ (5) $\sqrt{a^2b^2c^2}$ (6) $\sqrt{a^2b^4c^2}$

(a) Find the sum of the six results.

6. Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and find the numerical value of each of the following:

(1) $a\sqrt{x^2y^2}$ (2) $b\sqrt{x^4y^2}$ (3) $a\sqrt{x^4y^4}$

(4) $3a\sqrt{x^4}$ (5) $4b\sqrt{y^4}$ (6) $5a\sqrt{x^4y^4}$

(b) Find the sum of the six results.

Algebra.

306. To Find the Cube Root of Numbers Represented by Letters and Figures.

Explanation.

Since the cube root of a number is one of its three equal factors, the cube root of a^6, $(a \times a \times a \times a \times a \times a)$, is a^2, $(a \times a)$. The cube root of a^3 is a. The cube root of a^9 is a^3. Let $a = 2$, and verify each of the foregoing statements.

1. $\sqrt[3]{b^6} = ?$ $\quad \sqrt[3]{b^3} = ?$ $\quad \sqrt[3]{b^9} = ?$ $\quad$ Verify.

2. $\sqrt[3]{a^3b^3} = ?$ $\quad \sqrt[3]{a^6b^3} = ?$ $\quad \sqrt[3]{a^6b^6} = ?$ $\quad$ Verify.

3. $\sqrt{8a^6} = ?$ $\quad \sqrt[3]{27b^3} = ?$ $\quad \sqrt{64a^6} = ?$ $\quad$ Verify.

4. Let $a = 2$, $b = 3$, and $c = 5$, and find the numerical values of each of the following:

(1) $\sqrt[3]{a^3b^6}$ $\quad$ (2) $\sqrt[3]{8c^3}$ $\quad$ (3) $\sqrt[3]{27b^3}$

(4) $2\sqrt[3]{a^6b^3}$ $\quad$ (5) $3\sqrt{b^3c^3}$ $\quad$ (6) $4\sqrt[3]{b^6c^3}$

(a) Find the sum of the six results.

307. Miscellaneous Problems.

Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and find the numerical value of each of the following:

(1) $ab + \sqrt{x^2y^4}$ $\quad$ (2) $ab\sqrt{x^2y^4}$ $\quad$ (3) $2a\sqrt{x^4y^4}$

(4) $ab + \sqrt[3]{x^3y^6}$ $\quad$ (5) $ab\sqrt[3]{x^3y^6}$ $\quad$ (6) $2b\sqrt[3]{x^6y^6}$

(7) $\frac{1}{4}\sqrt{a^4b^2}$ $\quad$ (8) $\sqrt{\frac{1}{4}a^4b^2}$* $\quad$ (9) $\frac{3}{4}\sqrt{a^4x^2}$

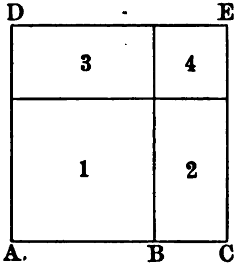
(b) Find the sum of the nine results.

* The factors of this number are $\frac{1}{2}$, $\frac{1}{2}$, a, a, a, a, b, b.

Geometry.

308. The Square of the Sum of Two Lines.

1. Study the diagram and *observe*—



(1) That the line AC is the sum of the lines AB and BC.

(2) That the square, 1, is the square of AB.

(3) That the rectangle, 2, is as long as AB and as wide as BC.

(4) That the rectangle, 3, is as long as AB and as wide as BC.

(5) That the square, 4, is the square of BC.

(6) That the square, $ACED$, is the square of the sum of AB and BC.

2. Since a similar diagram may be drawn with any two lines as a base, the following general statement may be made:

The square of the sum of two lines is equivalent to the square of the first plus twice the rectangle of the two lines plus the square of the second.

3. If the line AB is 10 inches and the line BC 5 inches, how many square inches in each part of the diagram, and how many in the sum of the parts?

4. Suppose the line AB is equal to the line BC; what is the shape of 2 and 3?

5. In the light of the above diagram study the following:

$14^2 = 196.$ $(10 + 4)^2 = 10^2 + 2\,(10 \times 4) + 4^2 = 196.$

$16^2 = ?$ $(10 + 6)^2 =$ ____________________

$25^2 = ?$ $(20 + 5)^2 =$ ____________________

309. Miscellaneous Review.

1. What is the square root of a^2b^2?
What is the square root of $3 \times 3 \times 5 \times 5$?

2. What is the cube root of a^3b^3?
What is the cube root of $2 \times 2 \times 2 \times 7 \times 7 \times 7$?

3. What is the square root of a^2b^4?
What is the square root of $5^2 \times 3^4$?

4. What is the cube root of a^6b^6?
What is the cube root of $3^6 \times 5^6$?

5. The area of a certain square floor is 784 square feet. How many feet in the perimeter of the floor?

6. The area of a certain square field is 40 acres. How many rods of fence will be required to enclose it?

7. The solid content of a certain cube is 216 cubic inches. How many square inches in one of its faces?

8. If there are 64 square inches in one face of a cube, how many cubic inches in its solid content?

9. The square of $(30 + 5)$ is how many more than the square of 30 plus the square of 5?

10. The square of $(40 + 3)$ is how many more than the square of 40 plus the square of 3?

11. The square of a is a^2; the square of $2a$ is $4a^2$. The square of two times a number is equal to how many times the square of the number itself?

12. The square of an 8-inch line equals how many times the square of a 4-inch line?

SQUARE ROOT.

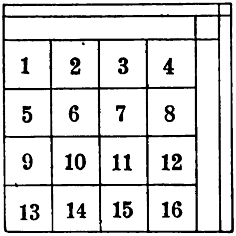
310. To Find the **Approximate Square Root** of Numbers that are not Perfect Squares.

Find the square root of 1795.

Regard the number as representing 1795 1-inch squares. These are to be arranged in the form of a square, and the length of its side noted. 100 1-inch squares = 1 10-inch square.

1700 1-inch squares = 17 10-inch squares.

But 16 of the 17 10-inch squares can be arranged in a square that is 4 by 4; that is, 40 inches by 40 inches. See diagram.



After making this square (40 inches by 40 inches) there are (1700 − 1600 + 95) 195 1-inch squares remaining. From these, additions are to be made to two sides of the square already formed. Each side is 40 inches; hence the additions must be made upon a base line of 80 inches. These additions can be as many inches wide as 80 is contained times in 195.* 195 ÷ 80 = 2+. The additions are 2 inches wide. These will require 2 times 80, + 2 times 2, = 164 square inches.

After making this square (42 in. by 42 in.) there are (195 − 164) 31 square inches remaining. If further additions are to be made to the square, the 31 square inches must be changed to tenth-inch squares. In each 1-inch square there are 100 tenth-inch squares; in 31 square inches there are 3100 tenth-inch squares. From these, additions are to be made upon two sides of the 42-inch square. 42 inches equal 420 tenth-inches. The additions must be made upon a base line (420 × 2) 840 tenth-inches long. These additions can be as many tenth-inches wide as 840 is contained times in 3100. 3100 ÷ 840 = 3+. The additions are 3 tenth-inches wide. These will require 3 times 840, + 3 times 3, = 2529 tenth-inch squares.

After making this square (42.3 by 42.3) there are (3100 − 2529) 571 tenth-inch squares remaining. (If further additions are to be made to the square, the 571 tenth-inch squares must be changed to hundredth-inch squares.) The square root of 1795, true to tenths, is 42.3.

* Allowance must be made for filling the little square shown at the upper right-hand corner of the diagram.

Square Root.

NOTE.—Pupils who have mastered the work on the preceding page will have no difficulty in discovering that the same result may be obtained by the following process:

Find the square root of 1795.

Operation.

$$
\begin{array}{rr|l}
 & \multicolumn{2}{l}{\overset{\frown}{17}\overset{\frown}{95}.^{\vee}(42.36} \\
 & \multicolumn{2}{l}{16} \\
\hline
40 \times 2 = & 80^{*} & 195 \\
 & 2 & 164 \\
\hline
420 \times 2 = & 840 & 3100 \\
 & 3 & 2529 \\
\hline
4230 \times 2 = & 8460 & 57100 \\
 & 6 & 50796 \\
\hline
 & & 6304
\end{array}
$$

Rule.

1. *Beginning with the decimal point, group the figures as far as possible into periods of two figures each.*

2. *Find the largest square in the left-hand period*† *and place its root at the right as the first figure of the complete root.*

3. *Subtract the square from the left-hand period and to the difference annex the next period. Regard this as a dividend.*

4. *Take* 2 *times* 10 *times the root already found as a trial divisor, and find how many times it is contained in the dividend. Write the quotient as the second figure of the root, and also as a part of the divisor. Multiply the entire divisor by the second figure of the root, subtract the product from the dividend, and proceed as before.*

PROBLEMS.

Find the approximate square root (true to tenths):

(1) 875.	(2) 1526.	(3) 2754.	(4) 4150.
(5) 624.	(6) 624.7.	(7) 62.47.	(8) 6.24.

(a) Find the sum of the eight results.

* The entire divisor is 80 and 2; that is, 82.

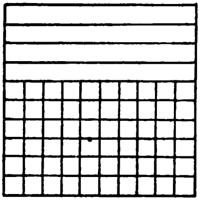
† The left-hand period may consist of either one or two figures.

Square Root.

311. To Find the Approximate Square Root of Decimals That Are Not Perfect Squares.

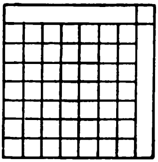
Find the square root of .6.

Regard the number as representing .6 of a 1-inch square. .6 of a 1-inch square = 60 tenth-inch squares.



But 49 of the 60 tenth-inch squares can be arranged in a square that is 7 by 7; that is, 7 tenths of an inch by 7 tenths of an inch.

After making this square there are (60 − 49) 11 tenth-inch squares remaining. If additions are to be made to the square, the 11 tenth-inch squares must be changed to hundredth-inch squares. In each tenth-inch square there are 100 hundredth-inch squares; in 11 tenth-inch squares there are 1100 hundredth-inch squares. From these, additions are to be made upon two sides of the .7-inch square. .7 = 70 hundredths. The additions must be made upon a base line (70×2) 140 hundredth-inches long. These additions can be as many hundredth-inches wide as 140 is contained times in 1100. $1100 \div 140 = 7+$. The additions are 7 hundredth-inches wide. These will require 7 times 140, + 7 times 7, = 1029 hundredth-inch squares.



After making this square (.77 by .77) there are (1100 − 1029) 71 hundredth-inch squares remaining. (If further additions are to be made to the square the 77 hundredth-inch squares must be changed to thousandth-inch squares.) The square root of .6, true to hundredths, is .77.

Note.—The work on this page should be first presented orally by the teacher. It must be given very slowly. Great care must be taken that pupils image each magnitude when its word-symbol is spoken by the teacher. Any attempt to move forward more rapidly than this can be done by the slowest pupil will result in failure so far as that pupil is concerned.

Square Root.

NOTE.—Pupils who have mastered the work on the preceding page will readily understand the following process. See rule on page 352.

1. Find the square root of .6.

Operation.

$$
\begin{array}{r|l}
 & .\overset{\frown}{60}\overset{\frown}{00}(.774 \\
 & 49 \\
\hline
70 \times 2 = 140 & 1100 \\
7 & 1029 \\
\hline
770 \times 2 = 1540 & 7100 \\
4 & 6176 \\
\hline
 & 924
\end{array}
$$

Observe—

1. *That in grouping decimals for the purpose of extracting the square root it is necessary to begin at the decimal point.*

2. *That the square root of any number of hundredths is a number of tenths; the square root of any number of ten-thousandths is a number of hundredths, etc.*

2. Find the square root of 54264.25.

$$
\begin{array}{r|l}
 & \overset{\frown}{5}\overset{\frown}{42}\overset{\frown}{64}.\overset{\frown}{25}(232.946 \\
 & 4 \\
\hline
20 \times 2 = 40 & 142 \\
3 & 129 \\
\hline
230 \times 2 = 460 & 1364 \\
2 & 924 \\
\hline
2320 \times 2 = 4640 & 440.25 \\
9 & 418.41 \\
\hline
23290 \times 2 = 46580 & 21.8400 \\
4 & 18.6336 \\
\hline
232940 \times 2 = 465880 & 3.206400 \\
6 & 2.795316 \\
\hline
 & .411084
\end{array}
$$

Observe that the trial divisor is always 2 times 10 times the part of the root already found.

Square Root.

312. The following numbers are perfect squares. Find their square roots by both the factor method and the method given on the four preceding pages.

(1) 6889	(2) 841	(3) 71824
(4) 1849	(5) 729	(6) 60516

(a) Find the sum of the six results.

(7) $\frac{81}{625}$	(8) $\frac{289}{625}$	(9) $\frac{576}{625}$
(10) $\frac{4489}{625}$	(11) $\frac{2916}{625}$	(12) $\frac{900}{625}$

(b) Find the sum of the six results.

(13) .81	(14) .0625	(15) .04
(16) 1.21	(17) .7921	(18) .0004

(c) Find the sum of the six results.

313. Miscellaneous.

1. The square of a number represented by one digit gives a number represented by —— or —— digits.

2. The square of a number represented by two digits gives a number represented by —— or —— digits.

3. The square of a number represented by three digits gives a number represented by —— or —— digits.

4. The square root of a perfect square represented by one or two digits is a number represented by ——— digit.

5. The square root of a perfect square represented by three or four digits is a number represented by ——— digits.

Square Root.

314. Miscellaneous Problems.

1. What is one of the two equal factors of 9216?

2. What is one of the four equal factors of 20736? *

3. If 7921 soldiers were arranged in a solid square, how many soldiers would there be on each side?

4. How many rods of fence will enclose a square field whose area is 40 acres?

5. How many rods long is one side of a square piece of land containing exactly one acre? †

6. If the surface of a cubical block is 150 square inches, what is the length of one edge of the cube?

7. How many rods of fence will enclose a square piece of land containing 4 acres 144 square rods?

8. Find the side of a square equal in area to a rectangle that is 15 ft. by 60 ft.

9. Compare the amount of fence required to enclose two fields each containing 10 acres: one field is square, and the other is 50 rods long and —— rods wide.

10. Find the area of the largest possible rectangle having a perimeter of 40 feet.

11. If a square piece of land is $\frac{1}{4}$ of a square mile, how much fence will be required to enclose it?

* To find one of the four equal factors of a number (the 4th root) extract the square root of the square root. Why? What is the fourth root of 81?

† Find the answer to problem 5, true to hundredths of a rod.

Algebra.

315. SQUARE ROOT AND AREA.

1. If a piece of land containing 768 square rods is three times as long as it is wide, how wide is it? *

$$\begin{aligned}
&\text{Let} && x = \text{the width,}\\
&\text{then} && 3x = \text{the length,}\\
&\text{and} && x(3x) \text{ or } 3x^2 = \text{the area.}\\
& && 3x^2 = 768\\
& && x^2 = 256\\
& && x = 16
\end{aligned}$$

2. If a certain room is twice as long as it is wide, and the area of the floor 968 square feet, what is the length and the breadth of the room?

3. One half of the length of Mr. Smith's farm is equal to its breadth. The farm contains 80 acres. How many rods of fence will be required to enclose it?

4. Each of four of the faces of a square prism is an oblong whose length is twice its breadth. The area of one of these oblongs is 72 square inches. What is the solid content of the prism.

5. The width of a certain field is to its length as 2 to 3. Its area is 600 square rods. The perimeter of the field is how many rods?

6. If $\frac{3}{4}$ of the length of an oblong equals the width and its area is 768 square inches, what is the length of the oblong?

7. If to $2\frac{1}{2}$ times the square of a number you add 15 the sum is 375. What is the number?

* To solve this problem arithmetically, one must discern that this piece of land can be divided into three equal squares, the side of each square being equal to the width of the piece.

Algebra.

316. Square Root and Proportion.

When the same number forms the second and the third term of a proportion it is called a **mean proportional**, of the first and the fourth term; thus, in the proportion $3:6::6:12$, 6 is a mean proportional of 3 and 12.

Example.

In the proportion $12:x::x:75$, find the value of x.

Since the product of the means equals the product of the extremes, x times x equals 12 times 75, or,

$$x^2 = 900.$$
$$x = 30.$$

Find the value of x in each of the following proportions:

1. $9:x::x:16$.	4. $12:x::x:48$.
2. $16:x::x:25$.	5. $5:x::x:125$.
3. $8:x::x:32$.	6. $36:x::x:49$.

(a) Find the sum of the six mean proportionals.

7. An estate was to be divided so that the ratio of A's part to B's would equal the ratio of B's part to C's. If A received \$8000 and C received \$18000, how much should B receive?

8. Find the mean proportional of $\frac{2}{5}$ and $1\frac{3}{5}$.

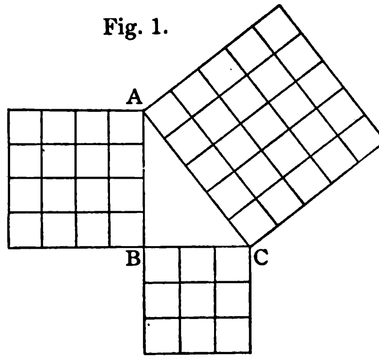
9. The ratio of the areas of two squares is as 4 to 9. What is the ratio of their lengths?

10. The area of the face of one cube is to the area of the face of another cube as 16 to 25. What is the ratio of the solid contents of the cubes?

Geometry.

317. RIGHT-TRIANGLES.

Fig. 1.



1. The longest side of a right-triangle is the **hypothenuse.** Either of the other sides may be regarded as the **base,** and the remaining side as the **perpendicular.**

2. Convince yourself by examination of the figures here given, and by careful measurements and paper cutting, that the square of the hypothenuse of a right-triangle is equivalent to the sum of the squares of the other two sides.

Fig. 2.

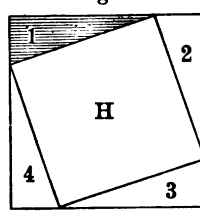
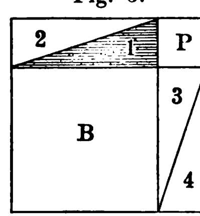


Fig. 3.



Figures 2 and 3 are equal squares. If from figure 2, the four right-triangles, 1, 2, 3, 4, be taken, H, the square of the hypothenuse, remains. If from figure 3, the four right-triangles (equal to the four right-triangles in figure 2) be taken, B, the square of the base, and P, the square of the perpendicular, remain. When equals are taken from equals the remainders are equal, therefore the square, H, equals the sum of the squares B and P.

3. To find the hypothenuse of a right-triangle when the base and perpendicular are given: *Square the base; square the perpendicular; extract the square root of the sum of these squares.*

318. Miscellaneous Review.

1. Find approximately the diagonal of a square whose side is 20 feet.*

2. Find approximately the distance diagonally across a rectangular floor, the length of the floor being 30 feet and its breadth 20 feet.

3. How long a ladder is required to reach to a window 25 feet high if the foot of the ladder is 6 feet from the building and the ground about the building level?

4. If the length of a rectangle is a, and its breadth b, what is the diagonal?

5. The base of a right triangle is 40 rods and its perpendicular, 60 rods. (a) What is its hypothenuse? (b) What is its area? (c) What is its perimeter?

6. The area of a certain square piece of land is $2\frac{1}{2}$ acres. (a) Find (in rods) its side. (b) Find its perimeter. (c) Find its diagonal, true to tenths of a rod.

7. The length of a rectangular piece of land is to its breadth as 4 to 3. Its area is 30 acres. (a) Find its breadth. (b) Find its perimeter. (c) Find the distance diagonally across it.

8. A certain piece of land is in the shape of a right-triangle. Its base is to its altitude as 3 to 4. Its area is 96 square rods. (a) Find the base. (b) Find the altitude. (c) Find the perimeter.

9. Find one of the two equal factors of 93025.

* From the study of right-triangles on page 359 it may be learned that the diagonal of a square is equal to the square root of twice the square of its side.

METRIC SYSTEM.

NOTE.—The teacher should present this subject orally before attempting the pages that follow. It will be well if this oral work can be commenced many weeks before this page is reached in the regular work of the class. A meter stick, a liter measure, and metric weights should be provided and each pupil should weigh and measure until he can *easily* think quantity in the units of this system *without reference to the units of any other system.*

319. All units in the metric system of measures and weights are derived from the primary unit known as the **meter.**

When the length of the primary unit of this system was determined it was supposed to be one ten-millionth of the distance from the equator to the pole. A pendulum that vibrates seconds is nearly one meter long.

In the names of the derived units of this system the prefix *deka* means 10; *hekto* means 100; *kilo* means 1000; *myria* means 10000; *deci* means tenth; *centi* means hundredth; *milli* means thousandth.

320. LINEAR MEASURE.

10 millimeters (mm.)	= 1 centimeter (cm.).*
10 centimeters	= 1 decimeter (dm.).
10 decimeters	= 1 meter (m.).
10 meters	= 1 dekameter (Dm.).
10 dekameters	= 1 hektometer (Hm.).
10 hektometers	= 1 kilometer (Km.).
10 kilometers	= 1 myriameter (Mm.).

* In the common pronunciation of these words the primary accent is on the first syllable and a secondary accent on the penultimate syllable; thus, cen'timéter. In the better pronunciation the accent is on the vowel preceding the letter m, that is, on the antepenultimate syllable; thus, centim'eter, dekam'eter, etc.

Metric System.

321. The names of the units of **surface measurement** are the same as those used for linear measurement, combined with the word *square;* thus, a surface equivalent to a square whose side is a meter is 1 **square meter.**

The pupil, if properly taught to this point, will be able, without difficulty, to fill the blanks in the table of—

SQUARE MEASURE.

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.).
—— square centimeters	= 1 square decimeter (sq dm.).
—— square decimeters	= 1 square meter (sq. m.).
—— square meters	= 1 square dekameter (sq. Dm.).
—— square dekameters	= 1 square hektometer (sq. Hm.).
—— square hektometers	= 1 square kilometer (sq. Km.).
—— square kilometers	= 1 square myriameter (sq. Mm.).

NOTE.—The special unit of surface measure for measuring land is equivalent to a square whose side is ten meters. This unit is called an **ar.**

100 centars (ca.)	= 1 ar (a.).
100 ars	= 1 hektar (Ha.).

EXERCISE.

1. In a square decimeter there are —— sq. cm.
2. In 2 square decimeters there are —— sq. cm.
3. In a 2-decimeter square there are —— sq. cm.
4. In a square meter there are —— sq. dm.
5. In 2 square meters there are —— sq. dm.
6. In a 2-meter square there are —— sq. dm.
7. In a square meter there are —— sq. cm.
8. In 2 square meters there are —— sq. cm.
9. In a 2-meter square there are —— sq. cm.

Metric System.

322. The names of the units of **volume measurement** are the same as those used for linear measurement, combined with the word *cubic;* thus, a volume equivalent to a cube whose edge is a meter is 1 **cubic meter.**

The pupil should be able easily to fill the blanks in the table of—

VOLUME MEASURE.

1000 cubic millimeters (cu. mm.)	= 1 cubic centimeter (cu. cm.).*
—— cubic centimeters	= 1 cubic decimeter (cu. dm.).
—— cubic decimeters	= 1 cubic meter (cu. m.).
—— cubic meters	= 1 cubic dekameter (cu. Dm.).
—— cubic dekameters	= 1 cubic hektometer (cu. Hm.).

NOTE 1.—The special unit of capacity for measuring liquids, grain, small fruits, etc., is the **liter.** It is equal to 1 cubic decimeter. 10 liters (l.) = 1 dekaliter (Dl.), and 1 tenth of a liter = 1 deciliter (dl.), etc.

NOTE 2.—The special unit for measuring wood is the **ster.** It is equal to 1 cubic meter.

EXERCISE.

1. A cubic meter equals —— liters.
2. A cubic meter equals —— cubic decimeters.
3. A cubic meter equals —— cubic centimeters.

4. A cubic decimeter equals —— cubic centimeters.
5. Two cubic decimeters equal —— cubic centimeters.
6. A 2-decimeter cube equals —— cubic centimeters.

7. A 5-centimeter square equals —— square centimeters.
8. A 5-centimeter cube equals —— cubic centimeters.

9. One tenth of a liter equals —— cubic centimeters.
10. One deciliter equals —— cubic centimeters.

* The abbreviation cc. is often used for cubic centimeter.

Metric System.

323. The primary unit of weight is the **gram.** This equals the weight of one cubic centimeter of pure water.

WEIGHT.

10 milligrams (mg.)	= 1 centigram (cg.).
10 centigrams	= 1 decigram (dg.).
10 decigrams	= 1 gram (g.).
10 grams	= 1 dekagram (Dg.).
10 dekagrams	= 1 hektogram (Hg.).
10 hektograms	= 1 kilogram (Kg.).

NOTE.—The special unit for the weight of very heavy articles is the **tonneau.** It equals the weight of a cubic meter of pure water, or 1000 kilograms.

EXERCISE.

1. The weight of 1 liter of water is —— grams.
2. The weight of 6 cubic centimeters of water is ——.
3. The weight of a cubic decimeter of water is ——.
4. One kilogram of water equals —— cubic centimeters.
5. One hektogram of water equals —— cc.
6. One dekagram of water equals —— cubic centimeters.
7. If the specific gravity of iron is 7.5, what is the weight of a cubic centimeter of iron?
8. If the specific gravity of cork is $\frac{1}{4}$, what is the weight of a cubic decimeter of cork?
9. If the specific gravity of oil is .9, what is the weight of a liter of oil?
10. What is the weight of a cubic meter of stone whose specific gravity is 2.5?
11. What is the weight of a cubic decimeter of wood whose specific gravity is .8?

Metric System.

324. Miscellaneous Problems.

1. Find the area of a rectangular surface that is 1 meter long and 6 decimeters wide. Make a diagram of this surface upon the blackboard.

2. Find the area of a rectangular surface that is 2 decimeters long and 5 centimeters wide. Make a diagram of this surface on your slate or paper.

3. Find the solid content of a 5-centimeter cube. A 5-centimeter cube is what part of a cubic decimeter?

4. Find the solid content of a 4-decimeter cube. A 4-decimeter cube is what part of a cubic meter?

5. Find the entire surface of a 4-centimeter cube. The surface of a 4 centimeter cube is what part of a square decimeter?

6. Find the area of a rectangular surface that is 2.4 yards by 5 yards; of a rectangular surface that is 2.4 meters by 5 meters.

7. Which is the larger of the two surfaces described in problem 6?

8. Find the area of a rectangular surface that is 3.5 yards by 2.5 yards; of a rectangular surface that is 3.5 meters by 2.5 meters.

9. Find the volume of a rectangular solid that is 3.4 feet by 3 feet by 2 feet; of a rectangular solid that is 3.4 meters by 3 meters by 2 meters.

10. Find the volume of a rectangular solid that is 3.5 meters by 2.3 meters by 4.6 meters.

11. What is the weight of a cubic decimeter of wood whose specific gravity is .5?

Metric System.

325. Miscellaneous Problems.

1. Estimate in meters the width of the lot upon which the school building stands. Measure it.

2. Estimate in centimeters the width of your desk. Measure it.

3. Estimate in square centimeters the area of a sheet of paper. Measure and compute.

4. Estimate in square meters the area of the blackboard. Measure and compute.

5. Estimate the number of cubic meters of air in the school room. Measure and compute.

6. Estimate in grams the weight of a teaspoonful of water. Weigh it.*

7. Estimate in kilograms your own weight.

8. Estimate in liters the capacity of a water pail.

9. Estimate in kilograms the weight of a gallon of water.

10. Estimate in kilometers the distance from the schoolhouse to your home.

326. Table of Equivalents.

Meter	a little more than 1 yard	39.37 inches.
Kilometer	nearly $\frac{5}{8}$ of a mile	3280.8+ feet.
Decimeter	nearly 4 inches	3.937 inches.
Ar	nearly $\frac{1}{40}$ of an acre	3.954 sq. rd.
Ster	a little more than $\frac{1}{4}$ cord	35.3+ cu. ft.
Liter	a little more than 1 liquid quart,	1.056+ qt.
Gram	nearly $15\frac{1}{2}$ grains	15.4+ grains.
Kilogram	nearly $2\frac{1}{5}$ pounds	2.204+ lb.

*Every school should be provided with scales, weights, and measures.

Algebra.

327. Metric Units in Algebraic Problems.

1. I am thinking of a rectangular surface. Its length is 5 times its breadth. Its area is 45 square decimeters. How long and how wide is the surface ?*

2. I am thinking of a triangular surface. Its base is three times its altitude. Its area is 8.64 square meters. What is the length of its base ?

3. I am thinking of a cube whose entire surface is 150 square centimeters. What is the length of one of its edges ?

4. The perimeter of a certain rectangle is 20.4 meters. Its length is twice its breadth. (a) Find its length and breadth. (b) Find its area.

5. The difference in the weight of two lead balls is 24 grams. The united weight of the two balls is 1 kilogram. (a) Find the weight of each ball. (b) Does the heavier ball weigh more or less than 1 pound ?

6. A merchant had three pieces of lace. In the second piece there were twice as many meters as in the first. In the third piece there were 6 meters more than in the second. In the three pieces there were 106 meters. (a) How many meters in each piece ? (b) Were there more or less than 53 yards in the second piece ?

7. John weighs 3.6 kilograms more than Henry. Together they weigh 83.6 kilograms. (a) Find the weight of each boy. (b) Does John weigh more or less than 90 pounds ?

* Let x = the number of decimeters in the breadth of the surface.

Algebra.

328. METRIC UNITS IN ALGEBRAIC PROBLEMS.

1. A ball rolling down a perfectly smooth and uniformly inclined plane rolls 3 times as far the 2nd second as the 1st; 5 times as far the 3rd second as the 1st; 7 times as far the 4th second as the first. If in 4 seconds it rolls 192 decimeters (a) how far did it roll in the 1st second? (b) In the 4th second? (c) Did it roll more or less than 48 inches in the first second?

2. I am thinking of a right-triangle. Its altitude is to its base as 3 to 4. The sum of its altitude and base is 14 centimeters. (a) Find the altitude. (b) Find the base. (c) Find the area. (d) Find the hypothenuse. (e) Is the hypothenuse more or less than 4 inches?

3. A freely falling* body falls three times as far the 2nd second of its fall as it does the 1st second. In two seconds it falls 19.6 meters. (a) How far does it fall in the 1st second? (b) In the 2nd second?

4. A freely falling body falls 3 times as far the 2nd minute of its fall as it does the 1st minute. In two minutes it falls 70560 meters. (a) How far does it fall in the 1st minute? (b) In the 2nd minute? (c) 70560 meters equals how many kilometers? (d) 70560 meters equals (approximately) how many miles?

5. A freely falling body falls 3 times as far the 2nd half-second as it does the 1st half-second. In one second it falls 4.9 meters. (a) How far does it fall in the 1st half-second? (b) In the 2nd half-second?

*** A freely falling body is a body falling in a perfect vacuum.**

Geometry.

329. The Circumference of a Circle.

1. Cut a 3-inch circle from cardboard. By rolling it upon a foot rule, measure its circumference.

2. Measure the diameter of a bicycle wheel; then by rolling it upon the ground or upon the school-room floor, measure its circumference.

3. In a similar manner measure the diameters and the circumferences of other wheels until you are convinced that the circumference of a circle is a little more than —— times its diameter.

4. The circumference of a circle is nearly $3\frac{1}{7}$ times the diameter; more accurately, it is 3.141592+ times the diameter.

Note.—It is a curious fact that the diameter of a circle being given in numbers, it is impossible to express in numbers its *exact* circumference. The circumference being given in numbers, it is impossible to express in numbers its *exact* diameter. In other words, *the exact ratio of the circumference to the diameter is not expressible.*

5. Find the approximate circumference of a 5-inch circle; of a 7-inch circle; of a 10-inch circle.*

6. Find the approximate diameter of a circle that is 6 ft. in circumference.*

7. The circumference of a 6-inch circle is how many times the circumference of a 3-inch circle?

8. The diameter of a circle whose circumference is 12 inches is what part of the diameter of a circle whose circumference is 24 inches?

*In the solution of such problems as these, the pupil may use, as the approximate ratio of the circumference to the diameter, 3.14.

CONTENTS—PART III.

PART III.

DENOMINATE NUMBERS.

Linear Measure.

NOTE.—Pupils who have mastered the Elementary Book and the preceding pages of this book have had much practice in the use of denominate numbers. In part to provide for ready reference and in part to give further application of the principles already presented, the subject is here treated as a whole.

331. The English and United States standard unit of length is the **Imperial yard** arbitrarily fixed by Act of Parliament and afterward adopted in the United States. It is about $\frac{360000}{391393}$ of the length of a pendulum that vibrates once a second at the level of the sea in the latitude of London. It is $\frac{3600}{3937}$ of a meter.

TABLE.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards	= 1 rod (rd.).
$16\frac{1}{2}$ feet	= 1 rod.
320 rods	= 1 mile (mi.).
1760 yards	= 1 mile.
5280 feet	= 1 mile.

1 fathom (used in measuring the depth of the sea)	= 6 feet.
1 knot (used in navigation)	= 1.15+ miles.
1 league (used in navigation)	= 3 knots.
1 hand (used in measuring the heights of horses)	= 4 inches.
1 chain (used by civil engineers)	= 100 feet.
1 chain (used by land surveyors)	= 66 feet.
1 pace (used in measuring approximately)	= $\frac{1}{6}$ of a rod.
1 barleycorn (used in grading length of shoes)	= $\frac{1}{3}$ of an inch.
1 furlong (a term nearly obsolete)	= $\frac{1}{8}$ of a mile.

Denominate Numbers—Linear Measure.

EXERCISE.

1. Mont Blanc is 15810 feet, or about —— miles high.
2. Mt. Everest is 29000 feet, or about —— miles high.
3. Commodore Dewey opened fire on the enemy at a distance of 5000 yards, or about —— miles.
4. My horse, measured over the front feet, is $16\frac{1}{4}$ hands, or —— feet —— inches high.
5. The vessel seemed to be about three leagues, or —— miles distant.
6. On sounding, they found the depth of the water to be 15 fathoms, or —— feet.
7. The cruiser made 20 knots, or about —— miles, an hour.
8. The length of the lot was 36 paces, or about —— rods.
9. 10000 feet is nearly —— miles.
10. 15000 feet is nearly —— miles.
11. 1000 yards is about —— of a mile.
12. 100 feet is —— rods —— foot.
13. 200 feet is —— rods —— feet.
14. 300 feet is —— rods —— feet.
15. A kilometer is about —— rods.
16. A Civil Engineer's chain is —— rods —— foot.

PROBLEMS.

1. A seven-foot drive wheel of a locomotive makes how many revolutions to the mile?
2. Which is the longest distance, 5 miles 319 rods 16 feet 6 inches, 5 miles 319 rods 5 yards 1 foot 6 inches, or 6 miles?
3. Reduce 40 rd. 4 ft. 5 in. to inches.

Denominate Numbers—Surface Measure.

332. The standard unit of surface measure is a **square yard** which is the equivalent of a 1-yard square. This unit, like the square foot, square inch, square rod, and square mile, is derived from the corresponding unit of linear measure.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.).
$272\frac{1}{4}$ square feet	= 1 square rod.
160 square rods	= 1 acre (A.).
4840 square yards	= 1 acre.
43560 square feet	= 1 acre.
640 acres	= 1 square mile (sq. mi.).

EXERCISE.

1. Show by a drawing that there are 144 square inches in a 1-foot square.

2. Show by a drawing that there are 9 square feet in a 1-yard square.

3. Show by a drawing that there are $30\frac{1}{4}$ square yards in a 1-rod square.

4. Estimate the number of square yards of blackboard in the room; the number of square feet of blackboard.

5. Estimate the number of square feet in the floor of the schoolroom; the number of square yards.

6. Estimate the square yards of plastering on the walls of the schoolroom.

7. Estimate the number of square rods in the schoolhouse lot. Is the lot more or less than $\frac{1}{2}$ of an acre?

Denominate Numbers—Surface Measure.

333. In the measurement of land it is more convenient to use a decimal scale; hence the invention of the **Gunter Chain.** This chain is 4 rods long and is divided into 100 links.

Observe that links are hundredths of chains.
Observe that square chains are tenths of acres.

1. Land, 3 chains by 4 chains contains —— acres.
2. Land, 5 chains by 4 chains contains —— acres.
3. Land, 3 chains by 8 chains contains —— acres.
4. Land, 5 chains by 7 chains contains —— acres.
5. Land, 8 chains by 6 chains contains —— acres.
6. Two chains 35 links equals —— chains.
7. Two chains 75 links equals —— chains.
8. Two chains 5 links equals —— chains.
9. Two chains 9 links equals —— chains.
10. Land, 4 ch. by 4.50 ch. contains —— acres.
11. Land, 5 ch. by 3.20 ch. contains —— acres.
12. Make a rule and find the number of acres in each of the following:

(1) Land, 12 chains 35 links by 9 chains 50 links.
(2) Land, 21 chains 8 links by 12 chains 30 links.
(3) Land, 32 chains 25 links by 15 chains 6 links.

(a) Find the sum of the area of the ten pieces of land described on this page.

To the Teacher.—A rod is exactly 25 links. A foot is about $1\frac{1}{2}$ links. Hence rods and feet can be easily changed to chains and links by regarding each 4 rods as 1 chain and each additional rod as 25 links and each additional foot as $1\frac{1}{2}$ links. The error in any one measurement never exceeds 2 inches. 9 rd. 12 ft. $=$ 2 chains 43 $(25+18)$ links.

Denominate Numbers—Surface Measure.

334. To determine the amount of **carpet** necessary for a given room several minor problems must be solved which can be best studied by means of an—

EXAMPLE.

1. How many yards of carpet must be purchased for a room 16 ft. by 20 ft. if the carpet is 1 yd. wide?

(1) How many breadths will be necessary if the carpet is put down lengthwise of the room? How much must be cut off or turned under from one breadth in this case?

(2) How many breadths will be necessary if the carpet is put down crosswise of the room? How much must be cut off or turned under from one breadth in this case?

(3) Make two diagrams of the room on a scale of 1 inch to the foot and show the breadths of carpet in each case.

(4) How many yards must be purchased in each case?

(5) If in the first case there is no waste in matching the figure and in the second case there is a waste of 8 inches on each breadth *except the first,* which plan of putting down the carpet will require the greater number of yards?

(6) If the carpet costs 90¢ a yard and the conditions are as stated in No. 5, what is the cost of the carpet in each case?

2. How many yards of carpet must be purchased for a room 16 ft. by 20 ft. if the carpet is $\frac{3}{4}$ of a yard wide and there is no waste in matching the figure?

3. How many yards of carpet must be purchased for a room that is 15 ft. 6 in. by 16 ft. 4 in. if the carpet is $\frac{3}{4}$ of a yard wide, is put down lengthwise of the room, and there is no waste in matching the figure?

Denominate Numbers.

335. PLASTERING AND PAPERING.

1. How many square yards of plastering in a room (walls and ceiling) that is 15 ft. by 18 ft. and 12 ft. high, an allowance of 12 square yards being made for openings?

NOTE.—In estimating the cost of plastering, allowance is made for "openings" (windows and doors) only when they are very large in proportion to the wall to be covered. Why are plasterers unwilling to deduct the entire area of all the openings?

2. At 24¢ a square yard how much will it cost to plaster a room that is 17 ft. by 20 ft. and 10 feet from the floor to the ceiling, deducting 16 square yards for openings?

3. How many "double rolls" of paper will be required for the walls of a room that is 14 ft. by 16 ft. and 11 ft. high above the baseboards, if an allowance of 1 full "double roll" is made for openings?

NOTE.—Wall paper is usually 18 inches wide. A "single roll" is 24 ft. long. A "double roll" is 48 ft. long. In papering a room 11 ft. high it would be safe to count on 4 full strips from each "double roll." The remnant would be valueless unless it could be used over windows or doors. Since each strip is 18 inches wide, a "double roll" will cover 72 inches (6 ft.) of wall measured horizontally.

4. At 12¢ a "single roll," how much will the paper cost for the walls of a room that is 12 ft. by 14 ft. and 7 ft. above the baseboards, if the area of the openings is equivalent to the surface of 2 "single rolls" of paper?

5. Find the cost, at 25¢ a square yard, of plastering the walls of a room that is 48 ft. by 60 ft. and 18 feet high, deducting 30 square yards for openings.

Denominate Numbers.

336. Farm Problems.

Find how many acres in—

1. A piece of land 1 rod by 160 rods.
2. A piece of land 7 rods by 160 rods.
3. A piece of land 13 rods by 160 rods.
4. A piece of land 22 feet by 160 rods.
5. A piece of land $8\frac{1}{4}$ yards by 160 rods.

(a) Find the sum of the five results.

6. A piece of land 8 rods by 80 rods.
7. A piece of land 17 rods by 80 rods.
8. A piece of land $37\frac{1}{2}$ rods by 80 rods.
9. A piece of land $618\frac{3}{4}$ feet by 80 rods.
10. A piece of land 550 yards by 80 rods.

(b) Find the sum of the five results.

11. A piece of land 12 rods by 40 rods.
12. A piece of land 27 rods by 40 rods.
13. A piece of land 46 rods by 20 rods.
14. A piece of land 36 rods by 20 rods.
15. A piece of land 264 feet by 20 rods.

(c) Find the sum of the five results.

16. A piece of land 1 rod by 1 mile.
17. A piece of land 11 rods by 1 mile.
18. A piece of land 66 feet by 1 mile.
19. A piece of land 99 yards by 1 mile.
20. A piece of land 198 feet by $\frac{1}{2}$ of a mile.

(d) Find the sum of the five results.

21. A piece of land $\frac{1}{2}$ of a mile long and as wide as the schoolroom.

Denominate Numbers.

337. Farm Problems.

1. A piece of land 1 foot wide and 43560 feet long is how many acres?

2. Change 43560 feet to miles.

3. A piece of land 1 foot wide must be how many miles in length to contain 1 acre?

4. Some country roads are 66 feet wide. How many acres in $8\frac{1}{4}$ miles of such road?

5. How many acres in 1 mile of road that is 4 rods wide?

6. A farmer walking behind a plow that makes a furrow 1 foot wide will travel how far in plowing 1 acre?

7. A farmer walking behind a plow that makes a furrow 16 inches wide will travel how far in plowing 1 acre?

8. If a mowing machine cuts a swath that averages 4 feet in width, how far does it move in cutting 1 acre?

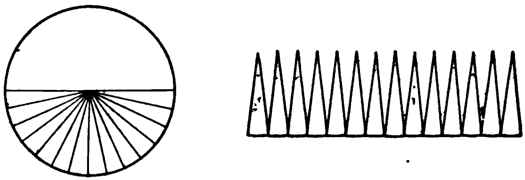
9. If potatoes are planted in rows that are 3 feet apart, (a) how many miles of row to each acre? (b) How many rods of row to each acre? (c) If 4 rods of row on the average yield 1 bushel, what is the yield per acre?

10. Strawberry plants are set in rows that are 2 feet apart. (a) How many miles of row to the acre? (b) How many rods of row to the acre? (c) How many feet of row to the acre?

11. If corn is planted in rows $3\frac{1}{2}$ feet apart and if the "hills" are $3\frac{1}{2}$ feet apart in the row, how many hills to each acre?

Geometry.

338. To Find the Area of a Circle.



1. Cut one half of a circular piece of paper as indicated in the diagram.

Observe that if the circle is cut into a very large number of parts and opened as shown in the figure, the circumference of the circle becomes, practically, a straight line.

Note.—Imagine the circle cut into an infinite number of parts and thus opened and the circumference to be a straight line.

Observe that a circle may be regarded as made up of an infinite number of triangles whose united bases equal the circumference and whose altitude equals the radius. Hence to find the area of a circle we have the following:

Rule I. *Multiply the circumference by* $\frac{1}{4}$ *of the diameter.*

2. It has already been stated that if the diameter of a circle is 1, its circumference is 3.141592. Hence the area of a circle whose diameter is 1 is $(3.141592 \times \frac{1}{4})$ .785398.

3. A circle whose diameter is 2, is 4 times as large as a circle whose diameter is 1; a circle whose diameter is 3, 9 times as large, etc. Hence to find the area of a circle we have also the following:

Rule II. *Multiply the square of the diameter by* .785398.

4. The approximate area may be found by taking $\frac{3}{4}$ (or .78) of the square of the diameter. (See Note 9, p. 445.)

339. Miscellaneous Problems.

1. Find the approximate area of a circle whose diameter is 20 feet.

2. What is the area of a circle whose diameter is 1 foot? 1 yard? 1 rod? 1 mile?

3. What is the area of a circle whose diameter is 2 feet? 2 yards? 2 rods? 2 miles?

4. A horse is so fastened with a rope halter that he can feed over a circle forty feet in diameter. Does he feed over more or less than 5 square rods?

5. Find the approximate length (in rods) of the side of a square containing 1 acre.

6. Find the approximate diameter (in rods) of a circle whose circumference is one mile.

7. Find the approximate area of the circle described in problem 6.

8. Find the approximate circumference of a circle whose diameter is 30 rods.

9. The expression "a bicycle geared to 68" means that the machine is so geared that it will move forward at each revolution of the pedal shaft as far as a 68-inch wheel would move forward at one revolution. How far does a bicycle "geared to 68" move forward at each revolution of the pedal shaft? A bicycle "geared to 70"?

10. What is the approximate circumference of the largest circle that can be drawn on the floor of a room 40 ft. by 40 ft. if at its nearest points the circumference is 2 feet from the edge of the floor?

VOLUME MEASURE.

340. The standard unit of volume measure is a **cubic yard,** which is the equivalent of a 1-yard cube. This unit, like the cubic foot and the cubic inch, is derived from the corresponding unit of linear measure.

CUBIC MEASURE.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).

EXERCISE.

1. Show by a drawing that there are 27 cu. ft. in a 1-yard cube.
2. How many cubic inches in 1 half of a cubic foot?
3. How many cubic inches in a $\frac{1}{2}$-foot cube?
4. How many cubic feet in 1 third of a cubic yard?
5. How many cubic feet in a $\frac{1}{3}$-yard cube?
6. Estimate in cubic feet the amount of air in the schoolroom.
7. Estimate in cubic yards the amount of air in the schoolroom.
8. Estimate in cubic inches the capacity of your dinner box.
9. Estimate in cubic feet the capacity of some wagon box.
10. Estimate in cubic inches the volume of the school globe.*

* A globe is a little more than $\frac{1}{2}$ of the smallest cube from which it could have been made. See note 10, p. 445.

Denominate Numbers—Volume Measure.

341. Wood is usually measured by the **cord.** A cord is a pile 4 feet wide, 4 feet high, and 8 feet long, or its equivalent. Hence—

128 cubic feet = 1 cord.

Problems.

1. Estimate the number of cords of wood that could be put upon the floor of the school room if the desks were removed and the wood piled to the depth of four feet.

2. If 4-foot wood is piled 6 feet high what must be the length of the pile to contain 100 cords?

3. How many cords of wood in a pile 8 feet wide, 8 feet high, and 16 feet long?

4. Compare the amount of wood in the pile described in problem 3, with the amount in a pile one half as wide, one half as high, and one half as long.

5. If I pay $1.10 a cord for sawing wood, cutting each 4-foot stick into 3 pieces, how much ought I to pay for cutting each 4-foot stick into 4 pieces?

6. A pile of wood 4 ft. high, 4 ft. wide, and 192 ft. long contains —— cords. How many cords in a pile 4 feet high, 192 feet long, and 46 inches wide?

7. A pile of wood is as wide as it is high and 32 feet long. It contains 9 cords. What is the width and height of the pile?

8. How many cords of 4-foot wood can be piled in a cellar that is 24 feet wide and 32 feet long, provided the pile is 4 feet high and one end of each 4-foot stick touches a wall of the cellar?

Denominate Numbers—Volume Measure.

342. Rough Stone is usually measured by the **cord.** A pile 4 feet high, 4 feet wide, and 8 feet long or its equivalent, is 1 cord.

NOTE.—One cord of good stone is sufficient for about 100 cubic feet of wall. Hence in estimates it is customary to use the number 100 instead of 128; that is, as many cords of stone will be required for a given wall as 100 cubic feet is contained times in the number of cubic feet in the wall.

PROBLEMS.

1. Estimate the number of cords of stone necessary for a cellar wall 18 inches thick, the inside dimensions of the cellar being 15 feet by 18 feet and 7 feet deep, no allowance being made for openings in the wall.

2. What are the outside dimensions of the wall of the cellar described in problem 1?

3. What length of wall 7 feet high and 18 inches thick is equivalent, so far as amount of stone is concerned, to the cellar wall described in problem 1?

4. If $\frac{3}{7}$ of the depth of the cellar described above is to be below the surface of the ground, how many cubic yards of earth must be excavated?

5. How many per cent less of stone will be required for a 16-inch wall than for an 18-inch wall?

6. Estimate the stone necessary for a wall 100 yards long, 11 feet high, and 2 feet thick.

7. If the specific gravity of stone is $2\frac{1}{2}$ and each cord is equivalent to 100 solid feet, how much does a cord of stone weigh?

8. If the specific gravity of a certain stone is $2\frac{1}{4}$, what is the weight of a block 8 feet by 2 feet by 2 feet?

Denominate Numbers—Volume Measure.

343. An ordinary **brick** is 2 in. by 4 in. by 8 in. and weighs about 4 pounds.

Problems.

1. How many bricks are equivalent to 1 cubic foot?

NOTE.—When bricks are laid in mortar in the usual way, about 22 bricks are required to make a cubic foot of wall.

2. Estimate the number of bricks necessary for a cellar wall 12 inches thick, the inside dimensions of the cellar being 15 feet by 18 feet, and 7 feet deep, no allowance being made for openings in the wall?

3. What are the outside dimensions of the wall of the cellar described in problem 2?

4. What length of wall 7 feet high and 12 inches thick is equivalent, so far as the number of bricks required is concerned, to the cellar wall described in problem 2?

5. If $\frac{3}{7}$ of the depth of the cellar described above is to be below the surface, how many cubic yards of earth must be excavated?

6. Estimate the number of bricks necessary for a wall 100 yards long, 11 feet high, and 1 foot thick.

7. If a brick is exactly 2 in. by 4 in. by 8 in. and weighs exactly $4\frac{1}{4}$ lbs. what is its specific gravity? (Note 16, p. 446.)

8. Find the approximate weight (in tons) of a pile of bricks as long as your school-room, 2 feet wide, and 4 feet high.

9. Find the approximate weight of a chimney, outside dimensions, 16 in. by 16 in., and 20 ft. high, the flue being 8 in. by 8 in.

Denominate Numbers—Lumber.

344. A **foot of lumber** is a board 1 foot square and 1 inch thick or its equivalent. (Note 11, p. 445.)

NOTE 1.—An exception to the foregoing is made in the measurement of boards less than 1 inch in thickness. A square foot of such boards is regarded as a foot of lumber, whatever the thickness.

EXERCISE.

Tell the number of feet of lumber in each of the following boards, the thickness in each case being one inch (or less):

1 in. wide and 12 ft. long.	2 in. wide and 12 ft. long.
3 in. wide and 12 ft. long.	4 in. wide and 12 ft. long.
7 in. wide and 12 ft. long.	13 in. wide and 12 ft. long.
9 in. wide and 12 ft. long.	12 in. wide and 12 ft. long.

(a) How many feet (of lumber) in the eight boards?

PROBLEMS.

1. How much lumber in 6, 12-ft., 1-in. boards whose widths are 11 in., 13 in., 9 in., 10 in., 12 in., and 14 in.?

2. How much lumber in 5, 12-ft., $\frac{3}{4}$-in. boards whose widths are 10 in., 12 in., 12 in., 11 in., and 14 in.?

3. How much lumber in 7, 12-ft., $\frac{1}{2}$-in. boards whose widths are 9 in., 8 in., 5 in., 7 in., 8 in., 6 in., and 9 in.?

4. How much lumber in 8, 12-ft., 1-in. boards each of which is 12 inches wide?

5. How much lumber in 54, 12-ft., 1-in. boards each of which is 6 inches wide?

(b) Find the sum of the five results.

Denominate Numbers—Lumber.

PROBLEMS.

NOTE 2.—**A 14-foot board** contains $\frac{1}{6}$ more lumber than a 12-ft. board of the same width and thickness. Hence to find the number of feet of lumber in 14-foot boards, find the number of feet in as many 12-foot boards and add to the result $\frac{1}{6}$ of itself.*

1. How much lumber in 5, 14-ft., 1-in. boards whose widths are 11 in., 12 in., 12 in., 15 in., and 10 in.?

2. How much lumber in a pile of 14-ft. boards whose united width is 8 feet 7 inches?

3. How much lumber in 56, 14-ft. boards each of which is 6 inches wide? †

4. How much lumber in 24, 14-ft. boards each of which is 12 inches wide?

(a) Find the sum of the four results.

PROBLEMS.

NOTE 3.—**A 16-foot board** contains $\frac{1}{3}$ more lumber than a 12-ft. board of the same width and thickness. Make a rule for finding the number of feet of lumber in 16-foot boards.

1. How much lumber in 5, 16-ft., 1-in. boards whose widths are 12 in., 10 in., 14 in., 13 in., and 12 in.?

2. How much lumber in a pile of 16-foot boards whose united width is 9 feet 8 inches?

3. How much lumber in 48, 16-ft. boards each of which is 6 inches wide?

4. How much lumber in 34, 16-ft. boards each of which is 12 inches wide?

(b) Find the sum of the four results.

* Take the nearest integral number of feet.

† How much lumber in *one* 14-foot board 6 inches wide?

Denominate Numbers—Lumber.

Problems.

Note 4.—**A $1\frac{1}{4}$-inch board** contains $\frac{1}{4}$ more lumber than a 1-inch board of the same width and length. A $1\frac{1}{2}$-inch board contains $\frac{1}{2}$ more lumber than a 1-inch board of the same width and length.

1. How much lumber in 4, 12-foot, $1\frac{1}{4}$-in. boards whose widths are 12 in., 13 in., 14 in., and 13 in.?

2. How much lumber in 4, 16-foot, $1\frac{1}{2}$-in. boards whose widths are 13 in., 16 in., 12 in., and 13 in.?

3. How much lumber in 4, 18-foot, $1\frac{1}{2}$-in. boards, each of which is 12 inches wide?

4. How much lumber in 4, 16-ft., $1\frac{1}{2}$-in. boards, each of which is 6 inches wide?

(a) Find the sum of the four results.

Problems.

Note 5.—A "**2 by 4, 12**" is a piece of lumber 2 in. thick, 4 in. wide, and 12 feet long.

Find the number of feet of lumber in each of the following items:

1. 16 pieces 2×4, 12.
2. 18 pieces 4×4, 12.
3. 25 pieces 2×8, 12.
4. 30 pieces 2×6, 12.
5. 20 pieces 4×6, 12.

(b) Find the sum of the five results.

Observe that *in a 12-foot piece of lumber* there are as many feet as there are square inches in the cross-section. A piece of lumber 1 in. by 1 in. and 12 feet long is 1 foot of lumber; a piece 2 in. by 2 in. is 4 feet of lumber; a piece 2 in by 3 in. is 6 feet of lumber, etc.

Denominate Numbers—Lumber.

PROBLEMS.

NOTE 6.—In the measurement of timbers of all sizes it is customary to consider each piece as containing the integral number of feet nearest to the actual content. Thus, a piece of 2 × 4, 14, actually contains $9\frac{1}{3}$ feet, but in all lumber yards it is counted as 9 feet. A piece of 2 × 4, 16, actually contains $10\frac{2}{3}$ feet, but it is counted as 11 feet.

Find the number of feet of lumber in each of the following items:

1. 16 pieces 2 × 4, 14.
2. 24 pieces 4 × 4, 14.
3. 32 pieces 2 × 8, 14.
4. 17 pieces 4 × 6, 14.
5. 15 pieces 8 × 8, 16.
6. 12 pieces 4 × 10, 16.
7. 14 pieces 8 × 12, 16.
8. 6 pieces 12 × 12, 24.

(a) Find the sum of the eight results.

PROBLEMS.

NOTE 7.—"Lumber at $15 per M," means that the lumber is sold at the rate of $15 per 1000 feet.

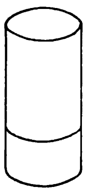
Find the cost:

1. 26, 16-foot, 6-in. fence boards @ $15 per M.
2. 34, 14-foot, 12-in. stock boards " $18 per M.
3. 20 pieces 2 × 4, 16, " $16 per M.
4. 14 pieces 4 × 6, 18, " $16 per M.
5. 25 pieces 4 × 6, 16, " $15 per M.
6. 18 pieces 4 × 4, 14, " $15 per M.

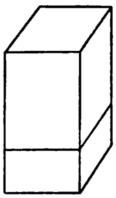
(b) Find the sum of the six results.

Geometry.

345. TO FIND THE SOLID CONTENT OF A **cylinder** OR OF A **right prism.***



Observe that in any cylinder or right prism the number of cubic units in one layer 1 unit high (as indicated in the diagrams) is equal to the number of square units in the area of the base. Thus, if there are $4\frac{1}{2}$ square units in the area of the base there are $4\frac{1}{2}$ cubic units in one layer. The content of the entire solid is as many times the cubic units in one layer, as the solid is linear units in height. Hence the rule as usually given: "*Multiply the area of the base by the altitude.*"



TO THE TEACHER.—This rule must be carefully interpreted by the pupil. He must not be allowed the misconception that area multiplied by any number can give solid content, except *through such interpretation as is suggested in the above observation.* (Note 12, p. 446.)

PROBLEMS.

1. Find the solid content of a square right prism whose base is 6 in. by 6 in., and whose altitude is 8 inches.

2. Find the approximate solid content of a cylinder 6 inches in diameter and 10 inches long.

3. Find the solid content of a triangular prism the area of whose base is 15 sq. in., and whose altitude is $11\frac{1}{2}$ inches.

4. Find the solid content of an hexagonal prism the area of whose base is 18 inches, and whose altitude is $10\frac{1}{2}$ inches.

* A right prism is a solid whose bases, or ends, are similar, equal, and parallel plane polygons, and whose lateral faces are perpendicular to its bases.

346. Miscellaneous Problems.

1. Find the solid content of an octagonal right prism the area of whose base is 24 square inches, and whose altitude is 15 inches.

2. What is the solid content of a cylinder, or of any right prism, the area of whose base is 30 square inches, and whose altitude is 12 inches?

3. How many cubic feet of earth must be removed to dig a well 6 feet in diameter and 20 feet deep?*

4. Find the approximate number of feet of $1\frac{1}{2}$-in. lumber required to make the lining of the sides of a cylindrical silo that is 20 feet in diameter and 30 feet deep.

5. Find the approximate number of cords of rough stone in a cylindrical pile that is 16 feet in diameter and six feet deep.

6. Find the approximate number of brick necessary for a solid cylindrical foundation that is 9 feet in diameter and 4 feet high.

7. If the average specific gravity of the brick and mortar used in the foundation described in problem 6, is 1.9, how much does the entire foundation weigh?

8. Find the weight in kilograms of a column of water 1 decimeter square and 10 meters deep.

9. Find the weight in pounds of 1000 feet of white pine 1-inch boards, the specific gravity being .6.

10. Find the weight of a load (1 cubic yard) of wet sand, the specific gravity being exactly 2.

* The exact number of cubic feet cannot be expressed in figures. An approximation that will answer many practical purposes may be obtained by regarding the circle (base) as $\frac{3}{4}$ of its circumscribed square. If an answer more nearly accurate is required use .78 instead of $\frac{3}{4}$.

Capacity.

347. The standard unit of capacity used in measuring liquids is a **gallon.** *A gallon equals* 231 *cubic inches.*

Liquid Measure.

4 gills (gi.)	=	1 pint (pt.).
2 pints	=	1 quart (qt.).
4 quarts	=	1 gallon (gal.).
$31\frac{1}{2}$ gallons	=	1 barrel (bbl.).

Observe that 1 cubic foot = nearly $7\frac{1}{2}$ gallons.

Observe that 4.2 cubic feet = nearly 1 barrel.

A kerosene barrel contains about 52 gallons. It equals nearly 7 cubic feet.

Problems.

1. Find the capacity (approximate or exact), in gallons, of a rectangular tank 3 ft. by 4 ft. by 8. ft.

2. Find the approximate capacity, in gallons, of a cylindrical tank 4 feet in diameter and 4 feet deep.

3. Find the approximate capacity, in barrels ($31\frac{1}{2}$ gal.), of a rectangular tank 2 ft. by 4 ft. by 12 ft.

4. Find the approximate capacity, in barrels ($31\frac{1}{2}$ gal.), of a cylindrical cistern 6 ft. in diameter and 6 ft. deep.

5. Find the approximate capacity, in barrels ($31\frac{1}{2}$ gal.), of a cylindrical cistern 12 ft. in diameter and 6 ft. deep.

6. Find the approximate capacity, in barrels ($31\frac{1}{2}$ gal.), of a cylindrical cistern 12 ft. in diameter and 12 ft. deep.

Denominate Numbers—Capacity.

348. The standard unit of capacity used in measuring grain, fruits, vegetables, lime, coal, etc., is a **bushel.** *A bushel equals* 2150.4 *cubic inches.*

NOTE.—In measuring large fruits, vegetables, lime, and coal, the unit is the "**heaped bushel.**" *A heaped bushel equals about* 1¼ "*stricken bushels.*"

DRY MEASURE.

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).
A bushel is nearly 1¼ cubic feet.
A "heaped bushel" is about 1½ cubic feet.

A "dry gallon" (4 quarts dry measure) equals 268.8 cubic inches.

Enough "**ear corn**" to make, when shelled, one bushel, occupies about 2¼ cubic feet. If the corn is inferior in quality it will occupy more space than this—sometimes 2½ cubic feet.

PROBLEMS.

1. Find the capacity in bushels of a wheat bin 8 ft. by 8 ft. by 10 ft.*

2. Give the dimensions of the smallest bin in which 1000 bushels of oats may be stored.

3. In a bin 12 feet square, there is rye to the depth of 7½ feet. How many bushels?

4. How many bushels of potatoes (without heaping the bin) may be stored in a bin that is 8 ft. by 4 ft. by 6 ft.?

5. If the corn is of excellent quality, how many bushels of "shelled corn" may be expected from a crib of ear corn, 8 ft. by 10 ft. by 80 ft.?

* For many practical purposes the approximate ratio (1¼) of the bushel to the cubic foot will give in such problems as these, results sufficiently accurate.

Denominate Numbers—Weight.

349. The standard unit of weight in common use is a **pound Avoirdupois.**

Avoirdupois Weight.

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).

The abbreviation for 1 hundredweight (100 lb.) is *cwt.*

Miscellaneous Weights.

1 gallon of water	= about $8\frac{1}{3}$ lb.
1 gallon of milk	= about 8.6 lb.
1 gallon of kerosene	= about $6\frac{1}{2}$ lb.
1 cubic foot of water	= $62\frac{1}{2}$ lb.
1 bushel of wheat	= 60 lb.
1 bushel of beans	= 60 lb.
1 bushel of clover seed	= 60 lb.
1 bushel of potatoes	= 60 lb.
1 bushel of shelled corn	= 56 lb.
1 bushel of ear corn	= 70 lb.*
1 bushel of rye	= 56 lb.
1 bushel of barley	= 48 lb.
1 bushel of oats	= 32 lb.
1 barrel of flour	= 196 lb.
1 barrel of beef or pork	= 200 lb.

Problems.

Find the cost—

1. Of 2650 lb. coal at $5.50 per ton.
2. Of 2650 lb. oats at 24¢ a bushel.
3. Of 3330 lb. wheat at 80¢ a bushel.
4. Of 4650 lb. potatoes at 42¢ a bushel.

(a) Find the sum of the four results.

*This means the amount of ear corn required to make 1 bushel of shelled corn.

Denominate Numbers—Weight.

PROBLEMS.

Find the cost—

1. Of 2560 lb. hay at \$7.50 per ton.
2. Of 1430 lb. straw at 30¢ per cwt.
3. Of $2\frac{1}{4}$ tons meal at $\frac{3}{4}$ of a cent a pound.
4. Of $1\frac{1}{2}$ tons corn husks at $1\frac{1}{4}$ cents a pound.
5. Of 3420 lb. hay at \$8.00 per ton.

(a) Find the sum of the five results.

Find the cost—

6. Of 2140 lb. oats at 24¢ a bushel.
7. Of 2140 lb. corn at 28¢ a bushel.
8. Of 2140 lb. wheat at 90¢ a bushel.
9. Of 2140 lb. barley at 36¢ a bushel.
10. Of 2140 lb. rye at 42¢ a bushel.

(b) Find the sum of the five results.

Find the cost—

11. Of 520 lb. clover seed at \$6.30 a bushel.
12. Of 520 lb. potatoes at 75¢ a bushel.
13. Of 520 lb. beans at \$2.15 a bushel.
14. Of 520 lb. corn at 20¢ a bushel.
15. Of 520 lb. ear corn at 35¢ a bushel.

(c) Find the sum of the five results.

Find the approximate weight—

16. Of a barrel of kerosene.
17. Of 1 quart of milk.
18. Of the oats that will fill a bin that is 4 ft. by 4 ft. by 9 ft.
19. Of the water that will fill a tank that is 2 ft. by 2 ft. by 12 ft.

Denominate Numbers—Weight.

350. Troy weight is used in weighing gold, silver, and jewels.

TROY WEIGHT.

24 grains (gr.)	=	1 pennyweight (pwt.).
20 pennyweights	=	1 ounce (oz.).
12 ounces	=	1 pound (lb.).

351. Apothecaries' weight is used in mixing medicines and in selling them at retail.

APOTHECARIES' WEIGHT.

20 grains (gr.)	=	1 scruple (℈).
3 scruples	=	1 dram (ʒ).
8 drams	=	1 ounce (℥).
12 ounces	=	1 pound (℔).

NOTE 1.—The pound Troy and the pound Apothecary are equal, each weighing 5760 grains. The pound Avoirdupois weighs 7000 Troy or Apothecary grains.

NOTE 2.—The ounce Troy and the ounce Apothecary are each 480 grains; the ounce Avoirdupois is 437½ grains.

QUERY.—Which is heavier, a pound of feathers or a pound of gold? An ounce of feathers or an ounce of gold?

PROBLEMS.

1. Change 5 lb. Avoirdupois weight to pounds, ounces, etc., Apothecaries' weight.

2. How many 5 gr. powders can be made from one Avoirdupois ounce of quinine?

3. One Avoirdupois ton of gold is how many Troy pounds?

4. Twenty-four Troy pounds equal how many Avoirdupois pounds?

Denominate Numbers—Time.

352. The standard units in the measurement of time are the **day** and the **year**.

NOTE 1.—The **solar day** is the interval between the time when the sun is on a given meridian and the time when it appears on that meridian again. These intervals (solar days) are not perfectly uniform. The average of these intervals is the **mean solar day**—the day noted by our watches and our clocks—the day, one twenty-fourth of which is called an hour.

NOTE 2.—The **solar year** is the time of one revolution of the earth around the sun, or nearly 365¼ mean solar days. The calendar year of 365 days is nearly 6 hours less than the solar year. Four years of 365 days each, would lack nearly 24 hours (4 times ¼ da.) of being equal to 4 solar years. Hence 1 day is added to 365 every fourth year, with the exception noted below.

NOTE 3.—The exact length of a solar year is 365 da. 5 hr. 48 min. 48 sec. Since this lacks 11 min. 12 sec. of being 365¼ days, it follows that if *every* fourth year should contain 366 days, in 400 years the calendar years would amount to 3 days (400 times 11 min. 12 sec.) more than the solar years. Hence three of the years that would otherwise contain 366 days, are made to contain 365 days. The years thus changed to 365 days are those ending the centuries (1800, 1900, 2000, 2100, etc.) unless the number denoting the year is exactly divisible by 400.

MEASURE OF TIME.

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (da.).
7 days	= 1 week (wk.).
365 days	= 1 common year.
52 wk. 1 da.	= 1 common year.
366 days	= 1 leap year.
52 wk. 2 da.	= 1 leap year.
12 months	= 1 year.

Denominate Numbers—Time.

EXERCISES.

1. If Jan. 1 of a common year is Monday, Feb. 1 is ——; March 1 is ——; April 1 is ——; May 1 is ——; June 1 is ——; July 1 is ——; August 1 is ——; September 1 is ——; October 1 is ——; November 1 is ——; December 1 is ——.

2. Jan. 1, 1899, was Sunday. Tell the day of the week for each of the following dates:

1900, January 1; February 1; February 8.
1901, January 1; February 1; February 9.
1902, January 1; February 1; March 1.
1903, January 1; February 1; March 10.

PROBLEMS.

1. How many days from Jan. 1, 1900, to Aug. 17, 1900?

2. Jan. 1, 1900, falls on Monday. Upon what day of the week does Aug. 17, 1900, fall?

3. What month begins on the same day of the week as January in every common year?

4. What months begin on the same day of the week as February in every common year?

5. If January begins on Sunday, (a) how many Sundays in the month? (b) How many Mondays? (c) How many Tuesdays? (d) How many Wednesdays?

6. If a common year begins on Sunday, (a) how many Sundays in the year? (b) How many Mondays?

7. (a) How many days old are you? (b) How many weeks old? (c) Upon what day of the week were you born?

Denominate Numbers—Circular Measure.

353. For the purpose of measurement, every circumference is supposed to be divided into 360 equal parts. Each of these parts is called an arc of 1 **degree.**

354. An **angle is measured** by regarding its vertex as the center of a circle, its sides being extended until they cut the circumference. The angle is measured by the arc lying between its sides. If the intercepted arc is an arc of 45 degrees, the angle is an angle of 45 degrees; if the arc is 20 degrees, the angle is 20 degrees, etc.

Circular and Angular Measure.

60 seconds (″)	=	1 minute (′).
60 minutes	=	1 degree (°).
360 degrees	=	1 circumference.

355. In geography, **a meridian** is a north and south line on the surface of the earth, extending from pole to pole.

356. Longitude is distance east or west in degrees (or parts of degrees) from a given meridian. Longitude is usually measured from the meridian that passes through Greenwich.

Problems.

Find difference in longitude between—

1. Rome, 12° 27′ east, and Washington, 70° 2′ 48″ west.
2. Washington and Chicago, 87° 37′ 30″ west.
3. Chicago and Denver, 104° 59′ 23″ west.
4. Denver and San Francisco, 122° 24′ 15″ west.
5. San Francisco and Berlin, 13° 23′ 53″ east.
6. Washington and Honolulu, 157° 50′ 36″ west.
7. Washington and Manila, 121° east.

Denominate Numbers—Longitude and Time.

357. *One degree of longitude corresponds to 4 minutes of time.*

EXPLANATORY.—The sun seems to move over 360 degrees of longitude in 24 hours; over 1 degree in $\frac{1}{360}$ of 24 hours = 4 minutes.

PROBLEMS.*

1. When it is noon at Greenwich, what is the time at Washington, 77° 2′ 48″?

2. When it is noon at Washington, what is the time at Greenwich?

3. When it is noon at Portland, Maine, 70° 15′ 40″, what is the time at San Francisco, 122° 24′ 15″?

4. When it is noon at San Francisco, what is the time at Portland, Maine?

358. Standard Railroad Time. Every railroad train in North America is run on the time of some one of the following meridians:

60th meridian (passing through Labrador).
75th meridian (passing near Philadelphia).
90th meridian (passing near St. Louis).
105th meridian (passing near Denver).
120th meridian (passing near Carson City).

Observe that each of the above numbers after the first is 15 greater than the one preceding it; and that 15 degrees of longitude correspond to 1 hour of time.

* At first the pupil should give an approximate answer to these problems, *considering integral degrees only.* Later, if thought advisable, he may take into the account the parts of degrees. One arc minute corresponds to $\frac{1}{60}$ of 4 minutes of time, or 4 seconds of time. One arc second corresponds to $\frac{1}{3600}$ of 4 minutes of time, or $\frac{4}{60}$ of one second of time. That is, each arc degree corresponds to 4 minutes of time; each arc minute, to 4 seconds of time; each arc second, to $\frac{4}{60}$ of a second of time. Hence multiplying by 4 the figures standing for degrees, minutes, and seconds, of longitude will give the figures standing for minutes, seconds, and 60ths of seconds, of time.

Denominate Numbers—Value.

359. The standard unit of value in the United States is the **dollar.**

UNITED STATES MONEY.

10 mills (m.)	= 1 cent (ct. or ¢).
10 cents	= 1 dime (d.).
10 dimes	= 1 dollar ($).

360. The standard unit of value in Great Britain and Ireland is the **pound.** Its value, reckoned in United States money, is $4.866½.

ENGLISH, OR STERLING, MONEY.

4 farthings (far.)	= 1 penny (*d.*).
12 pence	= 1 shilling (*s.*).
20 shillings	= 1 pound (£).
5 shillings	= 1 crown.
21 shillings	= 1 guinea.

361. The standard unit of value in France is the **franc.** Its value, reckoned in United States money, is 19.3¢.

362. The standard unit of value in Germany is the **mark.** Its value, reckoned in United States money, is 23.85¢.

363. The standard unit of value in Russia is the **ruble.** Its value, reckoned in United States money, is $.772.

PROBLEMS.

1. Find the value of a guinea in United States money.
2. Find the value of $1000 in English money.
3. Find the value of £5000 in United States money.
4. Find the value of £1000 in Russian money.
5. Find the value of 4000 marks in English money.
6. Find the value of 8000 francs in United States money.

SHORT METHODS.

Multiplication and Division.

Art. 1. To multiply a number by 50: *Multiply* $\frac{1}{2}$ *of the number by* 100. Why?

$46 \times 50 \qquad \frac{1}{2}$ of $46 \times 100 = 2300$

$47 \times 50 \qquad \frac{1}{2}$ of $47 \times 100 = 2350$

Multiply:

44 by 50	32 by 50	35 by 50
36 by 50	38 by 50	27 by 50
64 by 50	42 by 50	55 by 50
82 by 50	76 by 50	43 by 50
46 by 50	52 by 50	53 by 50

(a) Find the sum of the fifteen products.

Art. 2. To multiply a number by 51: *Take* 50 *times the number, to which add the number itself.* Why?

$48 \times 50 \qquad 50$ times $48 = 2400 \qquad 2400 + 48 = 2448$

$37 \times 51 \qquad 50$ times $37 = 1850 \qquad 1850 + 37 = 1887$

Multiply:

26 by 51	34 by 51	35 by 51
46 by 51	32 by 51	29 by 51
24 by 51	42 by 51	43 by 51
66 by 51	84 by 51	33 by 51
36 by 51	38 by 51	39 by 51

(b) Find the sum of the fifteen products.

Art. 3. To multiply a number by 52: *Take* 50 *times the number, to which add twice the number.* Why?

34×52	50 times $34 = 1700$	$1700 + 68 = 1768$
45×52	50 times $45 = 2250$	$2250 + 90 = 2340$

Multiply:

26 by 52	38 by 52	27 by 52
18 by 52	14 by 52	17 by 52
24 by 52	32 by 52	35 by 52
36 by 52	44 by 52	23 by 52

(c) Find the sum of the twelve products.

Art. 4. To multiply a number by 49: *Take* 50 *times the number, from which subtract the number itself.* Why?

24×49	50 times $24 = 1200$	$1200 - 24 = 1176$
33×49	50 times $33 = 1650$	$1650 - 33 = 1617$

Multiply:

18 by 49	22 by 49	27 by 49
28 by 49	34 by 49	35 by 49
16 by 49	46 by 49	43 by 49

(d) Find the sum of the nine products.

Art. 5. To multiply a number by $33\frac{1}{3}$: *Multiply* $\frac{1}{3}$ *of the number by* 100. Why?

$36 \times 33\frac{1}{3}$	$\frac{1}{3}$ of $36 \times 100 = 1200$
$37 \times 33\frac{1}{3}$	$\frac{1}{3}$ of $37 \times 100 = 1233\frac{1}{3}$

Multiply:

24 by $33\frac{1}{3}$	27 by $33\frac{1}{3}$	28 by $33\frac{1}{3}$
18 by $33\frac{1}{3}$	21 by $33\frac{1}{3}$	22 by $33\frac{1}{3}$
30 by $33\frac{1}{3}$	33 by $33\frac{1}{3}$	35 by $33\frac{1}{3}$

(e) Find the sum of the nine products.

Art. 6. To multiply a number by $34\frac{1}{3}$: *Take* $33\frac{1}{3}$ *times the number, to which add the number itself.* Why?

$24 \times 34\frac{1}{3}$	$33\frac{1}{3}$ times $24 = 800$	$800 + 24 = 824$
$25 \times 34\frac{1}{3}$	$33\frac{1}{3}$ times $25 = 833\frac{1}{3}$	$833\frac{1}{3} + 25 = 858\frac{1}{3}$

Multiply:

18 by $34\frac{1}{3}$	27 by $34\frac{1}{3}$	24 by $34\frac{1}{3}$
21 by $34\frac{1}{3}$	33 by $34\frac{1}{3}$	39 by $34\frac{1}{3}$
30 by $34\frac{1}{3}$	36 by $34\frac{1}{3}$	15 by $34\frac{1}{3}$
12 by $34\frac{1}{3}$	16 by $34\frac{1}{3}$	17 by $34\frac{1}{3}$

(f) Find the sum of the twelve products.

Art. 7. To multiply a number by $35\frac{1}{3}$: *Take* $33\frac{1}{3}$ *times the number, to which add twice the number.* Why?

$24 \times 35\frac{1}{3}$	$33\frac{1}{3}$ times $24 = 800$	$800 + 48 = 848$
$25 \times 35\frac{1}{3}$	$33\frac{1}{3}$ times $25 = 833\frac{1}{3}$	$833\frac{1}{3} + 50 = 883\frac{1}{3}$

Multiply:

21 by $35\frac{1}{3}$	30 by $35\frac{1}{3}$	27 by $35\frac{1}{3}$
18 by $35\frac{1}{3}$	66 by $35\frac{1}{3}$	36 by $35\frac{1}{3}$
33 by $35\frac{1}{3}$	39 by $35\frac{1}{3}$	31 by $35\frac{1}{3}$

(g) Find the sum of the nine products.

Art. 8. To multiply a number by $32\frac{1}{3}$: *Take* $33\frac{1}{3}$ *times the number, from which subtract the number itself.* Why?

$21 \times 32\frac{1}{3}$	$33\frac{1}{3}$ times $21 = 700$	$700 - 21 = 679$
$22 \times 32\frac{1}{3}$	$33\frac{1}{3}$ times $22 = 733\frac{1}{3}$	$733\frac{1}{3} - 22 = 711\frac{1}{3}$

Multiply:

15 by $32\frac{1}{3}$	21 by $32\frac{1}{3}$	18 by $32\frac{1}{3}$
24 by $32\frac{1}{3}$	30 by $32\frac{1}{3}$	27 by $32\frac{1}{3}$
33 by $32\frac{1}{3}$	39 by $32\frac{1}{3}$	36 by $32\frac{1}{3}$

(h) Find the sum of the nine products.

Art. 9. To multiply a number by 25: *Multiply $\frac{1}{4}$ of the number by* 100. Why?

$48 \times 25 \qquad \frac{1}{4}$ of $48 \times 100 = 1200$
$49 \times 25 \qquad \frac{1}{4}$ of $49 \times 100 = 1225$

Multiply:

36 by 25	32 by 25	33 by 25
40 by 25	28 by 25	29 by 25
24 by 25	16 by 25	19 by 25
52 by 25	48 by 25	50 by 25

(i) Find the sum of the twelve products.

Art. 10. To multiply a number by 26: *Take* 25 *times the number, to which add the number itself.* Why?

$36 \times 26 \qquad 25$ times $36 = 900 \qquad 900 + 36 = 936$
$37 \times 26 \qquad 25$ times $37 = 925 \qquad 925 + 37 = 963$

Multiply:

36 by 26	48 by 26	45 by 26
28 by 26	24 by 26	25 by 26
44 by 26	32 by 26	35 by 26

(j) Find the sum of the nine products.

Art. 11. To multiply a number by 27: *Take* 25 *times the number, to which add twice the number.* Why?

$36 \times 27 \qquad 25$ times $36 = 900 \qquad 900 + 72 = 972$
$37 \times 27 \qquad 25$ times $37 = 925 \qquad 925 + 74 = 999$

Multiply:

48 by 27	52 by 27	37 by 27
32 by 27	16 by 27	17 by 27
28 by 27	24 by 27	26 by 27

(k) Find the sum of the nine products.

Art. 12. To multiply a number by 24: *Take* 25 *times the number, from which subtract the number itself.* Why?

32×24	25 times $32 = 800$	$800 - 32 = 768$
33×24	25 times $33 = 825$	$825 - 33 = 792$

Multiply:

24 by 24	16 by 24	17 by 24
44 by 24	36 by 24	37 by 24
28 by 24	32 by 24	35 by 24
48 by 24	52 by 24	54 by 24

(l) Find the sum of the twelve products.

Art. 13. To multiply a number by 20: *Multiply* $\frac{1}{5}$ *of the number by* 100. How may a number be multiplied by 21? By 22? By 19?

35×21	20 times $35 = 700$	$700 + 35 = 735$

Multiply:

45 by 21	45 by 22	45 by 19
35 by 21	35 by 22	35 by 19
36 by 21	36 by 22	36 by 19

(m) Find the sum of the nine products.

Art. 14. To multiply a number by $16\frac{2}{3}$: *Multiply* $\frac{1}{6}$ *of the number by* 100. How may a number be multiplied by $17\frac{2}{3}$? By $18\frac{2}{3}$? By $15\frac{2}{3}$?

$24 \times 17\frac{2}{3}$	$16\frac{2}{3}$ times $24 = 400$	$400 + 24 = 424$

Multiply:

18 by $17\frac{2}{3}$	18 by $18\frac{2}{3}$	18 by $15\frac{2}{3}$
30 by $17\frac{2}{3}$	30 by $18\frac{2}{3}$	30 by $15\frac{2}{3}$
36 by $17\frac{2}{3}$	36 by $18\frac{2}{3}$	36 by $15\frac{2}{3}$

(n) Find the sum of the nine products.

Art. 15. To multiply a number by $12\frac{1}{2}$: *Multiply* $\frac{1}{8}$ *of the number by* 100. How may a number be multiplied by $13\frac{1}{2}$? By $14\frac{1}{2}$? By $11\frac{1}{2}$?

$32 \times 13\frac{1}{2}$ $12\frac{1}{2}$ times $32 = 400$ $400 + 32 = 432$

Multiply:

24 by $13\frac{1}{2}$	24 by $14\frac{1}{2}$	24 by $11\frac{1}{2}$
16 by $13\frac{1}{2}$	16 by $14\frac{1}{2}$	16 by $11\frac{1}{2}$
40 by $13\frac{1}{2}$	40 by $14\frac{1}{2}$	40 by $11\frac{1}{2}$

(o) Find the sum of the nine products.

Art. 16. To multiply a number by 125: *Multiply* $\frac{1}{8}$ *of the number by* 1000. How may a number be multiplied by 126? By 127? By 124?

96×125 $\frac{1}{8}$ of $96 \times 1000 = 12000$

96×126 125 times $96 = 12000$ $12000 + 96 = 12096$

Multiply:

120 by 126	120 by 127	120 by 124
320 by 126	320 by 127	320 by 124
240 by 126	240 by 127	240 by 124

(p) Find the sum of the nine products.

Art. 17. To multiply a number by 250: *Multiply* $\frac{1}{4}$ *of the number by* 1000. How may a number be multiplied by 251? By 252? By 249?

48×250 $\frac{1}{4}$ of $48 \times 1000 = 12000$

48×251 250 times $48 = 12000$ $12000 + 48 = 12048$

Multiply:

60 by 251	60 by 252	60 by 249
72 by 251	72 by 252	72 by 249

(q) Find the sum of the six products.

Art. 18. To square $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, etc.: *Multiply the integer by the integer plus* 1, *and add* $\frac{1}{4}$ *to the product.*

$$2\tfrac{1}{2} \times 2\tfrac{1}{2} = \overline{2 \text{ times } 2} + \overline{2 \text{ times } \tfrac{1}{2}} + \overline{\tfrac{1}{2} \text{ of } 2} + \overline{\tfrac{1}{2} \text{ of } \tfrac{1}{2}}$$

But $\overline{2 \text{ times } \frac{1}{2}} + \overline{\frac{1}{2} \text{ of } 2} = 1$ time 2; and $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$

$$\text{Hence, } 2\tfrac{1}{2} \times 2\tfrac{1}{2} = \overline{2 \times 3} + \tfrac{1}{4} = 6\tfrac{1}{4}$$

$$3\tfrac{1}{2} \times 3\tfrac{1}{2} = \overline{3 \times 4} + \tfrac{1}{4} = 12\tfrac{1}{4}$$

Multiply:

$4\frac{1}{2}$ by $4\frac{1}{2}$	$5\frac{1}{2}$ by $5\frac{1}{2}$	$6\frac{1}{2}$ by $6\frac{1}{2}$
$7\frac{1}{2}$ by $7\frac{1}{2}$	$8\frac{1}{2}$ by $8\frac{1}{2}$	$9\frac{1}{2}$ by $9\frac{1}{2}$
$1\frac{1}{2}$ by $1\frac{1}{2}$	$2\frac{1}{2}$ by $2\frac{1}{2}$	$3\frac{1}{2}$ by $3\frac{1}{2}$

(r) Find the sum of the nine products.

Art. 19. To square 25, 35, 45, etc.: *Multiply the tens' figure* by the tens' figure increased by* 1; *regard the product as hundreds, to which add* 25.

To explain this rule, think of 25 as 2 *tens and* $\frac{1}{2}$ *of a ten,* and apply the explanation given under Art. 18.

$$25 \times 25 = \overline{2 \times 3} \text{ hundred and } 25 = 625$$

$$35 \times 35 = \overline{3 \times 4} \text{ hundred and } 25 = 1225$$

$$45 \times 45 = \overline{4 \times 5} \text{ hundred and } 25 = 2025$$

Multiply:

55 by 55	65 by 65	75 by 75
85 by 85	95 by 95	15 by 15

(s) Find the sum of the six products.

*The author is aware that the expressions "*Multiply the tens' figure*" and "*the tens' figure increased by 1*" are tabooed by the hypercritical. But it is believed that neither obscurity nor misconception will arise from this use of the word *figure.* The word as here used clearly means the *form value of the figure*—the *number* which the figure by virtue of its shape represents.

Art. 20. To multiply $2\frac{1}{4}$ by $2\frac{3}{4}$, $3\frac{1}{5}$ by $3\frac{4}{5}$, etc.: *Multiply the integer by the integer plus* 1, *and to the product add the product of the fractions.*

Observe that this rule will apply only when the integer of the multiplicand and the integer of the multiplier are the same, and the sum of the fractions is 1.

$$2\tfrac{1}{4} \text{ by } 2\tfrac{3}{4} = \overline{2 \text{ times } 2} + \overline{2 \text{ times } \tfrac{1}{4}} + \overline{\tfrac{3}{4} \text{ of } 2} + \overline{\tfrac{3}{4} \text{ of } \tfrac{1}{4}}$$

$$\text{But } \overline{2 \text{ times } \tfrac{1}{4}} + \overline{\tfrac{3}{4} \text{ of } 2} = 1 \text{ time } 2, \text{ and } \overline{\tfrac{3}{4} \text{ of } \tfrac{1}{4}} = \tfrac{3}{16}$$

$$\text{Hence, } 2\tfrac{1}{4} \times 2\tfrac{3}{4} = \overline{2 \times 3} + \overline{\tfrac{3}{4} \text{ of } \tfrac{1}{4}} = 6\tfrac{3}{16}$$

$$3\tfrac{1}{5} \times 3\tfrac{4}{5} = \overline{3 \times 4} + \tfrac{4}{5} \text{ of } \tfrac{1}{5} = 12\tfrac{4}{25}$$

Multiply:

$4\frac{1}{3}$ by $4\frac{2}{3}$	$5\frac{1}{4}$ by $5\frac{3}{4}$	$6\frac{1}{3}$ by $6\frac{2}{3}$
$7\frac{1}{4}$ by $7\frac{3}{4}$	$8\frac{2}{3}$ by $8\frac{1}{3}$	$9\frac{1}{4}$ by $9\frac{3}{4}$

(t) Find the sum of the six products.

Art. 21. To multiply 24 by 26. 33 by 37, etc.: *Multiply the tens' figure by the tens' figure increased by* 1; *regard the product as hundreds, to which add the product of the units' figures.*

Observe that this rule will apply only when the tens' figure of the multiplicand and the tens' figure of the multiplier are alike, and the sum of the units' figures is 10.

$$22 \times 28 = \overline{2 \times 3} \text{ hundred and } 16 = \ \ 616$$

$$33 \times 37 = \overline{3 \times 4} \text{ hundred and } 21 = 1221$$

Multiply:

21 by 29	23 by 27	24 by 26
31 by 39	32 by 38	34 by 36
41 by 49	42 by 48	43 by 47

(u) Find the sum of the nine products.

Art. 22. To multiply a number by 15: *Multiply the number by* 10, *and to the product add* $\frac{1}{2}$ *of the product.*

64×15	10 times $64 = 640$	$640 + 320 = 960$
45×15	10 times $45 = 450$	$450 + 225 = 675$

Multiply:

24 by 15	32 by 15	35 by 15
46 by 15	34 by 15	43 by 15
82 by 15	66 by 15	75 by 15
37 by 15	41 by 15	39 by 15

(v) Find the sum of the twelve products.

Art. 23. To multiply a number by 99: *Take* 100 *times the number, from which subtract the number itself.* How may a number be multiplied by 98?

36×99	100 times $36 = 3600$	$3600 - 36 = 3564$
42×98	100 times $42 = 4200$	$4200 - 84 = 4116$

Multiply:

35 by 99	44 by 99	35 by 98
27 by 99	54 by 99	46 by 98
62 by 99	75 by 99	28 by 98

(w) Find the sum of the nine products.

Art. 24. To multiply a number by 75: *Multiply* $\frac{3}{4}$ *of the number by* 100. How may a number be multiplied by $66\frac{2}{3}$? By $62\frac{1}{2}$? By $87\frac{1}{2}$?

Multiply:

64 by 75	24 by $66\frac{2}{3}$	64 by $87\frac{1}{2}$
48 by 75	36 by $66\frac{2}{3}$	48 by $87\frac{1}{2}$
52 by 75	63 by $66\frac{2}{3}$	56 by $87\frac{1}{2}$
37 by 75	37 by $66\frac{2}{3}$	32 by $87\frac{1}{2}$

(x) Find the sum of the twelve products.

Art. 25. To divide a number by 25; by $33\frac{1}{3}$; by $12\frac{1}{2}$; by $16\frac{2}{3}$; by 20; by 50. (See pp. 212, 213, and 214, of this book.)

$$850 \div 25 = \overline{8 \text{ times } 4} + 2 = 34$$

$$933\tfrac{1}{3} \div 33\tfrac{1}{3} = \overline{9 \text{ times } 3} + 1 = 28$$

$$637\tfrac{1}{2} \div 12\tfrac{1}{2} = \overline{6 \text{ times } 8} + 3 = 51$$

$$750 \div 16\tfrac{2}{3} = \overline{7 \text{ times } 6} + 3 = 45$$

$$960 \div 20 = \overline{9 \text{ times } 5} + 3 = 48$$

$$450 \div 50 = \overline{4 \text{ times } 2} + 1 = 9$$

Divide:

1275 by 25	$1166\frac{2}{3}$ by $33\frac{1}{3}$	$762\frac{1}{2}$ by $12\frac{1}{2}$
950 by $16\frac{2}{3}$	880 by 20	950 by 50

(y) Find the sum of the six quotients.

NOTE.—Without a pencil, tell the integral quotient and the remainder resulting from the incomplete division of 1584 by 25.

Art. 26. To divide a number by 125; by 250: *Observe that* 125 *is contained in each thousand of a number,* 8 *times; that* 250 *is contained in each thousand of a number,* 4 *times.*

$$7125 \div 125 = \overline{7 \text{ times } 8} + 1 = 57$$

$$8500 \div 250 = \overline{8 \text{ times } 4} + 2 = 34$$

Divide:

13250 by 250	13500 by 250	13750 by 250
18000 by 250	18750 by 250	18500 by 250
12000 by 125	12500 by 125	12625 by 125
9125 by 125	9375 by 125	9875 by 125

(z) Find the sum of the twelve quotients.

NOTE.—Without a pencil, tell the integral quotient and the remainder resulting from the incomplete division of 15450 by 250.

Art. 27. When the same factor occurs in a dividend and in its divisor, it may be omitted from both without changing their ratio. Hence *all the factors that are common to a dividend and its divisor may be stricken out* (canceled) *and the quotient* (ratio) *be unchanged.*

Divide 180 by 42.

Operation No. 1.

$$42)180(4\tfrac{2}{7}$$
$$\underline{168}$$
$$\frac{12}{42} = \frac{2}{7}$$

Operation No. 2.

$$\frac{180}{42} = \frac{\cancel{2} \times 2 \times \cancel{3} \times 3 \times 5}{\cancel{2} \times \cancel{3} \times 7} = \frac{30}{7} = 4\frac{2}{7}$$

Observe that the striking out of the factors 2 and 3 from the dividend and its divisor does not change their ratio—the quotient.

II. Divide 420 by 35.

Operation No. 1.

$$35)420(12$$
$$\underline{35}$$
$$70$$
$$\underline{70}$$

Operation No. 2.

$$\frac{420}{35} = \frac{2 \times 2 \times 3 \times \cancel{5} \times \cancel{7}}{\cancel{5} \times \cancel{7}} = \frac{12}{1} = 12$$

Observe that if all the factors of one of the numbers are canceled, the number becomes 1 and not 0. The factor 5 is *5 times 1;* the factor 7, *7 times 1.* Hence in the above problem there really remain in the divisor, after the cancellation, the factors 1 and $1 = 1 \times 1 = 1$.

III. Divide $48 \times 8 \times 4 = 1536$ by $8 \times 4 \times 4 = 128$.

Operation No. 1.

$$128)1536(12$$
$$\underline{128}$$
$$256$$
$$\underline{256}$$

Operation No. 2.

$$\frac{\overset{6}{\cancel{48}} \times \overset{2}{\cancel{8}} \times \cancel{4}}{\cancel{8} \times \cancel{4} \times \cancel{4}} = \frac{12}{1} = 12$$

Observe that it is not necessary to obtain the *prime* factors of a dividend and its divisor to employ cancellation in finding the quotient. In the above the composite factor 8 is stricken out of the divisor and out of the 48 of the dividend.

IV. Divide $56 \times 35 = 1960$ by $15 \times 8 = 120$.

Operation No. 1.

$$\begin{array}{r|l} 120)1960(16\frac{1}{3} \\ 120 \\ \hline 760 \\ 720 \\ \hline \frac{40}{120} = \frac{1}{3} \end{array}$$

Operation No. 2.

$$\frac{\overset{7}{\cancel{56}} \times \overset{7}{\cancel{35}}}{\underset{8}{\cancel{15}} \times \cancel{8}} = \frac{49}{3} = 16\frac{1}{3}$$

Observe that in the above the factor 5 is stricken out of 15 and 35, and the factor 8 is stricken out of the divisor and out of the 56 of the dividend.

Miscellaneous Problems.

Note.—Employ "Short Methods" in the solution of the following problems.

How many cords of wood—

1. In a pile 32 feet by 8 feet by 4 feet? *
2. In a pile 40 feet by 16 feet by 6 feet?
3. In a pile 32 feet by 30 feet by 10 feet?

(aa) Find the sum of the three results.

How many acres of land—

4. In a piece 180 rods by 28 rods? †
5. In a piece 64 rods by 96 rods?
6. In a piece 136 rods by 32 rods?

(bb) Find the sum of the three results.

7. Multiply 64 by 96 and divide the product by $16 \times 24 \times 2$.

8. Multiply 250 by 72 and divide the product by $16\frac{2}{3} \times 3 \times 24$.

(cc) Find the sum of the two results.

* Think of a cord as 8 feet by 4 feet by 4 feet.

† Think of an acre as 40 rods by 4 rods.

Find the cost—

9. Of 346 acres of land at $50 per acre.
10. Of 346 acres of land at $51 per acre.
11. Of 346 acres of land at $52 per acre.
12. Of 346 acres of land at $49 per acre.
13. Of 254 acres of land at $51 per acre.
(dd) Find the sum of the five results.

14. Of 243 ft. iron pipe at $33\frac{1}{3}$¢ a foot.
15. Of 243 ft. iron pipe at $34\frac{1}{3}$¢ a foot.
16. Of 243 ft. iron pipe at $35\frac{1}{3}$¢ a foot.
17. Of 243 ft. iron pipe at $32\frac{1}{3}$¢ a foot.
18. Of 156 ft. iron pipe at $35\frac{1}{3}$¢ a foot.
(ee) Find the sum of the five results.

19. Of 260 lb. butter at 25¢ a pound.
20. Of 260 lb. butter at 26¢ a pound.
21. Of 260 lb. butter at 27¢ a pound.
22. Of 260 lb. butter at 24¢ a pound.
23. Of 184 lb. butter at 27¢ a pound.
(ff) Find the sum of the five results.

24. Of 350 lb. coffee at $12\frac{1}{2}$¢ a pound.
25. Of 350 lb. coffee at $13\frac{1}{2}$¢ a pound.
26. Of 350 lb. coffee at $14\frac{1}{2}$¢ a pound.
27. Of 350 lb. coffee at $11\frac{1}{2}$¢ a pound.
28. Of 330 lb. coffee at $16\frac{2}{3}$¢ a pound.
29. Of 330 lb. coffee at $17\frac{2}{3}$¢ a pound.
30. Of 330 lb. coffee at $15\frac{2}{3}$¢ a pound.
31. Of 240 lb. coffee at 25¢ a pound.
32. Of 240 lb. coffee at 26¢ a pound.
33. Of 240 lb. coffee at 27¢ a pound.
(gg) Find the sum of the ten results.

Find the cost—

34. Of $2\frac{1}{2}$ tons coal at $\$2\frac{1}{2}$ per ton.

35. Of $3\frac{1}{2}$ tons coal at $\$3\frac{1}{2}$ per ton.

36. Of $4\frac{1}{2}$ tons coal at $\$4\frac{1}{2}$ per ton.

(hh) Find the sum of the three results.

37. Of 25 tons of meal at $25 per ton.

38. Of 35 acres of land at $35 per acre.

39. Of 45 M. ft. of lumber at $45 per M.

(ii) Find the sum of the three results.

40. Of 23 yd. cloth at 27¢ a yard.

41. Of 36 yd. cloth at 34¢ a yard.

42. Of 42 yd. cloth at 48¢ a yard.

(jj) Find the sum of the three results.

43. Of 3240 ft. lumber at $15 per M.

44. Of 2460 ft. lumber at $15 per M.

45. Of 1620 ft. lumber at $16 per M.

(kk) Find the sum of the three results.

46. Of 99 lb. butter at 23¢ a pound.

47. Of 99 lb. butter at 28¢ a pound.

48. Of 98 lb. butter at 24¢ a pound.

(ll) Find the sum of the three results.

49. Paid $15.50 for ribbon at $16\frac{2}{3}$¢ a yard. How many yards did I buy?

50. Paid $24.75 for ribbon at $12\frac{1}{2}$¢ a yard. How many yards did I buy?

(mm) Find the sum of the two results.

PRACTICAL APPROXIMATIONS.

So far as practicable, solve the following problems without the aid of a pencil. At least, **exercise the judgment** on every problem *before making any figures.*

1. The specific gravity* of iron being about $7\frac{1}{4}$, how much does a cubic foot of it weigh? How much does a cubic inch of iron weigh?

2. A 4-inch iron ball weighs about —— pounds. A 2-inch iron ball weighs about —— pounds.

3. An iron rod, 1 inch in diameter and 12 feet long, weighs about —— pounds. An iron rod 2 inches in diameter and 12 feet long weighs about —— pounds.

4. A sheet of boiler iron, 8 feet square and $\frac{3}{8}$ of an inch thick, weighs about —— pounds.

5. What is the weight of the water that will fill a tank 2 feet wide, 2 feet deep, and 10 feet long?

6. The specific gravity of limestone is about $2\frac{1}{2}$. What is the weight of a piece of limestone that is 4 feet square and 3 inches thick?

7. The specific gravity of seasoned white pine is about .5; that is, a piece of white pine weighs about 5 tenths as much as the same bulk of water weighs. How much does a pine board 1 foot wide, 1 inch thick, and 12 feet long, weigh?

8. What is the weight of a stick of timber 12 inches by 12 inches and 20 feet long, if its specific gravity is .7?

9. What is the weight of 1000 feet of green lumber if its specific gravity is .9?

*When we say that the specific gravity of iron is about $7\frac{1}{4}$, we mean that it weighs about $7\frac{1}{4}$ times as much as water, the same bulk being considered.

10. The specific gravity of sand is about 2. How much does a load (27 cu. ft.) of it weigh?

11. If the specific gravity of granite is 2.7, how much does a cubic yard of it weigh?

12. The specific gravity of brick and mortar is nearly 2. What is the weight of a cubic foot of brick wall?

13. If a brick, 8 in. by 4 in. by 2 in., weighs $4\frac{1}{2}$ pounds, is its specific gravity more or less than 2? That is, does 1 cubic foot of bricks weigh more or less than exactly twice as much as 1 cubic foot of water?

14. If the specific gravity of Athens (Illinois) limestone is 2.4, and if a cord of it is equal to 100 solid feet, how much does a cord of this stone weigh?

15. If bricks, 8 in. long, 4 in. wide, and 2 in. thick, are laid on their largest face, how many bricks will be required for a walk 6 feet wide and 100 feet long, making some allowance for imperfect bricks and breakage in handling?

16. What is the capacity in gallons of a tank 5 feet long, 2 feet wide, and 2 feet deep?

17. Give the dimensions of a tank the capacity of which is 225 gallons.

18. A certain cylindrical tank is 8 feet in diameter. Each foot in depth will contain how many gallons?

19. One inch of rain-fall will give how many pounds of water on a horizontal surface 30 feet by 40 feet?

20. Two inches of rain-fall will give how many barrels ($31\frac{1}{2}$ gal.) of water on a horizontal surface 40 feet by 60 feet?

21. A one inch rain-fall will give how many tons of water to the acre?

22. Your school-house lot is what part of an acre?

23. A one inch rain-fall will give how many barrels ($31\frac{1}{2}$ gal.) of water on the school-house lot?

24. Your school-room floor is what part of an acre?

25. The distance around your school-room is what part of a mile?

26. How many bushels of oats would be required to fill your school-room to the depth of 3 feet?

27. A shed as large as your school-room would hold how many cords of wood?

28. Give the dimensions of a crib that will hold 1000 bushels of corn.

29. Give the dimensions of a pile of wood that contains 40 cords.

30. A piece of land 100 feet long and 5 rods wide is what part of an acre?

31. How many cubic inches in a cylinder 8 inches in diameter and 10 inches long?

32. How many cubic inches in an 8-inch sphere?

33. If 468.25 be multiplied by .5106 will the product be more or less than 234?*

34. If 484.079 be multiplied by .251, will the product be more or less than 121?

35. If 2480 be multiplied by .2479 will the product be more or less than 620?

36. If 6400 be multiplied by .74, the product will be how many less than $\frac{3}{4}$ of 6400?

37. If 4800 be multiplied by 1.6, the product will be how many more than $1\frac{1}{2}$ times 4800?

38. If 366.06 be divided by $\frac{3}{4}$, will the quotient be more or less than $\overline{366 + \frac{1}{3} \text{ of } 366}$?

* Observe that 234 is ½ of 468.

39. If 25.2314 be divided by $\frac{1}{2}$, will the quotient be more or less than 51?

40. If 250 be divided by .26, will the quotient be more or less than four times 250?

41. If cheese is worth 17 cents a pound, how much should be paid for (a) 2 lb. 3 oz.? (b) 3 lb. 5 oz.? (c) 1 lb. 7 oz.? (d) 4 lb. 9 oz. (e) 2 lb. 11 oz.?

42. If meat is worth 15 cents a pound, how much should be paid for (a) 1 lb. 4 oz.? (b) 2 lb. 6 oz.? (c) 1 lb. 7 oz.? (d) 2 lb. 9 oz.? (e) 3 lb. 13 oz.?

43. If cheese is worth $12\frac{1}{2}$ cents a pound, how much should be paid for (a) 2 lb. 8 oz.? (b) 2 lb. 4 oz.? (c) 3 lb. 1 oz.? (d) 4 lb. 9 oz.? (e) 4 lb. 15 oz.?

44. A wagon box 10 feet by 3 feet by 16 inches will contain how many bushels of shelled corn?

45. The surface of a sphere is equal to 4 times the area of a circle having the same diameter as the sphere. How many square inches in the surface of a 10-inch sphere?

46. How many square inches in the surface of a 20-inch sphere?

47. If from a cylinder of wood the largest possible cone be cut, exactly $\frac{2}{3}$ of the wood will be cut away. The solid content of a cone is therefore exactly $\frac{1}{3}$ of a cylinder having the same base and the same altitude (length). How many cubic inches in a cone the diameter of whose base is 8 inches and whose altitude is 12 inches?

48. How many bushels of grain in a conical pile whose diameter is 6 feet and whose altitude is 4 feet?

49. At 10 cents a square yard, what is the cost of painting the outer surface of a cylindrical standpipe whose diameter is 15 feet and whose altitude is 36 feet?

50. At 10 cents a square foot, what is the cost of lining a cylindrical tank (curved surface and bottom) whose diameter is 10 feet and whose depth is 12 feet?

51. A sphere is exactly $\frac{2}{3}$ and a cone exactly $\frac{1}{3}$ of a cylinder of the same dimensions. Find the solid content and compare the following:

(a) A 6-inch cube.

(b) A cylinder 6 in. in diameter and 6 in. long.

(c) A 6-inch sphere.

(d) A cone; base 6 in. in diameter, altitude 6 in.

52. A cubic foot of steel weighs 490 lb. (a) What is the weight of a cylinder of steel 1 foot in diameter and 1 foot long? (b) Of a sphere of steel 1 foot in diameter? (c) Of a cone of steel, base 1 foot in diameter, altitude 1 foot?

53. A circular piece of land 20 rods in diameter contains more or less than 2 acres?

54. At sight, give approximate answers to the following:

(a) Interest of $450.25 for 1 yr. 3 da. at 6%?

(b) Interest of $4000 for 29 da. at 6%?

(c) Interest of $250 for 1 yr. 8 mo. 5 da. at 6%?

(d) Interest of $500 for 1 yr. 11 mo. 16 da. at 6%?

(e) Interest of $751.27 for 2 yr. 6 mo. 1 da. at 4%?

(f) Interest of $149.75 for 1 yr. 7 mo. 29 da. at 6%?

(g) Interest of $298.97 for 1 yr. 6 mo. 4 da. at 8%?

(h) Interest of $495 for 15 da. at 6%?

(i) Interest of $1200 for 6 da. at 8%?

(j) Interest of $600 for 20 da. at 4%?

(k) Interest of $397.28 for 2 yr. 6 mo. at 6%?

(l) Interest of $5000 for 5 mo. 29 da. at 8%?

(m) Interest of $3000 for 2 mo. 29 da. at 7%?

(n) Interest of $4000 for 3 mo. 29 da. at 6%?

55. At sight, give answers to the following, that are *true to dollars;* then, with the aid of a pencil, if necessary, obtain answers that are *true to cents.*

(a) Cost of 2970 lb. coal at $4.50 per ton?[1]
(b) Cost of 3520 lb. hay at $11.50 per ton?[2]
(c) Cost of 1490 lb. straw at $4.25 per ton?
(d) Cost of 2460 lb. bran at $10.00 per ton?[3]
(e) Cost of 2310 lb. oil meal at $21 per ton?
(f) Cost of 2240 lb. beef at $6.10 per cwt.?
(g) Cost of 1560 lb. pork at $4.50 per cwt.?
(h) Cost of 2150 lb. flour at $3.05 per cwt.?
(i) Cost of 1200 lb. lard at $5.90 per cwt.?
(j) Cost of 1400 lb. tallow at $3.55 per cwt.?
(k) Cost of 1000 lb. nails at $3.05 per cwt.?
(l) Cost of 2240 ft. lumber at $15 per M.?[4]
(m) Cost of 4500 lath at $2.50 per M.?
(n) Cost of 6740 brick at $6.00 per M.?[5]
(o) Cost of 1997 ft. lumber at $27.50 per M.?
(p) Cost of 198 lb. butter at $27\frac{1}{2}$ cts. per lb.?[6]
(q) Cost of $203\frac{1}{2}$ lb. cheese at 16 cents per lb.?
(r) Cost of 2440 lb. oats at $24\frac{1}{4}$ cents per bu.?[7]
(s) Cost of 1680 lb. oats at $23\frac{1}{2}$ cents per bu.?

[1] 2970 lb. is nearly $1\frac{1}{2}$ tons.
[2] What is the cost of 3500 at $12 per ton?
[3] At $10 a ton, how much does 1 lb. cost?
[4] At $15 per M., how much is 1 foot worth?
[5] 6740 brick are nearly $6\frac{3}{4}$ M.
[6] 200 lb. butter at $27\frac{1}{2}$ cents is worth how much?
[7] At 24¢ a bushel, 1 lb. of oats is worth how much?

MISCELLANEOUS PROBLEMS.

NOTE.—The following problems are selected mainly from sets of **examination questions** supplied to the author for this purpose by one hundred school principals and superintendents.

1. Each edge of a cube is diminished by $\frac{1}{10}$ of its length.

(a) By what fraction of itself is the volume diminished?

(b) By what fraction of itself is the surface diminished?

2. How many cubical blocks, each edge of which is $\frac{1}{3}$ ft., are equivalent to a block 8 ft. long, 4 ft. wide, and 2 ft. thick?

3. A ladder 78 ft. long stands perpendicularly against a building. How far must it be pulled out at the foot that the top may be lowered 6 ft.?

4. A merchant sold $\frac{3}{8}$ of a quantity of cloth at a gain of 20% and the remainder at cost.

(a) His gain was what per cent of the cost?

(b) If he gained \$7.29 what was the cost of the goods?

5. What must I pay for 4% stock to get 5% on the investment?

6. The cubical content of one cube is eight times that of another:

(a) How does an edge of the first compare with an edge of the second?

(b) How does the surface of the first compare with the surface of the second?

7. A creditor receives \$1.50 for every \$4.00 that is due him and thereby loses \$301.05.

(a) What was the sum due him? (b) What per cent of the debt did he lose?

8. At $20 per M., board measure, what is the cost of the following: A stick of timber 30 feet long and 14 inches square, and a plank 18 feet long, 8 inches wide, and $2\frac{1}{2}$ inches thick?

9. A and B hire a pasture for $85; A puts in 8 cows and B puts in 12 cows. How much should each pay?

10. Simplify the following: $\frac{1}{3}$ of $\frac{7}{8}$ of $1\frac{1}{2}$.

11. Seven times John's property plus $32200 equals 21 times his property. How much is he worth?

12. Two men engage in business with a joint capital of $5000. The first year's gain was $1760, of which one received $1056. How much capital did each furnish?

13. Thirty-five per cent of the men in a regiment being sick, only 637 men were able to enter battle. How many men were there in the regiment?

14. A lawyer collected 80% of a debt of $2360 and charged 5% commission on the sum collected. How much did the creditor receive?

15. Write a negotiable note for $500, making yourself the payee and James J. Rogers the maker. Interest at the legal rate.

16. A speculator bought stock at 25% below par and sold it at 20% above par. He gained $1035. How much did he invest?

17. What is the rate per cent per annum if $712 gains $142.40 in 3 yr. 4 mo.?

18. A person asked for a lot of land, 40% more than it cost him, but finally reduced his price 15% of his asking price and sold it, making $9.50.

(a) What per cent did he make? (b) How much did the land cost him? (c) How much did he receive for it?

19. Purchased stock at a premium of 8 per cent. What rate of interest do I receive on the investment if it pays an annual dividend of 6%?

20. Find the volume of a cube the area of whose surface is 100.86 square inches.

21. How many apples must a boy buy and sell to make a profit of $9.30, if he buys at the rate of 5 for 3¢ and sells at the rate of 4 for 3¢?

22. Find the cost of 1875 lb. hay at $6.50 per ton.

23. What is the interest on $1200 from Sept. 21, 1898, to May 5, 1899, at 7% per annum?

24. The area of a square field is 10 acres. What is the distance diagonally across the field?

25. A "drummer" earns $2500 a year. One thousand dollars of this sum is a guaranteed salary. The remainder is his commission of 5% on his sales. What is the amount of his annual sales?

26. The area of a triangle is 325 square inches. Its base is 25 inches. What is its altitude?

27. Gave $6\frac{2}{3}$ lb. butter, worth 36¢ a pound, for $3\frac{1}{5}$ gal. oil. What was the cost of the oil per gallon?

28. A can build a certain wall in 10 days; B can build it in 12 days, and C in 15 days. In how many days can they build the wall working together?

29. A horse and a carriage together cost $550. The horse cost $\frac{3}{8}$ as much as the carriage. Find the cost of each?

30. A man willed $\frac{1}{3}$ of his property to his wife, $\frac{1}{3}$ of the remainder to his daughter, and the rest to his son. The difference between the wife's portion and the son's portion was 12480.33\frac{1}{3}$. How much was the man worth?

31. (a) How many loads, each containing a cubic yard, will be required to fill a street 150 feet long, 50 feet wide, and $2\frac{1}{2}$ feet deep? (b) How much will it cost at 18¢ per cubic yard?

32. A right triangle has two equal sides. Its hypothenuse is 100 rods long. (a) Find one of its two equal sides. (b) Find its area.

33. What is the ratio of the area of a circle to the area of its circumscribed square?

34. What is the ratio of the square of the radius to the square of the diameter of the same circle?

35. A man's tax is $37.50. The rate of tax is $1\frac{1}{5}\%$. Property is assessed at 30% of its value. What is the man's property worth?

36. A field containing 160 acres is 40 rods wide. At 45¢ a rod, how much less would it cost to fence a square field containing the same number of acres?

37. My agent in Baltimore having sold a consignment of grain, after taking out his commission at 3% and paying a freight bill of $1,125.00, sent me a draft for the amount due me—$19,536.00. For how much was the grain sold?

38. How many pickets 4 inches wide, placed 3 inches apart, are required to fence a garden 21 rods long and 14 rods wide?

39. At $6.30 a cord, what is the value of wood that can be piled under a shed 50 ft. long, 25 ft. wide, and 12 ft. high?

40. Find the curved surface of a cylinder 6 ft. in diameter and 12 ft. long.

41. (a) What is the bank discount and (b) what are the proceeds on a note for $125 payable in 90 days, the rate of discount being 8%?

42. If sugar that cost 5¢ a pound is sold at 18 lb. for a dollar, what is the gain per cent?

43. What is the area of a circle 30 inches in diameter?

44. What is the volume of a 12-inch globe?

45. What is the circumference of a circle that is 40 rods in diameter?

46. The "number belonging" in a certain school was 74. Five were absent in the forenoon and seven in the afternoon. What was the per cent of attendance for the day?

47. A rectangular piece of land 41 rods by 24 rods is how many acres?

48. Find the value of x in the following proportion: $17.5 : 25 :: x : 40$.

49. A circular $\frac{1}{2}$-mile race track encloses how many acres?

50. If the same number be added to the numerator and to the denominator of a proper fraction, will it make the fraction greater or less?

51. A certain roof is 40 feet long and, measured horizontally, 24 feet in width. A 2-inch rain-fall should give how many inches in depth in a cistern that receives the water from this roof, the cistern being 6 feet long and 4 feet wide?

52. A can do a piece of work in $\frac{1}{3}$ of a day. B can do the same amount of work in $\frac{1}{4}$ of a day. In what part of a day can both working together do the piece of work?

53. C can do a certain piece of work in 3 days. D can do the same amount of work in 4 days. In how long a time can both working together do the piece of work?

54. If the 6-foot drive wheel of a locomotive makes 840 revolutions in moving a certain distance, how many revolutions will a 7-foot wheel make in moving the same distance?

55. The circumference of one of my carriage wheels is 12 feet. The circumference of another wheel on the same carriage is 14 feet. How far has the carriage run when the smaller wheel has made exactly 300 more revolutions than the larger wheel?

56. Is the capacity of a cylindrical pail 6 inches in diameter and 5 inches deep more or less than $\frac{1}{2}$ of a gallon?

57. At what rate per cent must I invest $800 that in 1 year 6 months it will amount to $854?

58. If exactly $\frac{5}{6}$ of a stick of timber floating in the water is submerged, and if the timber is 12 inches by 12 inches and 30 feet long, how many pounds does it weigh?

NOTE.—If $\frac{5}{6}$ of the timber is submerged, it weighs $\frac{5}{6}$ as much as its own bulk of water.

59. How many pickets are required to inclose a square $2\frac{1}{2}$-acre lot if the pickets are 3 inches wide and 3 inches apart?

60. The boundary of a certain field is described as follows: Beginning at the northeast corner of section 14; thence south, 24 rods; thence west, 20 rods; thence south, 15 rods; thence west, 40 rods; thence south, 41 rods; thence west, 20 rods; thence north, 80 rods; thence east, 80 rods, to the place of beginning. How many acres in the field?

61. At 90 cents a yard, find the cost of carpeting a room that is 15 feet wide and 18 feet long, the carpet to run lengthwise of the room, there being a waste of 1 foot on each breadth, *except the first*, for matching; carpet 1 yd. wide.

62. The perimeter of a rectangular field is 144 rods and its length is twice its breadth. Find its area.

63. Change $\frac{2}{7}$ of a mile to a compound number made up of rods, feet, and inches.

64. Sold $\frac{3}{5}$ of a barrel of sugar for what $\frac{1}{2}$ of it cost. What was the per cent of loss?

65. Sold $\frac{1}{2}$ of a barrel of sugar for what $\frac{3}{5}$ of it cost. What was the per cent of gain?

66. If a merchant sells goods at a uniform profit of 20%, and his sales on a certain day amount to $60, his gain is how many dollars?

67. Find the cost of 600 ft. of gas pipe, list 28¢ a foot, at "55 and 3 10's off."*

68. Bought for "60 off" and sold for "50 off." What was the per cent of profit?

69. From $\frac{2}{3}$ of a certain number subtract $\frac{3}{5}$ of it and 27 remains. What is the number?

70. Divide the number 495 into two parts, the ratio of the parts being as 2 to 3.

71. Divide the number 187 into two parts, the ratio of the parts being as $\frac{2}{3}$ to $\frac{3}{4}$.

72. The product of a certain number multiplied by $1\frac{3}{8}$ is 352. What is the number?

73. Find the cost at $16 per M. of 2-in. plank for a floor 24 feet by 42 feet.

74. The perimeter of an oblong is 192 ft., and its length is twice its breadth. Find its area.

75. At $45 per M., find the cost of a board 16 feet long, 18 inches wide, and $1\frac{1}{4}$ inches thick.

76. A man bought a horse and a carriage for $315; he paid $2\frac{1}{2}$ times as much for the carriage as for the horse. Find the cost of each.

* "55 and 3 10's off," means "55 and 10 and 10 and 10 off."

77. A house and lot cost $5000. For how much per month must it rent to pay the owner a sum equal to 5% of its cost and $230 for insurance, taxes, and repairs?

78. If $4\frac{1}{2}$ lb. of butter can be made from 100 lb. of milk, how much butter per week can be made in a creamery that is receiving 15000 lb. of milk a day?

79. The area of a rectangular piece of land 36 rods long is 900 square rods. How many rods of fence required to enclose the field?

80. A shingle is 4 inches wide.* If shingles are laid $4\frac{1}{2}$ inches to the weather, each shingle practically covers —— square inches. Then —— shingles will cover 1 square foot. On account of waste and short measurements, it is necessary to purchase 9 shingles for every square foot to be covered, if they are to be laid $4\frac{1}{2}$ inches to the weather. Shingles are put up in bunches 20 inches wide and containing 50 courses. Hence each bunch contains 250 shingles. At the lumber yards parts of bunches are not offered for sale. How many bunches of shingles must I purchase for a double roof 35 feet long, rafters 16 feet long, the shingles to be laid $4\frac{1}{2}$ inches to the weather?

81. The square of a certain number is 576. What is its cube?

82. The area of one face of a cube is 64 square inches. What is the solid content of the cube?

83. In plowing an acre with a twelve-inch plow the man walking behind it travels ($43560 \div 5280$) $8\frac{1}{4}$ miles. What part of $8\frac{1}{4}$ miles will that man travel who plows an acre with a 14-inch plow? With a 16-inch plow?

* Shingles are not of uniform width; but in counting them at the lumber yards, every 4 inches in width is called 1 shingle.

84. A can do a piece of work in 12 days. A and B can do an equal amount of work in 8 days. In how long a time can B do the work?

85. The edge of one cube is $2\frac{1}{2}$ times as long as the edge of another cube. (a) The surface of the first cube is how many times the surface of the second cube? (b) The solid content of the first cube is how many times the solid content of the second cube?

86. At "50 and 10 off" the net cost was $29.34. Find the list price.

87. How many 160-acre farms in a township 6 miles long and 6 miles wide?

88. If there is a 4-rod road on every section line* of a township 6 miles square, (a) how many acres of the township in its roads? (b) How many acres of each square 160-acre farm are taken for roads?

89. What single discount is equivalent to "40 and 20 and 10 off"?

90. Estimate the weight of a 4-inch sod from an acre of ground.

91. A room 16 feet by 22 feet has a floor made of 4-inch tile. (a) How many tiles in the floor? (b) How many tiles in the border of four rows?

92. If I buy at 20% below list price and sell at 20% above list price, what is my per cent of gain?

93. Add two hundred and seven thousandths, and two hundred seven thousandths.

94. From nine hundred and eight ten-thousandths, subtract nine hundred eight ten-thousandths.

* A section is 1 mile square, and half of the width of the road is on each side of every section line.

95. Add two hundred seventy-five tenths, three hundred twenty-four hundredths, and five hundred thirty-six thousandths.

96. Multiply six hundred twenty-seven and forty-five thousandths by two and six tenths.

97. Divide one hundred forty-four by twelve hundredths.

98. If the hills of corn are $3\frac{1}{2}$ feet apart each way, (a) how many hills to the acre? (b) If the corn is cut and shocked, putting "8 hills square" in a shock, how many shocks to the acre? (c) If there are "16 hills square" in each shock, how many shocks to the acre?

99. If it is worth \$1.00 a cord to cut "4-foot wood" into 16-inch pieces, how much is it worth to cut "8-foot wood" into pieces of the same length?

100. An agent sold 1460 lb. butter at $23\frac{1}{2}$¢ a pound. If his commission for selling is 5% and he paid charges amounting to \$8.96, how much should he remit to the owner of the butter?

101. A lot 50 feet wide and 120 feet "deep" (long) was sold for \$450. This is equivalent to what price per acre?

102. Weight of wagon and hay, 4750 lb.; weight of wagon 1620 lb. How much is the hay worth at \$12.50 per ton?

103. How many acres in $5\frac{1}{2}$ miles of 4-rod road?

104. The specific gravity of ice is .92. (a) How much does a cubic foot of ice weigh? (b) How many tons of ice, if packed solid, can be stored in a building 12 feet square, the ice to be 8 feet deep?

105. How many tons in an acre of ice 15 inches thick?

106. On the first day of May the water-meter at the Illinois Institution for the Blind stood at 375,400 (cu. ft.); on

the first day of June, the reading was 477,700. Regarding each cubic foot as $7\frac{1}{2}$ gallons, (a) how many gallons of water were used in May? (b) What was the amount of the bill for water if the price was 12¢ per thousand gallons, with a discount of $16\frac{2}{3}$ per cent?

107. At $1000 an acre, find the value of a strip of land 4 feet by 125 feet.

108. If the specific gravity of iron is $7\frac{1}{4}$, how many cubic feet in 1 ton of iron?

109. A merchant marked goods 25% above the cost; he sold them at 25% below the marked price. What per cent did he lose?

110. The cube of a number divided by the number equals 1764. What is the number?

111. What is the edge of a cube whose entire surface is 6144 sq. inches?

112. Seven and one half feet are what part of a rod?

113. Peter has $12\frac{1}{2}$% more money than Paul; together they have $6.97. How much money has each?

114. Change the following to a common fraction in its lowest terms: $.27\frac{3}{11}$.

115. Answer the following at sight:

(a) Interest of $48 for 2 mo. at 6%.
(b) Interest of $375 for 2 mo. at 6%.
(c) Interest of $240 for 4 mo. at 6%.
(d) Interest of $330 for 4 mo. at 6%.

116. Answer the following at sight:

(a) Divide 125 by .5.
(b) Divide 125 by .05.
(c) Divide 12.5 by .5.
(d) Divide .125 by 5.

 6

SET I.

COOK COUNTY, ILLINOIS.

Eighth Grade Examination for County Superintendent's Diploma.

June, 1897. O. T. Bright, Supt.

Time, 9:30 to 12.

1. What is the ratio of (a) .2% to 2%? (b) $5 \div .5 = ?$ (c) $.05 \div 5 = ?$ (d) $.5 \div .05 = ?$ (e) $.005 \div .5 = ?$ (f) $.05 \div .005 = ?$

2. A girl spelled 95% of 60 words. How many words did she miss?

3. When a vessel sails 160 miles a day, she completes her voyage in 14 days. In what time would she complete it if she sailed 196 miles a day?

4. A three-inch cube was painted on all sides. It was then cut into inch cubes. (a) How many of the inch cubes were painted on three sides? (b) How many on two sides? (c) How many on one side? (d) How many were not painted at all?

5. Fill the blanks in the following:

A boy had a fish pole 15 feet long. A piece equal to 20% of its length was broken off while catching fish.

(a) The part remaining was ——% of the whole pole.

(b) The part broken off was ——% of the part remaining.

(c) The part remaining was ——% of the part broken off.

6. The N. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of a certain section of land was fenced off into four equal fields. (a) What is the shortest length of fence necessary? (b) How much land in each field?

7. A straight pole 72 feet high is broken 20 feet from the ground, but is not detached. How far from the foot will the top reach?

SET II.

NEW HAVEN (CONN.) PUBLIC SCHOOLS.

Entrance Examination to the High Schools.

Fall, 1897. C. N. Kendall, Supt.

(Answer 10 *entire* questions.)

1. (a) 13 oz. is what per cent. of 5 lb. avoirdupois?

(b) A man added 18 cows to his herd, thereby increasing the number 25 per cent. How many cows has he now?

2. If lead pencils that cost 3 cents each are sold for 5 cents each, what is the per cent of profit?

3. Find the interest on a note for \$330 at 6 per cent, given August 3, last year, and due to-day.

4. Write a negotiable, interest-bearing, promissory note.

5. (a) Divide $\frac{3}{4}$ of $\frac{5}{6}$ of $7\frac{1}{5}$ by $3\frac{2}{9}$.

(b) Subtract $8\frac{3}{16}$ from the sum of $5\frac{1}{3}$, $2\frac{1}{5}$, $4\frac{7}{12}$.

6. A rectangular field is $86\frac{1}{2}$ rods long and 46.875 rods wide. How much wheat will it produce at the rate of 20 bushels per acre?

7. A rectangular park, the sides of which are respectively 45 rods and 60 rods long, has a walk crossing it from corner to corner. How long is the walk?

8. If $\frac{3}{4}$ of 9 bushels of wheat cost \$$13\frac{1}{2}$, what will $\frac{5}{8}$ of a bushel cost?

9. If hay sells for \$14 a ton at a loss of $12\frac{1}{2}$ per cent, what must it sell for to gain 15 per cent?

10. How many pounds of cotton at $7\frac{1}{4}$ cents a pound can a broker buy for \$9,225, and retain his commission of $2\frac{1}{2}$ per cent?

11. Find the proceeds of a 3 months' note for \$500 discounted at a bank at 6 per cent.

12. If a building 20 feet high casts a shadow of 6 feet, what length of shadow will a church spire 114 feet high cast?

SET III.

COOK COUNTY, ILLINOIS.

Applicants for Teachers' Second Grade Certificates.

June, 1898. O. T. Bright, Supt.

"Mental Arithmetic." Time, 20 minutes.

Fill the blanks:

1. By a sale of goods I lost $12\frac{1}{2}\%$. The cost was ——% of the selling price.

2. The circumference of a 3-in. circle is ——% of its radius.

3. A five-inch square is ——% greater than a four-inch square.

4. I buy apples four for three cents and sell them three for four cents. I gain —— per cent.

5. A and B are 78 miles apart and walk toward each other; A walks 3 miles an hour and B $3\frac{1}{2}$ miles an hour. When they meet B will have walked —— miles.

6. A stick of timber 15 inches square and 32 feet long contains —— board feet.

7. A horse trots $23\frac{3}{4}$ miles in $2\frac{1}{2}$ hours. His rate per hour is —— miles.

8. \$154 was divided among A, B, C, and D, in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. C got —— dollars.

9. A school contains 50 pupils:
   Monday, 3 were absent in the forenoon,
   Tuesday, 2 were absent all day,
   Wednesday 3 absent forenoon and 2 afternoon,
   Thursday, 4 absent all day,
   Friday, all present

The per cent of attendance for the week was ——.

10. 64 gal. of wine and 16 gal. of water were mixed. One pint of the mixture contained —— of a gal. of water.

SET IV.

COOK COUNTY, ILLINOIS.

Applicants for Teachers' Second Grade Certificate.

August, 1895. O. T. BRIGHT, Supt.

Time, 60 minutes.

1. If a quarter section of land has fenced within it the largest possible circular lot, how many acres of the quarter section will remain outside of the circle?

2. Find the value of the following lumber at $15 per M.:

20 pieces 2 x 4, 18 ft. long.
20 pieces 4 x 4, 12 ft. long.
20 pieces 3 x 10, 16 ft. long.
45 16 ft. stock boards, 15 inches wide.

3. (a) What sum invested in 8% bonds at $33\frac{1}{3}$% premium will yield an annual income of $1200?

(b) What if the bonds were $33\frac{1}{3}$ discount?

4. Find the value of a piece of land 20 ft. x 40 rods at $1000 per acre.

5. What is the ratio of $3\frac{7}{8}$ to $\frac{5}{6}$? Answer in per cent.

SET V.

Examination Department of the University of the State of New York, January, 1898.

100 credits. Necessary to pass, 75.

Time, 9:15 a. m. to 12:15 p. m.

Answer the first five questions and five of the others, but *no more*. If more than five of the others are answered only the first five answers will be considered. Give all operations (except mental ones) necessary to find results. Reduce each result to its simplest form and mark it *Ans*. Each complete answer will receive 10 credits.

1. Define numerator, denominator, divisor, factor, proportion.

2. Find the weight in kilograms of a stone 1 meter square

and .4 of a meter thick, assuming that the stone is $2\frac{1}{2}$ times as heavy as water.

3. Simplify $\dfrac{3 \times \frac{2}{7} \times 4.2}{\frac{5}{18} \times \frac{20}{27}}$

4. Find the interest on $375 at $4\frac{1}{2}\%$ from July 1, last year, to the present time.

5. Multiply 65.15 by 3.14159 and divide the result by 57.296, finding a result correct to three decimal places.

6. Find the cost at $50 an acre of a rectangular field 1650 feet long and 825 feet wide.

7. Find the time required to fill a cistern 8 feet square and 5 feet deep by a pipe which admits water at the rate of 1 quart a second.

8. Make a receipted bill of the following: J. L. Robbins & Co. sold this day to Samuel Jones 8 yards of cloth at $37\frac{1}{2}$ cents, 24 yards of calico at $8\frac{1}{3}$ cents, 1 dozen handkerchiefs at $12\frac{1}{2}$ cents each, and 3 dozen towels at $2.50 a dozen.

9. Find the cost of four sticks of timber, each 8 inches by 10 inches and 30 feet long, at $15 a 1000 feet board measure.

10. Find the least common multiple of 153, 204, and 510.

11. If 4% bonds to the amount of $8000 face value are bought at $92\frac{1}{2}\%$, find the cost of the bonds, and the rate of income on the investment.

12. If 3 men can do a piece of work in 8 days of 10 hours each, how many men will be required to do the same work in 6 days of 8 hours each? (Solve by proportion.)

13. By selling a horse for $144, a profit of 60 per cent is made; find the cost of the horse.

14. The diameter of a bicycle wheel is 28 inches; find the number of revolutions it makes in going 1 mile.

15. Find the square root of 7, correct to three decimal places.

SET VI.

Examination for Admission to State High Schools, Minnesota, 1898.

Time two hours.

Answer any six—no more. If more are attempted and the student does not designate which six he wishes to be graded upon, the first six answers will be taken.

1. a (2) What is the ratio of 2 to .90?

b (2) What is the ratio of $\frac{3}{4}$ to $\frac{3}{10}$?

c (2) What is the ratio of $\frac{3}{10}$ to $\frac{3}{4}$?

d (2) What is the ratio of 90% to .09?

e (2) What is the ratio of 70% to 50%?

2. a (5) What is the cubical content of a cellar 15 ft. wide, 20 ft. long, and 10 ft. deep? (In the solution express all operations in the form of equations.)

b (1) What unit (or units) of measure did you use in the example?

c (4) Describe the unit of measure used in measuring boards.

3. (10) A merchant sells an overcoat for $22; a suit of clothes for $23; a hat for $5. On the overcoat he makes 10% of the cost; on the suit 15%, and on the hat 25%. What per cent of the cost of the goods does he make on the entire sale?

4. a (5) A man bought a watch and a chain for $70. One-half of the cost of the watch equals $\frac{2}{3}$ of the cost of the chain. What was the cost of each?

b (5) Analyze.

5. (10) The rates at which A, B, and C work are to each other as 2, 3, and 4. What integers will indicate the time it will take each to do a certain piece of work?

6. (10) How long a rope must a horse have in order that he may graze over an acre of land, if he be tied to a stake in the center of a field?

7. (10) B buys bank stock at 78 and sells it at 84. C buys railroad stock at 70 and sells at 75. Each buys the same number of shares, and B makes $1000 more than C. How much money did B invest?

8. (10) Make (5) and solve (5) a problem in the solution of which it will be necessary to extract the square root.

9. a (5) What is the interest on $700 for 1 yr. 5 mo. and 10 da. at 7% per annum?

b (5) Analyze.

10. (10) A servant is engaged for a year for $280 and a suit of clothes. He leaves at the end of six months and receives $130 and the suit. What is the value of the suit? (An algebraic solution is allowed for this problem.)

11. a (5) How long will it take $1560 at 5% simple interest to gain $426.83⅓?

b (5) Analyze.

SET VII.

Examination for State Certificates. Illinois, 1898.

Time, two hours.

1. (a) Every fraction is a ratio. Explain.

(b) Every integral number is a ratio. Explain.

2. In the report of the Committee of Ten it is recommended that "the course in arithmetic be at the same time abridged and enriched."

(a) Tell what abridgment you regard as important.

(b) Tell what enrichment you consider essential.

3. (a) Tell what sense-magnitudes you prefer to use in presenting to third grade pupils the subject of fractions. (b) At what stage of the work do you think sense-magnitudes should give place to imaginative magnitudes?

4. When and to what extent should pupils in the grades be required to memorize definitions of mathematical terms? (b) When and to what extent should pupils be required to memorize directions for performing operations?

5. Mention all the standard linear units with which you are familiar, and give the ratio of each (either exact or approximate) to some other linear unit.

6. What is the weight of 1000 feet of white pine boards (1 inch in thickness) if the specific gravity of the boards is .6?

7. The foundation of my house is 32 feet square on the outside. The house is 20 feet high to the plates and the roof has the usual eave-projections. Give approximately the number of barrels ($31\frac{1}{2}$ gal.) of water that will fall upon this roof in one year, if the rain-fall is $34\frac{1}{2}$ inches.

8. Give approximately the following ratios:

(a) Of the circumference to the diameter of a circle.

(b) Of the diagonal to the side of a square.

(c) Of the area of a circle to the area of its circumscribed square.

(d) Of the area of a circle to the area of its inscribed square.

9. What single discount is equal to a discount of 45 per cent and "5 10's," *i. e.*, to "45 and 10 and 10 and 10 and 10 and 10 off," from the list price?

10. If money is worth 6 per cent annual interest now and prospectively, what is the actual cash value of a note of $1000 running two years and drawing 5 per cent interest, payable annually?

VIII. The Bank Test.*

To the Teacher.—Below are figures representing 51 sums of money. Procure 51 blank checks and cause them to be filled, using the sums here given. Draw one check from the 51 checks and give the remaining 50 to a pupil to transcribe the sums and find their amount. When the pupil obtains a result the teacher can quickly determine whether it is correct by comparing it with the sum of the 51 checks, *less the sum named on the check drawn out.* Before the checks are given to the second pupil, the check removed should be replaced and another withdrawn. Thus, although each pupil should obtain a result differing from that obtained by the pupil preceding him, its accuracy can be quickly tested by the teacher.

To the Pupil.—Can you, *on first trial*, transcribe the sums named on 50 checks and find the amount accurately in 30 minutes?

$324.56	$565.60	$123.20	$75.00
$234.50	$525.40	$312.95	$190.35
$46.45	$112.00	$86.50	$250.00
$325.00	$86.74	$91.23	$50.00
$302.26	$59.29	$12.65	$8.25
$7.75	$875.00	$1.50	$431.05
$201.45	$34.36	$85.40	$90.00
$130.25	$212.24	$230.94	$642.45
$71.20	$708.30	$60.00	$75.00
$1250.25	$6.50	$500.00	$2324.45
$9.10	$101.50	$36.09	$275.00
$150.00	$2.50	$1008.60	$140.65
$256.74	$987.84	$50.00	

* In a leading bank in Chicago, it is customary to test applicants for positions as accountants by placing before them 150 checks, requiring each applicant to copy the sums named on the checks and find their amount. The author of this book is informed that the average inexperienced applicant does this in about 30 minutes, with some errors, however, both in transcribing and in footing. An expert accountant can do this amount of work *accurately* in 6 minutes.

IX. Curious Comparisons.

1. If a pig whose girth is 2 feet weighs 50 lb., what is the weight of a similarly proportioned pig whose girth is 4 feet?

2. If a disk of dough 15 inches in diameter is sufficient for 20 doughnuts, how many such doughnuts can be made from a disk 30 inches in diameter?

3. The bore of a 10-inch gun is how many times as large as the bore of a 2-inch gun?

4. The ball of a 10-inch gun is how many times as large as the ball of a 2-inch gun?

5. A square, a pentagon, a hexagon, an octagon, and a circle have equal perimeters. (a) Which has the greatest area? (b) Which has the least area?

6. The capacity of a cistern 6 feet in diameter and 6 feet deep is about 40 barrels. What is the capacity of a cistern 12 feet in diameter and 12 feet deep?

7. A $2\frac{1}{2}$-inch pipe is how many times as large as a 1-inch pipe?

8. If a man 6 feet tall weighs 190 lb., how much would a similarly proportioned giant 12 feet tall weigh?

9. In a certain orchard the trees are 15 feet apart each way and there are 800 trees. How many trees in an orchard of equal size, the trees being 30 feet apart each way?

10. A ball of yarn 3 inches in diameter is sufficient for one mitten. How many mittens can be made from a ball 6 inches in diameter?

11. A grindstone was originally 30 inches in diameter. It has been worn until it is but 15 inches in diameter. What part of the stone has been worn away?

12. A square and an oblong have equal areas. Which has the greater perimeter?

X. Puzzling Problems.

1. If a person traveling as expeditiously as possible from Boston to San Francisco, should mail a letter to his friend in Boston every day at noon, how often would the letters be received in Boston?

2. If a man and a boy, the boy doing exactly one-half as much work as the man, can hoe one and one-half acres of corn in one and one-half days, how many acres can 6 men hoe in 6 days?

3. John and James sold apples together. The first day they sold 60 apples at the rate of 5 apples for 2 cents, and received 24 cents. The second day they divided the apples. John took 30 of the larger apples and sold them at the rate of 2 for 1 cent. James took the remaining 30 apples and sold them at the rate of 3 for 1 cent. They received 25 cents. Why did they receive one cent more the second day than the first?

4. A pile of four-foot wood stands upon a hill-side. The pile is 8 feet long (measured on the ground), and 4 feet high (measured vertically). Does the pile contain one cord?

5. A man had shingles enough to cover his house if he laid them 4 inches to the weather. He laid them $4\frac{1}{2}$ inches to the weather. What part of the shingles provided remained? Explain.

6. If on a line of railroad connecting Chicago and San Francisco one passenger train leaves Chicago daily at 6 o'clock a. m., and makes the journey to San Francisco in exactly five days, and one train leaves San Francisco daily at 6 o'clock p. m., and makes the journey to Chicago in exactly five days, (a) a person taking the train at Chicago will meet how many passenger trains while going to San Francisco? (b) How many trains of passenger cars required to equip the road?

EXPLANATORY NOTES.

Note 1. The forty-five **primary facts of addition** are as follows:

1	2	2	3	3	4	3	4	5	4	5	6	4	5	6
1	1	2	1	2	1	3	2	1	3	2	1	4	3	2
2	3	4	4	5	5	6	6	6	7	7	7	8	8	8

7	5	6	7	8	5	6	7	8	9	6	7	8	9	6
1	4	3	2	1	5	4	3	2	1	5	4	3	2	6
8	9	9	9	9	10	10	10	10	10	11	11	11	11	12

7	8	9	**7**	**8**	**9**	7	**8**	**9**	**8**	**9**	8	**9**	**9**	9
5	4	3	**6**	**5**	**4**	7	**6**	**5**	**7**	**6**	8	**7**	**8**	9
12	12	12	**13**	**13**	**13**	14	**14**	**14**	**15**	**15**	16	**16**	**17**	18

The nine facts in full-faced type should receive special attention. Pupils seldom fail to memorize the other thirty-six facts.

Note 2. There are eighty-one **primary facts of subtraction**; that is, two for every primary fact of addition except the 1st, 3rd, 7th, 13th, 21st, 30th, 37th, 42nd, and 45th. The facts of subtraction should be learned while learning the facts of addition. If a pupil really knows that 8 and 9 equal 17, he knows also that 17 less $8 = 9$, and 17 less $9 = 8$.

Note 3. When the **sign of multiplication** is followed by a fraction, it indicates that *a certain part* of the number preceding the sign is to be repeated as many times as there are units in the numerator of the fraction following the sign; thus, $12 \times \frac{3}{4}$, means, that 1 fourth of 12 is to be repeated 3 times; $50 \times .5$, means, that 1 tenth of 50 is to be repeated 5 times.

Note 4. **This sign,** $\times$, is sometimes so used that it means *times*, thus, $3 \times \$6$, must be read, *three times six dollars*. 3×6, may be read, *three multiplied by six* or *three times six*. As employed in this

book, *the sign never means times.* Instead of 3 × \$6, the author prefers \$6 × 3. It is believed that the restriction of this sign to one use and to one meaning, at least in the first years of arithmetical study, will promote clearness of thought and accuracy in expression.

Note 5. Without danger of ambiguity, **the sign**, ×, is sometimes used in this book and elsewhere in place of the word *by;* thus, 1 pc. of 2 × 4, 12 (to be read, *1 pc. of 2 by 4, 12*) means, a piece of lumber 2 inches thick, 4 inches wide, and 12 feet long.

Note 6. Besides those problems in which either the multiplicand or the multiplier is 1, and which require no effort on the part of the pupil beyond learning to count, there are sixty-four **primary facts of multiplication** that must be perfectly memorized before the pupil can acquire facility in the process. They are as follows:

2 times 2 = 4	3 times 2 = 6	4 times 2 = 8	5 times 2 = 10
2 " 3 = 6	3 " 3 = 9	4 " 3 = 12	5 " 3 = 15
2 " 4 = 8	3 " 4 = 12	4 " 4 = 16	5 " 4 = 20
2 " 5 = 10	3 " 5 = 15	4 " 5 = 20	5 " 5 = 25
2 " 6 = 12	3 " 6 = 18	4 " 6 = 24	5 " 6 = 30
2 " 7 = 14	3 " 7 = 21	4 " 7 = 28	5 " 7 = 35
2 " 8 = 16	3 " 8 = 24	4 " 8 = 32	5 " 8 = 40
2 " 9 = 18	3 " 9 = 27	4 " 9 = 36	5 " 9 = 45

6 times 2 = 12	7 times 2 = 14	8 times 2 = 16	9 times 2 = 18
6 " 3 = 18	7 " 3 = 21	8 " 3 = 24	9 " 3 = 27
6 " 4 = 24	7 " 4 = 28	8 " 4 = 32	9 " 4 = 36
6 " 5 = 30	7 " 5 = 35	8 " 5 = 40	9 " 5 = 45
6 " 6 = 36	7 " 6 = 42	8 " 6 = 48	9 " 6 = 54
6 " 7 = 42	7 " 7 = 49	8 " 7 = 56	9 " 7 = 63
6 " 8 = 48	7 " 8 = 56	8 " 8 = 64	9 " 8 = 72
6 " 9 = 54	7 " 9 = 63	8 " 9 = 72	9 " 9 = 81

Although a knowledge of the "elevens" and "twelves" of the table as it is usually given is convenient and helpful, it will be observed that it is not a necessity in the process of multiplication. The facts given above include all that are essentially fundamental.

Note 7. **The sign ÷**, which is read *divided by*, **has two meanings** in concrete problems, which correspond to the two cases in division. In one case it means, *find how many times the divisor is contained in the dividend;* in the other case it means, *find one of a certain number of equal parts into which the dividend is supposed to be divided.* In each case there is division into equal parts. In the first case, the quotient tells *the number of parts.* In the second case, the quotient tells *the size of one part.*

$18 ÷ $2, means, *find how many times $2 are contained in $18.*

$18 ÷ 2, means, *find 1 half of $18.* (See foot-note, p. 192.)

Note 8. There are, in a sense, 128 **primary facts of division,**— two for each one of the sixty-four facts of multiplication. These facts are so closely related to the facts of multiplication that they should be learned in connection with the multiplication table. If a child really perceives that five fours (:: :: :: :: ::) are 20, he will also know that 4 is contained in 20 five times, and that 1 fifth of 20 is 4.

Note 9. If from a square piece of paper, the largest possible circle be cut, a little less than ¼ of the paper will be cut away. Hence a circle is a little more than ¾ (.78 +) of its circumscribed square.

Observe that the diameter of a circle is equal to the side of its circumscribed square.

Note 10. If from a cube of wood, the largest possible sphere be cut, a little less than ½ of the wood will be cut away. Hence a sphere is a little more than ½ (.52 +) of its circumscribed cube.

Observe that the diameter of a sphere is equal to the edge of its circumscribed cube.

Note 11. A piece of board 1 inch wide, 1 inch thick, and 12 feet long, is 1 ft. of lumber. Hence the number of feet of lumber in any 12-foot stick, is equal to the number of square inches in its cross-section.

Note 12. If from a square right prism of wood the largest possible cylinder be cut, a little less than ¼ of the wood will be cut away.

Hence a cylinder is a little more than ¾ (.78+) of its circumscribed square right prism.

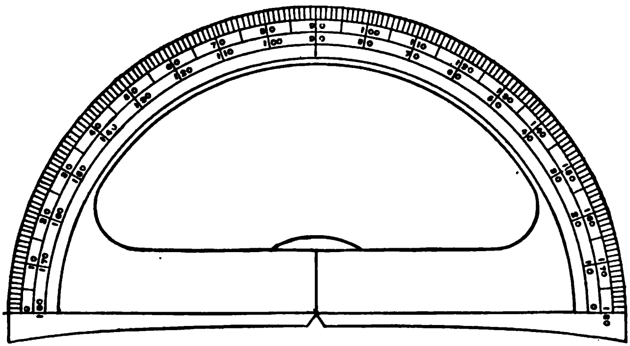
Note 13. If from a cylinder of wood whose diameter equals its altitude the largest possible sphere be cut, exactly ⅓ of the wood will be cut away. Hence a sphere is exactly ⅔ of a cylinder whose diameter and altitude are each equal to the diameter of the sphere.

Note 14. If from a cylinder of wood the largest possible cone be cut, exactly ⅔ of the wood will be cut away. Hence a cone is exactly ⅓ of a cylinder of equal diameter and altitude.

Note 15. If from a square right prism of wood the largest possible pyramid be cut, exactly ⅔ of the wood will be cut away. Hence a square pyramid is exactly ⅓ of a square right prism of equal base and altitude.

Note 16. The specific gravity of a liquid or solid is the ratio of its weight to the weight of an equal bulk of pure water.

PROTRACTOR.



Carefully paste this sheet upon card-board; then cut out the protractor with a sharp knife and preserve it for use in measuring and in constructing angles. See pages 239, 249, 259, etc.

Hall, Frank H. 81145

Complete arithmetic.

	DATE	NAME	DATE